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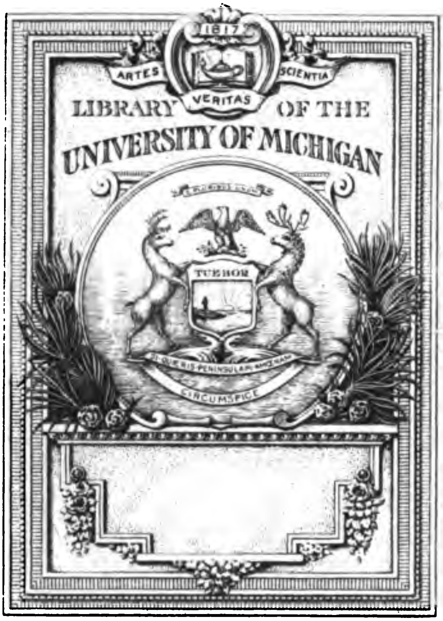

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arnard, Frederick Augustus Porter,
1809-1889.

A treatise on arithmetic ... in
hich the principles of the science
re inductively developed ... com-
ining written arithmetic with cop-
ous mental exercises ... By Fred-
rick A. P. Barnard ...

Hartford,

Packard & Butler

1830

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PREFACE.

A preface is not unfrequently made the vehicle of conveying to the public an author's apology, for having presumed to think and write upon a subject, on which many others have thought and written before him. Such an apology, however, while it betrays a want of that manly confidence, with which every one, who is conscious of having striven, with integrity of purpose, to promote the good of his fellows, should look forward to the award of an intelligent public; is in itself, unnecessary, because ineffectual to accomplish the object, for which it is intended. In the present enlightened age, works are judged of, by their own intrinsic merit, and not by what their authors may choose to say in their favor: and nothing can be more vain, than to attempt, by fair pretences, to palm off upon the public an inferior production. Influenced by considerations like these, the author of the following treatise was about to lay it before the public, without apology, and with hardly a prefatory remark; confident, that if it possess merit, this will be discovered and approved; if otherwise, that whatever he could say, would be a mere waste of breath.

He deems it due, however, to those, whose interesting task it is, "to rear the tender thought," to state in what respects he has chosen to differ from other arithmeticians, and also, briefly to assign his reasons for so doing. These are the following:

1. The book commences in a style likely to attract and fix the attention of the pupil, and which, for its simplicity, is calculated to prevent the prejudices, which the young are so apt to conceive against mathematical studies. It is believed that, by commencing below the level of the pupil's capacity, and thus rendering it certain that every thing is understood in the outset, important advantages are gained; among which it is not the least, that the learner becomes encouraged to *expect* that he shall understand the parts which succeed.

2. In passing from the mental to the written exercises, it will be observed that the transition is so gradually made, as to render the resemblances and the differences entirely distinct. While the same general principle is seen obviously to pervade the whole of the same class of examples, the facilities of calculation afforded by the scheme of notation, which are of course mostly peculiar to written arithmetic, may easily be distinguished.

3. In developing the principles of those operations, embraced under what are usually called the ground rules, it has been an object to illustrate each step of the process separately. This is an important distinction between the present and all former treatises on the subject. Thus, in addition, the first examples only differ from the preceding mental exercises, in requiring the numbers concerned to

be written. In the beginning, a single order, only, is employed; afterwards, two, three, and so on, the greatest simplicity being still preserved, in order that the mind of the pupil may be occupied with nothing, except the mere arrangement of his numbers. Afterwards, larger numbers are used, containing *many orders*, but so contrived, that there shall be no necessity of carrying from one order to another. Finally, the pupil is taught to *carry*, by means of a series of examples, commencing again with the greatest simplicity, and advancing to the more difficult combinations. The learner is at first, however, required actually to *set down* the number carried, until he cannot fail to perceive the advantage of adding it *mentally*. All these things are usually explained in a single example, and that often consisting of numbers so great that the learner can have no conception of their amount, nor, of course, of the principles concerned in the process. The consequence is, that the young beginner, perplexed by the number and variety of things which he is required to remember without comprehending, and tired of the irksome task of performing operations which are to him only not completely mechanical, becomes disgusted with the study, and imbibes prejudices against all kinds of mathematical pursuits, which continue to the end of his life.

4. This treatise is much more *full* than any designed for the same purpose which has preceded it. A glance at the table of contents, will be sufficient to show, that the variety of subjects treated of, is much greater than is usual, and that much useful information is given on numerous topics, collaterally connected with the main subject.

5. The abbreviations in calculation are thought to be a valuable addition to a practical work on this subject. These abbreviations will often furnish a convenient mode of verification or proof of an operation performed by the common method.

6. The articles on circulating decimals, are believed to contain much which will interest. It is delightful to contemplate the beauty and harmony of numbers, as they are exemplified in the perfect conformity even of this apparently anomalous class of quantities, to fixed and unchangeable laws. No book, before the present, has given a popular exposition of the subject, and it is thought that in this part of the work, even teachers may in some instances meet with new ideas.

7. INTEREST is treated with the fulness which its importance demands. The method by decimal multipliers, which is becoming so deservedly popular from its simplicity, has been adopted and minutely explained.

8. The author is inclined to think that his method of developing the principles of ratio and proportion, will meet approbation. It is certain, that the subject of compound proportion, can no where be found treated with equal simplicity. The process by means of a single fraction, is likewise original, and is exceedingly concise. The author has employed this process with his classes, and has found it eminently successful. Pupils learn it with great facility, and when it is once familiar, they solve by means of it, the most complex questions, almost as easily as the most simple.

9. Of the extraction of the square and cube roots, an analytic investigation is here, for the first time, given. By this means, it is hoped, that this difficult subject will be rendered more intelligible.

10. In general, there is a simplicity in the explanations and illustrations which it is believed will not be found elsewhere. This arises from the fact that they are the result of EXPERIMENT, upon minds of every degree of native acuteness, and energy, and in every stage of cultivation.

11. By the arrangement, TECHNICAL TERMS are introduced only after the pupil has been made thoroughly acquainted with the processes in which it is necessary to apply them, or with the circumstances which render it proper that they should be employed. We all know how difficult it is to form correct ideas, even of sensible objects, from description merely; and it is by no means surprising, that when the subject of a description is an abstract term, or some one of the numerous technicalities of science, it should be impossible for a child to form any idea of what is intended by the language. But when a name is applied to something already familiar, the ideas attached to it are perfectly distinct and definite. It was in a similar manner that the child first learned his mother tongue, and it is in this way that we ourselves are from time to time, adding to our stock of language.

12. By a similar arrangement, the rules are made to succeed the knowledge of the processes, which they describe. For mere practice, therefore, they are unnecessary, and indeed, they are not intended, except in a very few instances, to serve as the learner's guides. They are merely given, as furnishing concise and accurate language for the expression of ideas supposed to be previously familiar.

13. The rules will be found more brief and more easy to be committed in this book than in any which the author has seen. It has been a prominent object to make them so. And since the pupil is not expected, and ought not to be suffered to depend on them as a guide, conciseness has been attainable, without the sacrifice of any desirable end.

These constitute the prominent points of difference between the present and former treatises on this subject. In no one of these particulars has any deviation been made from other writers, without long and patient observation of the effect of different methods of instruction upon the youthful mind. This is the test to which the author has subjected all his proposed improvements, and the uniform success which has attended his experiments, leads him to look with confidence for the approbation of the public.

HARTFORD, JULY 30, 1830.

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ERRATA.

In printing a work on a scientific subject, entirely from manuscript, perfect accuracy is not to be expected. Accordingly, some errors will undoubtedly be discovered in this book; but it is believed that the following are all, which will be likely to occasion perplexity. Page 60. *Ans.* ex. 10, for 328, read 378.—p. 71, in ex. 4, for 4 men read 1 man. p. 151, line 13, for 9 read 6—line 15, for multiplier, read multiple.—p. 229, in ex. 5, insert at 5 per cent.—p. 236, ex. 2, *Ans.* should be $446,985—.—p. 237. *Ans.* by Mass. method should be $11,370.47175.—p. 244, ex. 43, for 65, read 650.—p. 249, line 6, for miles, read hours.—p. 250 l. 30, for 3, read 315.—p. 284, in *Ans.* to ex. 1, for $\frac{11}{27}\frac{31}{30}$ read $\frac{29}{35}$.

In a small part of the impression, p. 82. 3d line in ex. 87, for 65 read 75.—p. 261, line 3, for products, read quotients.—p. 283, line 38, for 3 read 7, three times.

In § LXXXIX, the rule called the Massachusetts rule is erroneously stated to have been *established by law* in that state. It has been made the rule of the state of New-York, by the decision of Chancellor Kent. The statement was made on the authority of a number of writers. For this correction, as well as for other important suggestions, the author is indebted to Professor Dewey, now Principal of the Berkshire Gymnasium.

TO TEACHERS.

It is not to be expected that any book, however perfect, should supersede the necessity of patient and laborious effort on the part of the teacher. The narrow compass into which it is necessary to compress all the important particulars, embraced in a whole science, in order to form a text book, suited to the wants of schools, renders it impossible for the author to go into that minuteness of detail, or to avail himself of that fulness of illustration, which it is the province of the living teacher to supply. On this account, many of the explanations and reasonings, contained in the following treatise, though more full than those on the same subjects in the common books, can only be regarded as furnishing hints, which the teacher may follow out at greater length.

Pupils should, by all means, be arranged for instruction in CLASSES. By this means, not only is a spirit of emulation excited, which very soon interests the learner in his study, but the teacher is enabled to devote more time to the instruction of each individual, and to the illustration of every subject. Classes may consist of as many as twelve or sixteen pupils with advantage. In clearing up any difficulties which may arise, the instructor should always *avoid direct explanations.* The method which the author has pursued, and which he most strongly recommends to all teachers, is to lead the pupil to make the desired discovery himself, by means of questions adapted to the case. These should commence with the utmost simplicity, and approach the difficulty by degrees. By a skilful use of this method, the author has seen many a Gordian knot unravelled, and many a sorrowful countenance brightened into smiles. It will often be well, to ask questions to the whole class at once.

Some may, perhaps, be here led to inquire, why the interrogative plan is not adopted *in the book.* To this it is replied, that the advantage of the method consists in rendering questions so gradually progressive that the pupil may answer hem with ease and without asststance. A body of questions of this kind, embracing *the whole* science of arithmetic, would be too voluminous for use; and on the other hand, if the questions are made fewer, and answers *furnished*, no plan can be more calculated to produce pernicious effects on the mind of the learner. The pupil should never be allowed to know the forms of the questions to be used in examining him; much less should he be furnished with the language of his answer.

With respect to the method of using this book, the author recommends that all young pupils, when taking up the subject for the first time, should study with attention the introductory exercises: afterwards the mental arithmetic alone, as far at least as fractions; then that they should perform some of the simplest examples in written arithmetic, under each of the ground rules: and finally, that they should begin anew and take the whole in order. Even in this perusal, however, they should omit the observations for advanced pupils, and *the rules* every where. These should be reserved for the final review, the latter being only intended to supply the pupil with accurate *language*, for the expression of his own ideas.

When *examples* are recited, the whole operation should be brought in on the slate, and parts of the process should be read by each pupil in turn. If there are many examples, it will be sufficient to read a small part of each. By this means it will be rendered certain that every pupil understands the subject, and at the same time a great temptation to deceit in copying answers, will be removed. This plan renders a separate *key* unnecessary, and might even dispense with answers altogether. For when a difference is perceived between two calculations, a moment's comparison, at the point where the variation commenced, will determine which is right.

The author makes the above suggestions as the result of his own experience. It is impossible, in the narrow limits allowed him, to exhibit, in full, his views on the best mode of teaching the science, or of using the following treatise; but for experienced instructors this will be unnecessary, and for those, to whom the employment is new, the above observations will not be entirely useless.

RECOMMENDATIONS.

From D. **OLMSTED**, Professor of Mathematics and Natural Philosophy in YALE COLLEGE.

I have examined an Elementary Treatise on Arithmetic, by Mr. **FREDERICK A. P. BARNARD**, and can cheerfully recommend it to the attention ~~of the public~~, ~~particularly of~~ Instructors, as a work of much merit, exhibiting an able and luminous view of the science of numbers.

DENISON OLMSTED.

YALE COLLEGE, July 22, 1830.

From C. **DEWEY**, Principal of the **BERKSHIRE GYMNASIUM**.

BERKSHIRE GYMNASIUM,
Pittsfield, July 23, 1830.

Mr. F. A. P. BARNARD.

Dear Sir—I have examined your "Arithmetic" with some attention, and cheerfully say, that it is, in my opinion, worthy of a high place among the improved systems of Arithmetic in our country. Though it cannot be, in many respects, an original work, it happily combines the improvements that have been made, and exhibits them in a clear and easy point of view: It developes the principles of the rules generally with much simplicity; the definitions are clear and concise, and the rules are stated, after the principles are explained, with brevity; many important practical abbreviations of calculations are introduced; mental exercises, as well as those for the slate, are employed in great abundance; and it is distinguished by a fulness in the subjects treated of, and in many interesting and important related ones, which belong to few works of the kind. The first part of the work, which is designed for the use of the quite young pupil, might well be published, after some enlargement, by itself. Without expressing unqualified approbation of every part of the work, there can be no doubt that it is fitted to be extensively useful.

With much esteem, &c.

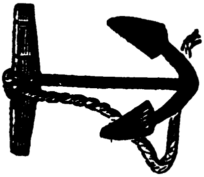
C. DEWEY.

ARITHMETIC.

INTRODUCTORY EXERCISES.

NUMERATION.

§ I. ARTICLE I. Here is an anchor.



How many anchors do you see here? *Ans.* ONE.

How many ropes are there, fastened to the anchor?

Now there is a *single* anchor, and a *single* rope; and each of these *single things*, you tell me, is ONE. Then, what is a *single thing* called? Hence,

A SINGLE THING OF ANY KIND IS CALLED ONE THING, OR ONE. Also A SINGLE THING OF ANY KIND IS OFTEN CALLED A UNIT, OR UNITY.

Thus, the anchor in the picture above is a unit.

If there were another anchor in the picture, how many anchors would there be? *Ans.* TWO.

How many sheaves of wheat are there here?

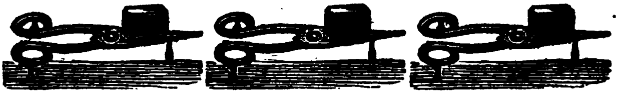
How many hands have you? How many thumbs? How many eyes? How many feet?

When you see one thing, and one more together, what do you call them? Hence,

ONE UNIT AND ONE MORE ARE CALLED TWO; that is, ONE AND ONE ARE TWO.

If there were one more sheaf in the last picture, how many sheaves would there be? *Ans.* THREE.

How many pairs of snuffers are there here?

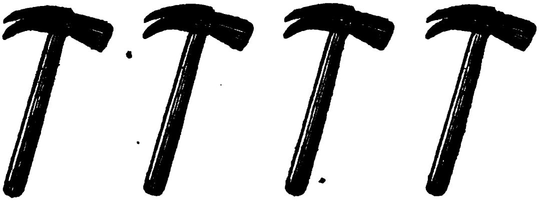


If you had another hand, how many hands would you have? If you had another eye, how many eyes would you have? When you see two things and one more together, what do you call them? Hence,

TWO UNITS AND ONE MORE ARE CALLED THREE; that is, TWO AND ONE ARE THREE.

If there were another pair of snuffers in the last picture, how many would there be? *Ans.* FOUR.

How many hammers are there here?



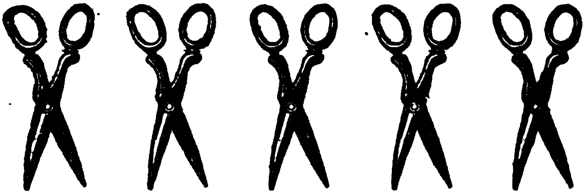
If you count your hands and feet together, how many have you?

When you see three things and one more together, what do you call them? Hence,

THREE UNITS AND ONE MORE ARE CALLED FOUR; that is, THREE AND ONE ARE FOUR.

If there were another hammer in the last picture, how many would there be? *Ans.* FIVE.

How many pairs of scissors are there here?



When you see four things and one more together, what do you call them? Hence,

FOUR UNITS AND ONE MORE ARE CALLED FIVE; that is, FOUR AND ONE ARE FIVE.

If there were another pair of scissors in the last picture, how many would there be? *Ans.* SIX.

How many houses are there here?

When you see five things and one more together, what do you call them? Hence,

FIVE UNITS AND ONE MORE ARE CALLED SIX; that is, FIVE AND ONE ARE SIX.

If there were another house in the last picture, how many would there be? *Ans.* SEVEN.

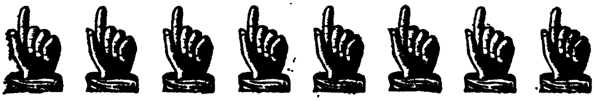
How many ships are there here?

When you see six things and one more together, what do you call them? Hence,

SIX UNITS AND ONE MORE ARE CALLED SEVEN; that is, SIX AND ONE ARE SEVEN.

If there were another ship in the last picture, how many would there be? *Ans.* EIGHT.

How many hands are there here?



When you see seven things and one more together, what do you call them? Hence,

SEVEN UNITS AND ONE MORE ARE CALLED EIGHT; that is, SEVEN AND ONE ARE EIGHT.

If there were another hand in the last picture, how many would there be? *Ans.* NINE.

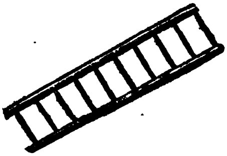
How many faces are there here?

When you see eight things and one more together, what do you call them? Hence,

EIGHT UNITS AND ONE MORE ARE CALLED NINE; that is, EIGHT AND ONE ARE NINE.

If there were another face in the last picture, how many would there be? *Ans.* TEN.

How many rounds are there in this ladder?



When you see nine things and one more together, what do you call them? Hence,

NINE UNITS AND ONE MORE ARE CALLED TEN; that is, NINE AND ONE ARE TEN.

Now count from one to ten.

Ten and one are eleven. Eleven and one are twelve. Twelve and one are thirteen. Thirteen and one are fourteen. Fourteen and one are fifteen. Fifteen and one are sixteen. Sixteen and one are seventeen. Seventeen and one are eighteen. Eighteen and one are nineteen. Nineteen and one are twenty.

In the same manner, by adding one every time, we obtain twenty one, twenty two, twenty three, twenty four, twenty five, twenty six, twenty seven, twenty eight, twenty nine, thirty.

Afterwards, we have thirty one, thirty two, thirty three, and so on; then, forty, forty one, forty two, and so on;

then fifty, fifty one, and so on; then sixty; then seventy; then eighty; then ninety; then ONE HUNDRED.

Now count from ten to twenty. From twenty to thirty. From thirty to forty. From forty to fifty. From fifty to sixty. From sixty to seventy. From seventy to eighty. From eighty to ninety. From ninety to one hundred.

Now count from one to twenty. From ten to thirty. From thirty to fifty. From seventy to ninety. From sixty to one hundred.

Count ten. Count fifteen. Eighteen. Twenty one. Thirteen. Seventeen. Twenty eight. Nineteen. Twenty five. Eleven.

ART. II. NOTE TO TEACHERS. Let the pupil read the following sentences, substituting the *names* of the figures, for the *figures themselves*. Very young pupils will need to be informed what the names are.

Read the following sentences. The house had 1 chimney. I saw only 1 man. The church has but 1 spire. In walking out 1 or two days ago, I saw a wagon with 1 man in it, drawn by 1 horse. Then 1 means ONE.

Read the following. I have 2 hands. 1 and 1 are 2. My lesson is 2 pages long. I have 2 brothers and 1 sister. I have never seen a man with 2 heads. 1 of my brothers has 2 dogs. There are 2 large trees in my father's garden. Then 2 means TWO.

Read the following. George has 3 books. 2 and 1 are 3. When I was 3 years old I could not read. Men never have 3 eyes. 3 miles is a long distance to walk. Then, 3 means THREE.

Read the following. There are 4 boys below me in my class. 4 days ago I was unwell. 3 and 1 are 4. My father bought me 4 oranges yesterday. Then 4 means FOUR.

Read the following. My brother is 5 years older than I am, but my sister is 2 years younger. 4 and 1 are 5. 5 is more than 3. I gave 5 apples for 2 peaches. 5 boys sit on the bench with me. Then 5 means FIVE.

Read the following. I saw 6 boxes in my father's store. 5 and 1 are 6. There are 6 tumblers on the table. 4 of them are empty and 2 are full. I have 6 marbles. Then 6 means SIX.

Read the following. There are 7 days in one week. 6 and 1 are 7. I have 6 cents. If I had 1 more, I should have 7. There are houses in Edinburgh more than 7 stories high. Then 7 means SEVEN.

Read the following. There are 8 desks in this school room. 7 would be sufficient. 1 desk is large enough for 8 boys. 7 and 1 are 8. In 6 or 8 days I shall have a new book. Then 8 means EIGHT.

Read the following. I have 9 quills. 8 and 1 are 9. I rode 9 miles yesterday. I shall soon be 9 years old. Then 9 means NINE.

1 man, 2 dogs, 3 cats, 4 mice, 5 birds, 6 hats, 7 pens, 8 ducks, 9 crows.

one	two	three	four	five	six	seven	eight	nine
1	2	3	4	5	6	7	8	9

These are called FIGURES.

Write the figure seven. Eight. Six. Three. Five. Nine. One. Two. Four.

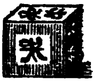
◆

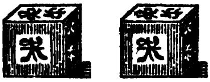
ADDITION.

§II. ART. I. 1. One axe is in one place, and one in another. How many in both?

2. Then 1 and 1 are how many?

3. One tea-box is in one place, and two are in another. How many in both?



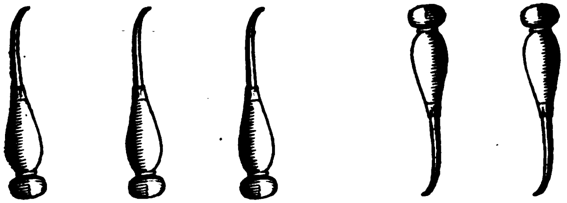


4. Then 1 and 2 are how many?

5. Two trees are in one place, and two in another. How many in all?

6. Then 2 and 2 are how many?

7. Three awls are in one place, and two in another. How many in all?



8. Then 3 and 2 are how many?

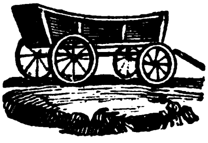
9. Three ships are in one place, and three in another. How many in all?

10. Then 3 and 3 are how many?

11. Three houses are in one place, and four in another. How many in all?

12. 3 and 4 are how many?

13. There are four wheels on one wagon and four on another. How many on both?



14. 4 and 4 are how many?

15. How many marks are four marks and five marks?

| | | | | | | | |

16. 4 and 5 are how many?

17. How many stars are four stars and six stars?

* * * *   * * * * * *

18. 4 and 6 are how many?

[The pupil should now be directed to count the characters which follow, in the same manner as the pictures above.]

19. William had 3 marbles and George had 4. How many had both?

⊖ ⊖ ⊖   ⊖ ⊖ ⊖ ⊖

20. 3 and 4 are how many?

21. A boy gave 5 cents for an orange, and 6 for a pint of pea-nuts. How many cents did he give for both?

⊖ ⊖ ⊖ ⊖ ⊖   ⊖ ⊖ ⊖ ⊖ ⊖ ⊖

22. 5 and 6 are how many?

23. A man had 7 acres of ground, and he bought 4 more. How many had he then?

⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖   ⊖ ⊖ ⊖ ⊖

24. 7 and 4 are how many?

25. 5 men were hunting, and they met 7 more. How many were there then?

⊖ ⊖ ⊖ ⊖ ⊖   ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖

26. 5 and 7 are how many?

27. A man gave 5 dollars for a chair, and 8 dollars for a table. What did he give for both?

⊖ ⊖ ⊖ ⊖ ⊖   ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖

28. 5 and 8 are how many?

29. A man had 9 labourers working in a field, and he sent 7 more there. How many were there then?

⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖   ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖

30. 9 and 7 are how many?

31. Thomas gave 8 cents for an orange, and had 9 cents left. How many cents had he at first?

⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖   ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖

32. 8 and 9 are how many?

33. If you have 9 cents, and receive ten more, how many will you have?

⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖   ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖

34. 9 and ten are how many?

35. If you buy a cake worth eleven cents, and an orange worth 7, how many cents must you pay for both?

⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖   ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖

36. Eleven and 7 are how many?

37. In one class there are 8 boys, and in another twelve. How many in both classes?

⬭⬭⬭⬭⬭⬭⬭⬭ ⬭⬭⬭⬭⬭⬭⬭⬭⬭⬭⬭⬭

38. 8 and twelve are how many?

39. One man owns 5 head of cattle, and another thirteen. How many do both own?

⬭⬭⬭⬭⬭ ⬭⬭⬭⬭⬭⬭⬭⬭⬭⬭⬭⬭⬭

40. 5 and thirteen are how many?

[The teacher should here require the pupil to close his book, and give the answers to the preceding questions, without counting. This should likewise be done in the articles which follow.]

41. If you have 2 cents in one hand and 2 in the other, how many have you in both? 2 and 2 are how many?

42. If you have 3 marbles in one hand and one in the other, how many have you in both? 3 and 1 are how many?

43. If you have 3 apples in one hand and 2 in the other, how many have you in both? 3 and 2 are how many?

44. If you have 4 pins in one hand and 3 in the other, how many have you in both? 4 and 3 are how many?

45. If you have 3 marbles in one hand and 5 in the other, how many have you in both? 3 and 5 are how many?

46. A boy gave 4 cents for a little book, and 5 cents for some paper. How much did he give for both? 4 and 5 are how many?

47. If you have eight cents and your father gives you 3 more, how many will you have? 8 and 3 are how many?

48. If there are 7 apples on the table, and you put 4 more there with them, how many will there be? 7 and 4 are how many?

49. If you have 6 apples, and afterwards have 5 more given you, how many will you have? 6 and 5 are how many?

50. If it is 8 o'clock when school begins, and the school continues 4 hours, what time will it be when you are dismissed? 8 and 4 are how many?

51. If a top cost 9 cents, and a toy-book 3 cents, what do both cost? 9 and 3 are how many?

52. If you have ten pins on one sleeve, and 2 on the other, how many have you on both? Ten and 2 are how many?

53. 7 and 2 are how many? 7 and 3? 7 and 4? 7 and 5? 7 and 6? 7 and 7? 7 and 8? 7 and 9?

54. 8 and 2 are how many? 8 and 3? 8 and 4? 8 and 5? 8 and 6? 8 and 7? 8 and 8? 8 and 9?

55. A boy had 2 cents. His father gave him 2 more, and his mother 1. How many had he then?

⊖ ⊖   ⊖ ⊖   ⊖

56. 2 and 2 and 1 are how many?

57. If you have 3 marbles in one hand, 3 in the other, and 2 in your pocket, how many have you in all?

⊖ ⊖ ⊖   ⊖ ⊖ ⊖   ⊖ ⊖

58. 3 and 3 and 2 are how many?

59. If you have 4 cents in one pocket, 3 in another, and 4 in your hand, how many have you in all?

⊖ ⊖ ⊖ ⊖   ⊖ ⊖ ⊖   ⊖ ⊖ ⊖ ⊖

60. 4 and 3 and 4 are how many?

61. If you have 6 pins on one sleeve, 7 on the other, and 3 in your hand, how many have you in all?

⊖ ⊖ ⊖ ⊖ ⊖ ⊖   ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖   ⊖ ⊖ ⊖

62. 6 and 7 and 3 are how many?

63. If your father pays 6 cents for the postage of a letter, 8 cents for a pamphlet, and 5 cents for some newspapers, how much does he pay in all?

⊖ ⊖ ⊖ ⊖ ⊖ ⊖   ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖   ⊖ ⊖ ⊖ ⊖ ⊖

64. 6 and 8 and 5 are how many?

65. If you give 4 cents for an orange, 9 cents for some apples, and spend 3 cents in candy, how much do you spend in all?

⊖ ⊖ ⊖ ⊖   ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖ ⊖   ⊖ ⊖ ⊖

66. 4 and 9 and 3 are how many?

67. If a jewsharp cost you 5 cents, an apple 1 cent, and half a pint of pea-nuts 3 cents, how much do the whole cost? 5 and 1 and 3 are how many?

68. If a skein of thread cost 3 cents, a skein of silk 6 cents, and some needles 4 cents, what cost the whole? 3 and 6 and 4 are how many?

69. If a watch cost 8 dollars, a chain 3 dollars, and a seal 4 dollars, what cost the whole? 8 and 3 and 4 are how many?

70. 2 and 2 and 2 are how many? 3 and 2 and 1? 4 and 1 and 2? 3 and 2 and 3? 5 and 1 and 3? 5 and 2 and 2?

71. 3 and 2 and 4 are how many? 3 and 3 and 4? 4 and 2 and 4? 4 and 3 and 4? 3 and 2 and 5? 4 and 3 and 5?

72. 4 and 4 and 1 are how many? 4 and 4 and 2? 3 and 5 and 2? 4 and 3 and 3? 5 and 3 and 2? 5 and 3 and 1?

73. 5 and 2 and 3 are how many? 5 and 3 and 3? 6 and 2 and 3? 6 and 3 and 1? 6 and 4 and 1? 6 and 5 and 1? 6 and 5 and 2?

74. 7 and 2 and 1 are how many? 7 and 2 and 2? 7 and 3 and 2? 7 and 4 and 1? 7 and 4 and 2? 7 and 3 and 3? 7 and 6 and 3?

75. 8 and 1 and 2 are how many? 8 and 2 and 2? 8 and 2 and 3? 8 and 4 and 3? 8 and 5 and 4? 8 and 5 and 5? 8 and 6 and 4?

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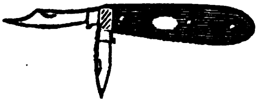
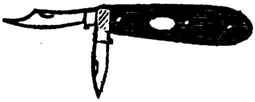
MULTIPLICATION.

ART. II. 1. If one penknife has twoblades, how many blades have two penknives?





2. Then 2 times 2, or *twice* 2 are how many?

3. If one fork has 3 points, how many points have two forks?

4. Then, 2 times 3, or *twice* 3 are how many?

5. One stage has four horses. How many horses have two stages?

6. Then 2 times 4, or *twice* 4 are how meny?

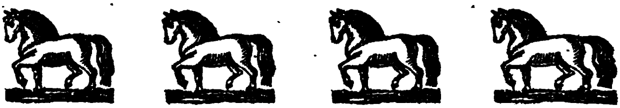
7. Here are three boats, and each boat contains three men. How many men in all?

8. 3 times 3 are how many?

9. One ship has three masts. How many masts have four ships?

10. 4 times 3 are how many?

11. One horse has four legs. How many legs have four horses?



12. 4 times 4 are how many?

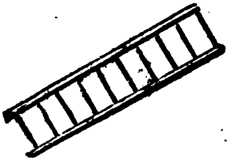
13. One chair has four legs. How many legs have five chairs?

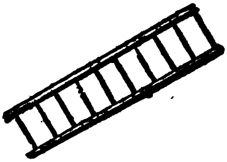
14. 5 times 4 are how many?

15. On one hand you have five fingers, if you count the thumb. How many fingers on seven hands?

16. 7 times 5 are how many?

17. If in one ladder there are ten rounds, how many rounds are there in two ladders?





18. 2 times ten are how many?

19. One man has two hands. How many hands have six men?

○○ ○○ ○○ ○○ ○○ ○○

20. 6 times 2 are how many?

21. If a knife have three blades, how many blades have four knives?

○○○ ○○○ ○○○ ○○○

22. 4 times 3 are how many?

23. If your lesson for one day be three pages, how much will that be for five days?

○○○ ○○○ ○○○ ○○○ ○○○

24. 5 times 3 are how many?

25. If one toy-book cost you four cents, how much must you give for three toy-books.

○○○○ ○○○○ ○○○○

26. 3 times 4 are how many?

27. If an ink-stand cost five cents, what will three ink-stands cost?

○○○○○ ○○○○○ ○○○○○

28. 3 times 5 are how many?

29. If a box of wafers cost six cents, what will three boxes cost?

○○○○○○○ ○○○○○○○ ○○○○○○○

30. 3 times 6 are how many?

31. A chair has four legs, how many legs have seven chairs?

○○○○ ○○○○ ○○○○ ○○○○ ○○○○ ○○○○ ○○○○

32. 7 times 4 are how many?

33. If a barrel of flour cost eight dollars, what will three barrels cost?

34. 3 times 8 are how many?

35. If a yard of ribbon cost nine cents, what cost four yards? 4 times 9 are how many?

36. A man bought 3 acres of ground, and gave 5 dollars an acre; what did he give for the whole? 3 times 5 are how many?

37. What cost 4 yards of cloth at 4 dollars a yard? 4 times 4 are how many?

38. If one yard of ribbon cost 3 cents, what cost 2 yards? What cost 3? What cost 4? What cost 5? What cost 6? What cost 7? What cost 8? What cost 9?

39. Twice 3 are how many? 3 times 3? 4 times 3? 5 times 3? 6 times 3? 7 times 3? 8 times 3? 9 tim[illegible]?

40. If one gallon of wine cost 4 dollars, what cos[illegible] What cost 3? What cost 4? 5? 6? 7? 8? 9?

41. Twice 4 are how many? 3 times 4? 4 times 4? 5 times 4? 6 times 4? 7 times 4? 8 times 4? 9 times 4?

42. If a yard of tape cost 2 cents, what cost 2 yards? What cost 3? 4? 5? 6? 7? 8? 9?

43. 2 times 2 are how many? 3 times 2? 4 times 2? 5 times 2? 6 times 2? 7 times 2? 8 times 2? 9 times 2?

44. If a barrel of flour cost 5 dollars, what cost 2 barrels? What cost 3? 4? 5? 6? 7? 8? 9?

45. 2 times 5 are how many? 3 times 5? 4 times 5? 5 times 5? 6 times 5? 7 times 5? 8 times 5? 9 times 5?

46. There are 7 days in 1 week; How many days are there in 2 weeks? In 3? In 4? In 5? 6? 7? 8? 9?

47. 2 times 7 are how many? 3 times 7? 4 times 7? 5 times 7? 6 times 7? 7 times 7? 8 times 7? 9 times 7?

48. If an orange cost 6 cents, what cost 2 oranges? What cost 3? What cost 4? 5? 6? 7? 8? 9?

49. Twice 6 are how many? 3 times 6? 4 times 6? 5 times 6? 6 times 6? 7 times 6? 8 times 6? 9 times 6?

50. If 8 boys can sit on one bench in the school-room, how many boys can sit on 2 benches? How many on 3? How many on 4? How many on 5? On 6? On 7? 8? 9?

51. Twice 8 are how many? 3 times 8? 4 times 8? 5 times 8? 6 times 8? 7 times 8? 8 times 8? 9 times 8?

52. If sugar is 9 dollars a hundred weight, what will 2 hundred weight cost ? What will 3 ? What will 4 ? 5 ? 6 ? 7 ? 8 ? 9 ?

53. 2 times 9 are how many ? 3 times 9 ? 4 times 9 ? 5 times 9 ? 6 times 9 ? 7 times 9 ? 8 times 9 ? 9 times 9 ?

54. 7 times 3 are how many ? 6 times 4 ? 5 times 5 ? 3 times 6 ? 8 times 3 ? 8 times 4 ? 5 times 6 ? 5 times 7 ?

55. 9 times 2 are how many ? 9 times 3 ? 9 times 5 ? 8 times 5 ? 8 times 6 ? 7 times 5 ? 7 times 6 ? 7 times 8 ?

56. 7 times 7 are how many ? 7 times 9 ? 7 times 2 ? 7 times 3 ; 6 times 3 ? 6 times 5 ? 6 times 6 ? 6 times 8 ?

57. 5 times 2 are how many ? 5 times 9 ? 5 times 8 ? 5 times 4 ? 4 times 3 ? 4 times 4 ? 4 times 5 ? 4 times 6 ?

58. Twice ten are how many ? 3 times ten ? 4 times ten ? 5 times ten ? 6 times ten ? 7 times ten ? 8 times ten ? [illegible]mes ten ?

59. Twice eleven are how many ? 3 times eleven ? 4 times eleven ? 5 times eleven ? 6 times eleven ? 7 times eleven ? 8 times eleven ? 9 times eleven ?

60. 2 times twelve are how many ? 3 times twelve ? 4 times twelve ? 5 times twelve ? 6 times twelve ? 7 times twelve ? 8 times twelve ? 9 times twelve ?

61. If 7 oranges cost 5 cents apiece, and 4 lemons 3 cents apiece, what cost the whole ? 7 times 5 and 4 times 3 are how many ?

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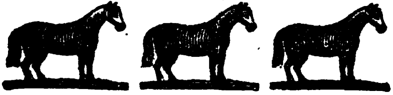
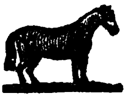
SUBTRACTION.

Art. III. 1. Three sheep were together, but one went away from the rest. How many were there left ?

2. 1 from 3 leaves how many ?

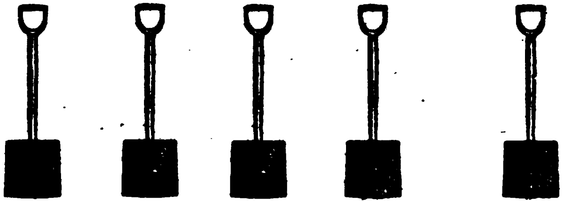
3. Four horses were together, but one went away from the rest. How many were left ?





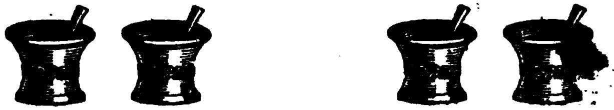
4. 1 from 4 leaves how many ?

5. If you take one shovel from five shovels, how many shovels are left ?



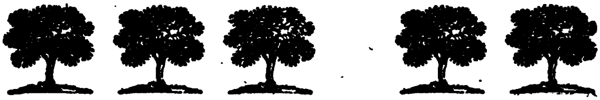
6. 1 from 5 leaves how many ?

7. If you take two mortars from four mortars, how many mortars are left ?



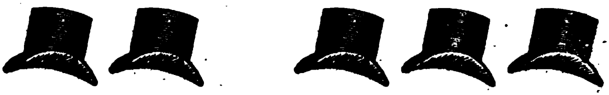
8. 2 from 4 leave how many ?

9. If you take 2 trees from 5 trees, how many are left ?



10. 2 from 5 leave how many ?

11. If you take 3 hats from 5 hats, how many are left ?

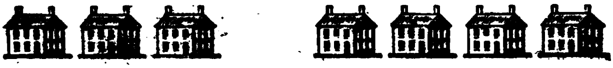


12. 3 from 5 leave how many ?

13. Take 3 ships from 6 ships ; how many are left ?

14. 3 from 6 leave how many ?

15. Take 4 houses from 7 houses ; how many are left ?



16. 4 from 7 leave how many ?

17. Take 4 marks from 8 marks; how many are left?

| | | | | | | |

18. 4 from 8 leave how many?

19. If there are 9 barrels together, and you take away four, how many will be left?

⬭ ⬭ ⬭ ⬭ ⬭   ⬭ ⬭ ⬭ ⬭

20. 4 from 9 leave how many?

21. If from ten tea-cups you take away 4, how many will be left?

⬭ ⬭ ⬭ ⬭ ⬭ ⬭   ⬭ ⬭ ⬭ ⬭

22. 4 from ten leave how many?

23. A man had 8 sheep and sold 2; how many had he left?

⬭ ⬭ ⬭ ⬭ ⬭ ⬭   ⬭ ⬭

24. 2 from 8 leave how many?

25. A man had 7 barrels of cider and sold 5; how many had he left?

⬭ ⬭   ⬭ ⬭ ⬭ ⬭ ⬭

26. 5 from 7 leave how many?

27. There were 6 trees standing together, and a man cut down 4; how many were left?

⬭ ⬭   ⬭ ⬭ ⬭ ⬭

28. 4 from 6 leave how many?

29. 8 apples were hanging on one limb; a boy gathered 5; how many were left?

⬭ ⬭ ⬭   ⬭ ⬭ ⬭ ⬭ ⬭

30. 5 from 8 leave how many?

31. James had 9 marbles, and in playing he lost 5; how many had he left?

⬭ ⬭ ⬭ ⬭   ⬭ ⬭ ⬭ ⬭ ⬭

32. 5 from 9 leave how many?

33. Thomas had ten books in his desk, and he took out 5; how many were left?

⬭ ⬭ ⬭ ⬭ ⬭   ⬭ ⬭ ⬭ ⬭ ⬭

34. 5 from ten leave how many?

35. William had ten pins on his sleeve, and he took away 6; how many were left?

⬭ ⬭ ⬭ ⬭   ⬭ ⬭ ⬭ ⬭ ⬭ ⬭

36. 6 from ten leave how many?

37. If I have twelve books, and sell 5 of them, how many have I left?

⬭ ⬭ ⬭ ⬭ ⬭ ⬭ ⬭   ⬭ ⬭ ⬭ ⬭ ⬭

38. 5 from twelve leave how many?

39. If you have eleven sticks of candy, and give away 4, how many have you left?

⬭ ⬭ ⬭ ⬭ ⬭ ⬭ ⬭   ⬭ ⬭ ⬭ ⬭

40. 4 from eleven leave how many?

41. If you have thirteen cents, and spend 6 of them, how many have you left?

⬭ ⬭ ⬭ ⬭ ⬭ ⬭ ⬭   ⬭ ⬭ ⬭ ⬭ ⬭ ⬭

42. 6 from thirteen leave how many?

43. If you have fourteen apples, and give away 8, how many have you left?

⬭ ⬭ ⬭ ⬭ ⬭ ⬭   ⬭ ⬭ ⬭ ⬭ ⬭ ⬭ ⬭ ⬭

44. 8 from fourteen leave how many?

45. If you have 5 oranges, and give away 3, how many have you left? 3 from 5 leave how many?

46. If you have 5 quills, and make pens of 4, how many are left? 4 from 5 leave how many?

47. If you have 6 apples, and eat 2, how many have you left? 2 from 6 leave how many?

48. If you have 7 marbles, and give away 5, how many have you left? 5 from 7 leave how many?

49. If you have 7 pins on your sleeve, and take away 6, how many will be left? 6 from 7 leave how many?

50. A man had 8 horses, and sold 5; how many had he left? 5 from 8 leave how many?

51. A man had 8 acres of land, and sold 6; how many had he left? 6 from 8 leave how many?

52. A merchant had 9 yards of cloth in one piece and sold 4 yards; how many yards were left? 4 from 9 leave how many?

53. A grocer sold 5 gallons of molasses out of 9 gallons; how many gallons were left? 5 from 9 leave how many?

54. An iron merchant sold 6 tons of iron out of 9 tons; how many tons had he left? 6 from 9 leave how many?

55. George had 9 toy-books, and gave away 7; how many had he left? 7 from 9 leave how many?

56. Thomas had 9 quarts of cherries, and sold 8 quarts; how many had he left? 8 from 9 leave how many?

57. If you have 8 school-books, and lose 2, how many have you left? 2 from 8 leave how many?

58. If you have 8 pencils, and use up 3 of them, how many have you left? 3 from 8 leave how many?

59. If you have 8 pens, and spoil 7 of them, how many have you left? 7 from 8 leave how many?

60. 2 from 4 leave how many? 2 from 3? 1 from 2? 1 from 3? 1 from 4? 2 from 5? 2 from 7? 2 from 6? 2 from 8? 2 from 9?

61. 1 from 5 leaves how many? 1 from 6? 1 from 7? 1 from 8? 2 from ten? 3 from 4? 3 from 6? 3 from 5? 3 from 7? 3 from 8?

62. 3 from 9 leave how many? 4 from 5? 4 from 4? 4 from 6? 4 from 7? 4 from 9? 4 from 8? 3 from ten? 3 from eleven?

63. 5 from 5 leave how many? 5 from 7? 5 from 6? 5 from ten? 5 from 9? 5 from 8? 5 from eleven? 5 from twelve?

64. 6 from 7 leave how many? 6 from 8? 6 from 6? 6 from 9? 6 from ten? 6 from eleven? 6 from twelve? 6 *from thirteen*?

65. 7 from 7 leave how many? 7 from 8? 7 from ten? 7 from 9? 7 from eleven? 7 from twelve? 7 from thirteen?

66. 8 from ten leave how many? 8 from 9? 8 from 8? 8 from eleven? 8 from twelve? 8 from thirteen? 8 from fourteen?

67. 9 from 9 leave how many? 9 from eleven? 9 from twelve? 9 from ten? 9 from fourteen? 9 from thirteen? 9 from fifteen? 9 from sixteen? 9 from eighteen? 9 from seventeen? 9 from nineteen?

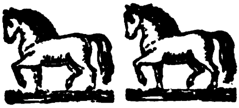
◆

DIVISION.

Art. iv. 1. A man had two trees which he wished to set out in different places, putting one in each place; in how many places could he set them out?

2. 1 is contained in 2 how many times?

3. A man had four horses, which he wished to harness in spans; that is, in *twos*. How man span did he have?



4. 2 are in 4 how many times?

5. A man owned six houses in several places; and he owned two in a place. How many places were there?

6. 2 are in 6 how many times?

7. A boy counted the hands of all the persons in a company, and found there were 8. How many persons were there?

8. 2 are in 8 how many times?

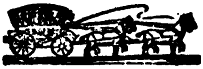
9. 6 men went sailing in boats, and 3 went in each boat. How many boats did they take?

10. 3 are in 6 how many times?

11. In one ship are 3 masts. How many ships will twelve masts supply?

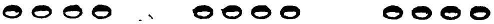
12. 3 are in twelve how many times?

13. 4 horses are usually harnessed to a stage. How many stages will 8 horses supply?



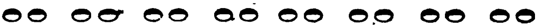
14. 4 are in 8 how many times?

15. William had twelve marbles, which he divided among his friends, giving 4 to each. To how many did he give them?



16. 4 are in twelve how many times?

17. There were sixteen pages in James's writing book, and he wrote 2 pages a day. How many days did it take him to write it through?

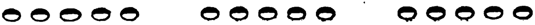


18. 2 are in sixteen how many times?

19. George had twelve apples, which he gave to his little brothers, giving 3 to each. How many brothers had he?

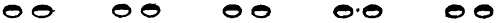
20. 3 are in twelve how many times?

21. A man sold several acres of land for 5 dollars an acre, and the whole brought him fifteen dollars. How many acres did he sell?



22. 5 are in fifteen how many times?

23. A man had ten oxen, and yoked them into pairs. How many pairs did he have?

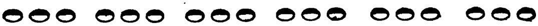


24. 2 are in ten how many times?

25. A gentleman divided twelve cents among his children, giving them 2 cents apiece. How many children had he?

26. 2 are in twelve how many times?

27. If I divide eighteen boys into classes of 3, how many classes will there be?



28. 3 are in eighteen how many times?

29. If I divide fourteen boys into classes of 7, how many classes will there be?

30. 7 are in fourteen how many times?

31. If I divide sixteen boys into classes of 8, how many classes will there be?

32. 8 are in sixteen how many times?

33. If I divide eighteen boys into classes of 9, how many classes will there be ?

○○○○○○○○○ ○○○○○○○○○

34. 9 are in eighteen how many times ?

35. A man divided eighteen cents among some boys, giving each boy 6 cents. How many boys were there ?

○○○○○○ ○○○○○○ ○○○○○○

36. 6 are in eighteen how many times ?

37. A boy had twenty-four sheets of paper, which he made into writing books, putting 4 sheets into a book. How many books did he make ?

○○○○ ○○○○ ○○○○ ○○○○ ○○○○ ○○○○

38. 4 are in twenty-four how many times ?

39. If an acre of ground cost 3 dollars, how much can I buy with eighteen dollars ? 3 are in eighteen how many times ?

40. If a toy-book cost 3 cents, how many can I buy with twenty one cents ? 3 are in twenty one how many times ?

41. If a pail will hold 4 gallons of water, how many pailfuls will a tub contain, which holds sixteen gallons ? 4 are in sixteen how many times ?

42. 4 quarts make a gallon. How many gallons are there in twenty quarts ? 4 are in twenty how many times ?

43. How many gallons are there in twelve quarts ? In 8 quarts ? In twenty four quarts ? In twenty eight quarts ? In thirty two quarts ? In thirty six quarts ? In forty quarts ?

44. 2 pints make a quart. How many quarts are there in 4 pints ? In 6 pints ? In 8 pints ? In ten pints ? In twelve pints ? In fourteen pints ? In sixteen pints ? In eighteen pints ? In twenty pints ? In twenty two pints ?

45. If a yard of ribbon cost 3 cents, how many yards will 6 cents buy ? Will 9 cents ? Will twelve cents ? Will fifteen cents ? Will eighteen ? Will twenty one ? Will twenty four ? Will twenty seven ? Will thirty ? Will thirty three ? Will thirty six ?

46. How many times 5 in ten ? In fifteen ? In twenty ? In twenty five ? In thirty ? In thirty five ? In forty ? In forty five ? In fifty ? In fifty five ? In sixty ?

47. How many times 6 in twelve? In eighteen? In twenty four? In thirty? In thirty six? In forty two? In forty eight? In fifty four? In sixty? In sixty six? In seventy two?

48. How many times 7 in fourteen? In twenty one? In twenty eight? In thirty five? In forty two? In forty nine? In fifty six? In sixty three? In seventy? In seventy seven? In eighty four? In ninety one?

49. How many times 8 in sixteen? In twenty four? In thirty two? In forty? In forty eight? In fifty six? In sixty four? In seventy two? In eighty? In eighty eight? In ninety six? In one hundred and four?

50. How many times 9 in eighteen? In twenty seven? In thirty six? In forty five? In fifty four? In sixty three? In seventy two? In eighty one? In ninety? In ninety nine? In one hundred and eight? In one hundred and seventeen?

51. How many times ten in twenty? In thirty? In forty? In fifty? In sixty? In seventy? In eighty? In ninety? In one hundred? In one hundred and ten? In one hundred and twenty? In one hundred and thirty?

52. How many times eleven in twenty two? In thirty three? In forty four? In fifty five? In sixty six? In seventy seven? In eighty eight? In ninety nine? In one hundred and ten? In one hundred and twenty one? In one hundred and thirty two?

53. How many times twelve in twenty four? In thirty six? In forty eight? In sixty? In seventy two? In eighty four? In ninety six? In one hundred and eight? In one hundred and twenty? In one hundred and thirty two? In one hundred and forty four?

Note. The preceding lessons may be omitted by the more advanced pupils who study this book.

1 6 16 1 6
1 1 1

ARITHMETIC.

NUMERATION.

§ III. For those who omit the preceding exercises, we here repeat the definition of a unit, § I. Art i.

A single thing of any kind is called a unit or unity.

You know very well what numbers are. When you say, John has *three* marbles, you tell me what *number* of marbles John has. When you say, my sister has *two* oranges, but I have only *one*, you tell me the *number* of oranges your sister has, and also the *number* you have yourself. As you understand this, we will not trouble you at present with a definition of number. The preceding exercises have required you to use numbers; and you will recollect that the names of a few of the first of these are, ONE, TWO, THREE, FOUR, FIVE, SIX, SEVEN, EIGHT, NINE; which are represented by the *characters*, 1, 2, 3, 4, 5, 6, 7, 8, 9. Besides these, the character 0, is employed, which means *nought* or *nothing;* that is, it has no value in itself. It is, however, very useful, as will be seen hereafter. It is usually called CYPHER or ZERO.

We have no *single* character, to represent any number greater than nine. Those already given are sufficient, as we shall see, to express any numbers, however large. The first nine are often called DIGITS. The whole are called FIGURES. The first nine are also called SIGNIFICANT FIGURES, because they have some *value* or *signify* something. The cypher is not a significant figure, because it has *no value*, or signifies *nothing*. Before we proceed to explain how larger numbers are written, we wish your very diligent attention to the following.

Here is a picture of an auction, or public sale of goods.

This is a *single* picture, and it is therefore *a unit.*

But there are several men in the picture, and each *single man* is also *a unit*. Hence, it appears, that, as *one picture* may contain *several men*, so

A UNIT OF ONE KIND MAY CONTAIN SEVERAL UNITS OF ANOTHER KIND.

Here is a basket of flowers.

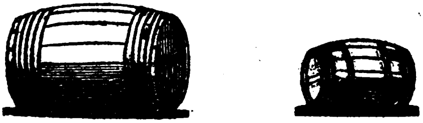
This is a *single basket*, and is therefore *a unit*. But it contains several flowers, and *each flower* is likewise *a unit*. Here we see, exactly as before, that a unit of one kind may contain several units of another.

We have similar examples in measures. One gallon contains four quarts; that is, *one* unit of one kind contains *four* units of another kind. One yard contains three feet; that is, *one* unit of one kind contains *three* units of another kind.

So, likewise in weights. One pound contains sixteen ounces. One hundred weight contains four quarters, &c.

So likewise in coins. One dollar contains one hundred cents. One eagle contains ten dollars, &c.

Moreover, suppose I have a barrel of a certain size. I may employ a man to make another, as many times larger as I please. Thus, as in the picture, I may have one made three times larger. Hence, we see, that



A UNIT OF ONE KIND MAY BE MADE TO CONTAIN AS MANY UNITS OF ANOTHER KIND AS WE CHOOSE.

In like manner, if I draw on paper, a short line, I may easily draw another, ten times as long.

Here, then, I make a unit of the latter kind, ten times as great, as one of the former. But I might have made it nine times, or eleven times, or twelve times, or any number of times as great.

Now, to come to the point at which we are aiming, we have only to apply this to numbers: for, it is plain, that, *in numbers, as well as in any thing else, we may have one kind of units, which shall contain several of another kind.*

Suppose, then, in numbers, we make *one unit* of a larger kind equal to *ten* of a smaller. *This is the way, in which numbers are actually reckoned.* Now, as the characters 1, 2, &c. stand for units of any kind, we may use them to stand for units of the larger kind, as well as for those of the smaller, if we can contrive any way of distinguishing one kind from the other. This we will do at present, by using different kinds of type. The larger units shall be expressed by the figures **1**, **2**, **3**, &c. and the smaller, by the common figures 1, 2, 3, &c. Then **1** is ten times 1, **2** is ten times 2, &c.

In the same manner, we may have a different kind of figures still, as 𝟙, 𝟚, 𝟛, &c. to stand for units of a larger kind than either of these; each of which shall be equal to ten of the kind denoted by the figures **1**, **2**, **3**, &c. Then, since 𝟙 stands for ten times **1**, or ten **1**s, and each of these ten **1**s stands for ten 1s, 𝟙 is equal to *ten times ten* 1s, or one hundred 1s.

We might have another kind still, each of which should be equal to ten 𝟙s, and of course equal to one thousand 1s, and so on. Here we have several kinds of units. For convenience, it is best to call the smallest, represented by 1, 2, 3, &c. *units of the first order*, the next, represented by **1**, **2**, **3**, &c., *units of the second order*, and those represented by 𝟙, 𝟚, 𝟛, &c., *units of the third order*.

THE FIRST ORDER MAY BE CALLED UNITS, SIMPLY; THE SECOND ORDER, TENS; AND THE THIRD ORDER, HUNDREDS; TAKING THEIR NAMES FROM THEIR VALUES.

[Let the pupil be now required to write the following numbers, placing units on the right; tens, next towards the left; and hundreds, next; as in the examples, whose answers are given. The figures, used in the book, should be imitated.]

Write one unit of the second order, (or one ten,) and one unit of the first order, (or one unit.) *Ans.* 11.

Write one unit of the third order, (or one hundred,) and one ten, and one unit. *Ans.* 111.

Write one hundred, one ten, and five units. *Ans.* 115.

Write two hundreds, three tens, and seven units. *Ans.* 237.

Three hundreds, five tens, and eight units. *Ans.* 358.

Two hundreds, eight tens, and nine units. Seven hundreds, six tens, and three units. Two tens, and two units. Nine tens, and six units. Four hundreds, and six tens, and four units. Five hundreds, five tens, and five units. Nine hundreds, seven tens, and three units. Four hundreds, eight tens, and four units. Eight hundreds, and nine tens, and nine units. Two hundreds, six tens, and three units. One hundreds, two tens, and three units. Two hundreds, five tens, and seven units. One ten, and three units, Seven tens, and three units. Nine hundreds, nine tens, and nine units.

In all these examples, units of the first order, come first on the right; those of the second come second; those of the third come third; so that if the *same kind of figures were used for all*, we should know the several orders, by their *places*.

Where do units of the first order come? Where do units of the second order come? Where do those of the third order come? Then,

THE NUMBER OR NAME OF ANY ORDER, IS THE SAME WITH THE NUMBER OR NAME OF ITS PLACE, COUNTING FROM THE RIGHT.

If we used the same figures for all the orders, how would you know the order to which any figure belonged?

Then, using the common figures, write the following numbers.

One ten, and one unit, or eleven.	Ans.	11.
One ten, and two units, or twelve.		12.
One ten, and three units, or thirteen.		13.
Fourteen, or one ten, and four units.		14.
Fifteen, or one ten, and five units.		15.
Sixteen, or one ten, and six units.		16.
Seventeen, or one ten, and seven units.		17.
Eighteen, or one ten, and eight units.		18.
Nineteen, or one ten, and nine units.		19.

One hundred and thirty-four, or one hundred, and three tens, and four units. *Ans*. 134.

Seven hundred and sixty-three, or seven hundreds, and six tens, and three units.

Now write one ten, or one ten and no units. Ans. 10. Here we see the use of the 0, or cypher; for if we were to write nothing but 1, it would read one unit of the first order, or one unit simply;

because it would be in the first place. Now the cypher has no value, but it makes the 1 stand in the second place, and, therefore, makes it a unit of the second order, which is equal to ten units of the first.

How do you write one ten? Why do you use the cypher? Why do you not use more cyphers than one? Has the cypher any value? Does the use of it affect the value of other figures?

Write two tens, or twenty.	20.
Three tens, or thirty.	30.
Four tens, or forty.	40.
Five tens, or fifty.	50.
Sixty, or six tens.	60.
Seventy, or seven tens.	70.
Eighty, or eight tens.	80.
Ninety, or nine tens.	90.
Ten tens, or one hundred.	

Ten tens, are ten units of the second order, which are equal to one of the third order, or one hundred, written 100

Why do you use two cyphers here?

If I were to annex another cypher to 100, (as 1000,) in what place would the 1 stand? *Ans. The fourth place.* It would, then, be a unit of what order? *Ans. The fourth order.*

A UNIT OF THE FOURTH ORDER IS CALLED A THOUSAND.

If a cypher were annexed to 1000, (as 10000) to what order would the 1 belong?

UNITS OF THE FIFTH ORDER ARE CALLED TENS OF THOUSANDS.

Another cypher would make the 1, a uuit of what order?

UNITS OF THE SEVENTH ORDER ARE CALLED MILLIONS.

Read the following numbers.

5	Five, or five units.
30	Thirty, or three tens.
35	Thirty-five.
	79 63 81 97 83 76 49 87 95
400	Four hundreds.
435	Four hundred and thirty-five.
	553 984 659 334 695 843
7000	Seven thousands.
7435	Seven thousand, four hundred, and thirty-five.
	2761 1561 4322 5112 7782
80000	Eighty thousand. or eight tens of thousands.
87435	Eighty-seven thousand, four hundred, and thirty-five.
	55534 60203 88905 76352 99122
500000	Five hundred thousand, or five hundreds of thousands.
587435	Five hundred and eighty-seven thousands, four hundred, and thirty-five.

679458 234567 320320 427427

2000000 Two millions.

2587435 Two million, five hundred and eighty-seven thousand, four hundred and thirty-five.

2305061 8910002 8395421 7878978

2004 17 40002 10203 1000001 2222222

813 617 23904 237801 603545 7897989 6304026

[The teacher should require the pupil to read these numbers, both by the familiar names, and also, by the names of the orders; as 6 tens of thousands, instead of 60 thousand, &c. So, likewise, in *writing* the following numbers, the pupil should change the common names into the same terms.]

Write the following numbers in figures.

Three - - - - - - - - - - - 3

Thirty, or three tens - - - - - - - - 30

Thirty-three, or three tens, and three units.

Forty-seven. Eighty-eight. Fifty-six. Thirty-five.

Eight hundred - - - - - - - - - - 800

Eight hundred and thirty-three. Six hundred and ninety-eight. Five hundred and fifty-five. Eight hundred and fifteen.

Two thousand - - - - - - - - - - 2000

Two thousand, eight hundred, and thirty-three.

Four thousand, nine hundred, and eighty-one.

Sixty thousand, or six tens of thousands, - - - - 60000

Sixty thousand, eight hundred, and thirty-three.

Five hundred thousand, or five hundreds of thousands, 500000

Five hundred and sixty-two thousand, eight hundred, and thirty-three.

One million - - - - - - - - - - 1000000

One million, five hundred and sixty-two thousand, eight hundred, and thirty-three.

Two hundred and three. Two thousand and three.

Twenty thousand, two hundred, and four.

Three million, eighty-five thousand, and twenty-one.

Six hundred and fifty eight thousand, nine hundred and seventy eight.

Three hundred and sixty-five.

Nine Million three hundred and thirty-seven thousand, five hundred and fifty-four.

Four hundred and eighty-eight thousand six hundred and seventy five.

Forty-four thousand, eight hundred and ninety-nine.

Five hundred and fifty-three.

Seven hundred and ninety-one.

◆

§ IV. How many orders of figures have you learned? What is the seventh? The sixth? Fifth? Fourth? Third? Second? First?

The other orders, as high as the 24th, may be learned from the following

NUMERATION TABLE.

ORDERS.	NAMES.		PERIODS.
24	Hund. of Sex.	5	VIII. Sextillions.
23	T. of Sex.	5	
22	Sex.	5	
21	Hund. of Quin.	5	VII. Quintillions.
20	T. of Quin.	5	
19	Quintillions.	5	
18	Hund. of Quad.	5	VI. Quadrillions.
17	T. of Quad.	5	
16	Quadrillions.	5	
15	Hund. of Trill.	5	V. Trillions.
14	T. of Trill.	5	
13	Trillions.	5	
12	Hund. of Bill.	5	IV. Billions.
11	T. of Bill.	5	
10	Billions.	5	
9	Hund. of Mil.	5	III. Millions.
8	T. of Mil.	5	
7	Millions.	5	
6	Hundred of Th.	5	II. Thousands.
5	Tens of Th.	5	
4	Thousands.	5	
3	Hundreds.	5	I. Units.
2	Tens.	5	
1	Units.	5	

In order to read large numbers, it is very convenient to divide them into periods of three figures each, begining at the right. The first is called the periods of units, because it contains units, and tens of units, and hundreds of units. The second, the period of thousands, because it contains thousands, and tens of thousands, and hundreds of thousands, and so on. The names of the periods are given above.

In reading, you should read each period as though it stood alone, and then annex the name of the period.

Thus, to read the period of thousands above, first read it as though it stood alone, and it will be, five hundred and fifty-five. Then annex the name of the period, and it will be, five hundred and fifty-five thousand.

Let the pupil point off and read the following numbers.

7 8 6 4 3 7 9 2 5 8 4 2 1 9
5 1 1 1 1 9 9 9 9 3 8 7 6 5 3
1 2 3 4 5 6 7 8 9 0 0 0 0 0 2 4
7 6 0 0 0 0 7 6 0 0 0 0 7 6
9 1 8 7 5 6 7 8 1 2 3 5 4 1 0 0 1
8 0 9 0 1 0 3 0 4 0 0 0 2 3 4 5
7 8 9 0 0 0 6 7 8 0 0 5 6 7 0 2
3 4 2 3 1 2 5 6 1 6 5 8 7 1 2 4
8 9 6 7 4 5 0 0 0 2 0 3 0 1 0 2 1 0 7 8 0 8

It is likewise most convenient to *write* numbers by periods.

Write three hundred and five thousand, four hundred and fifty-three.

How many periods are here mentioned, and what are they?

Write the period of thousands. *Ans.* 305 thousands.

Write the period of units. *Ans.* 453.

Which ought to stand first?

Then write them both together. *Ans.* 305,453.

Write three hundred and five million, four hundred and fifty three. Write first the millions' period.

There are no thousands; how will you write the thousand period? *Ans.* 000 thousands.

Write the units period. *Ans.* 453.

Arrange the periods in order. *Ans.* 305,000,453.

Write sixteen thousand and thirty. *Ans.* 16,030.

Then, *to write numbers in figures*, BEGIN WITH THE HIGHEST PERIOD AND WRITE EACH PERIOD AS THOUGH IT WAS THE ONLY ONE TO BE WRITTEN. PLACE CYPHERS WHEREVER A PERIOD OR AN ORDER IS OMITTED.

In this manner let the pupil write the following numbers.

Write seven millions, six hundred and sixty-two thousand, five hundred and ninety-three.

Four billions, twenty million, two hundred thousand and five.

Sixty-seven trillions, five hundred and five thousand.

Thirty-five billions and four.

Four hundred and seventy-nine quadrillions, twenty-three million and sixty.

One thousand and three.

One hundred thousand, five hundred and forty.

Seventy-seven million, four hundred and forty-two thousand, five hundred and nineteen.

Sixty five trillions, seventy four billions, eighty four millions, ninety four thousand, one hundred and five. Two quadrillions and fifty. Nine quintillions and thirty nine. Seven sextillions, eighty five millions, three hundred and four. Five sextillions, three hundred and seventy nine quintillions, four hundred and forty seven quadrillions, eight hundred and twenty three trillions, four hundred and thirty seven billions nine hundred and eighty six millions five hundred and thirty six thousand four hundred and seventy nine. Two sextillions and two.

◆

§ V. Where the figure 1 stands alone, what is its value? *Ans.* One unit. When the figure 2 stands alone, what is its value? *Ans.* Two units. When 3 stands alone what is its value? When 4? 5? 6? 7? 8? 9?

THE VALUE OF A FIGURE, STANDING ALONE, IS CALLED ITS SIMPLE VALUE.

If the figure 1 has a cypher at its right hand, what is its value? *Ans.* One ten.

If the figure 3 has two cyphers at the right, what is its value?

What is the value of 7 and four cyphers, or 70000? Of 800000? Of 2000? Of 5000000?

Is the value of the figures in these last examples, the same with their simple value, or is it greater, or is it less?

Why is it greater? *Ans.* Because it stands in a higher place. Then,

FIGURES HAVE A VALUE WHICH DEPENDS UPON THEIR PLACE, DIFFERENT FROM THEIR SIMPLE VALUE.

THE VALUE WHICH A FIGURE DERIVES FROM ITS PLACE, IS CALLED ITS LOCAL VALUE.

How many units of the first order are equal to one of the second?

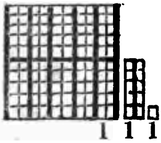
How many of the second are equal to one of the third?

How many of the third to one of the fourth? Of the 4th to one of the 5th? Of the 6th to one of the 7th? Of the 9th to one of the 10th? &c. Then

IT TAKES TEN UNITS OF ANY ORDER TO MAKE ONE OF THE NEXT HIGHER; And,

NUMBERS INCREASE FROM RIGHT TO LEFT IN TENFOLD PROPORTION.

Here is a picture to illustrate this tenfold increase.



The little square on the right, represents a unit of the first order, and corresponds to the 1 under it. The next diagram represents a unit of the second order, and corresponds likewise to 1, *in the second place.* The third diagram represents a unit of the third order, and corresponds to 1, in the third place.

You see, then, how rapidly numbers increase. If we were to go on, only two orders higher, that is, to the fifth order, the page would not be large enough to contain the diagram for that order.

You have now been learning to write numbers in figures. This is called NOTATION.

You have, also, been learning to read numbers written in figures. This is called NUMERATION. Then

NOTATION IS THE ART OF WRITING NUMBERS IN FIGURES, And, NUMERATION IS THE ART OF READING NUMBERS, EXPRESSED IN FIGURES.

* * *

OBSERVATIONS ON NUMERATION, FOR ADVANCED PUPILS.

§ VI. The names thirteen, fourteen, fifteen, &c. seem plainly to be derived from three and ten, four and ten, five and ten, &c., with a slight change in the words. One, two, three, four, and so on, up to ten, are primitive words; that is, they are derived, or formed from no other words in our language. Eleven and twelve, seem likewise to be primitive words, though Mr. Webster thinks that they may have come originally from the words, *one left*, (after ten,) and *two left*; twenty, thirty, forty, &c., come from two tens, three tens, &c. All the other numbers up to a hundred, are combinations of those already mentioned, without alteration. Hundred, thousand, and million, are primitive, *in our language*. Billion, trillion, quadrillion, &c., are formed by combining the Latin numerals with the termination *illion*. These names go on much farther than they are given above. After sextillions, we have septillions, octillions, nonillions, decillions, undecillions, duodecillions, tredecillions, &c., but there is seldom occasion to use these terms.

There is a reason why so many derivative names are employed in Numeration. For, otherwise, there would be as many distinct names as numbers, and, it would manifestly be very difficult to remember them all. Indeed, it would be impossible ever to learn them. For, if they were extended, only to 100,000 they would form a considerably greater body of words, than all the rest of the language put together; and, if a man were to employ himself twelve hours in a day in studying them, it would take him nearly twenty years merely to *read* the names as far as one trillion. Besides, we must discontinue giving names somewhere; and wherever we stop, there is still room to form more. Now, on the plan of numeration in use, the first ten names together with *hundred* and *thousand*, (making twelve in all,) are sufficient to express all *ordinary* numbers. After that, we only need the names of the higher *periods*. And if it were necessary so far to simplify, we might even dispense with these.

We have ten characters, to be used in writing numbers. The cypher has no value. All the others have a positive value, and are therefore called *significant* figures. Since characters stop at 9, some means must be contrived to continue the notation of numbers beyond, or the characters are of no use. For this reason, the first number, too great to be expressed by a *single character*, is considered a unit of a *higher order*, and is denoted by the figure 1, removed to *another place*. When we obtain too many of these *new units* to be expressed by a single character, we make *another order of units still*, and so on. Hence, we see, that it was not *necessary* to stop at 9. Nor was it necessary to continue our characters as far as 9. We might have stopped at 8, or 7, or 6, at pleasure. If we had gone beyond 9, we might have expressed any given number with fewer figures; if we had stopped short of 9, we should have been obliged to use more than are at present necessary.

The origin of the system of counting by tens, seems to have been, that men counted *on their fingers*, before writing was invented. Thus, instead of *thirty*, they said, *three times all their fingers and thumbs*, that is, three *tens*. So that our scheme of tenfold, or *decimal* notation, as it is called, owes its origin, probably, to chance. It would be more convenient, on many accounts, to reckon by *twelves*, instead of by *tens;* but the decimal notation is now too firmly established to be shaken. The number by which we reckon, that is, which expresses the ratio of increase in any system of Notation, is called the RADIX of that system. The Radix of the decimal Notation is 10.

The word *digit* comes from the Latin *digitus*, which means finger, and is applied to figures, because men formerly counted, as before stated, by their fingers. It properly belongs to the cypher, as well as to the other characters, but custom has restricted it to the significant figures.

The mode of dividing numbers into periods of *three* figures, is called the FRENCH mode of dividing, because the French arithmeticians first used it. The E[illegible] to divide into periods of *six* figures, and these, sometim[illegible] half periods of three figures.

As far as the order of *Hundreds of Millions*, both systems employ the same names for the several orders; but, after that, the English goes on with *thousands of millions*, *tens of thousands of millions*, and *hundreds of thousands of millions;* instead of *Billions*, *tens of Billions*, and *hundreds of Billions*, and, thus, brings the *Billion* order, where the French has the *Trillion*. The French is much most convenient. In the English, there is no *thousand period*, the period of *units* extending as far as millions, and, thereby embracing all the thousands. No other period is lost, but, as more figures are put into each, the higher periods are carried farther to the left. Of course, the names *Billion*, *Trillion*, &c, stand for much larger numbers in the English system, than in the French. An English Billion, for example, is a thousand French Billions, and an English Trillion is a million of French Trillions.

A perfect knowledge of the difference between the two may be gained by comparing the following Tables.

FRENCH METHOD.		ENGLISH METHOD.	
1st Period.	Units.	Units.	1st Period.
	Tens.	Tens.	
	Hundreds.	Hundreds.	
2nd Period.	Thousands.	Thousands.	
	Tens of Thousands.	Tens of Thousands.	
	Hundreds of Th.	Hundreds of Th.	

French Method.		English Method.	
3rd Period.	Millions.	Millions.	2nd Period.
	Tens of M.	Tens of M.	
	Hundreds of M.	Hundreds of M.	
4th Period.	Billions.	Thousands of M.	
	Tens of B.	Tens of Th. of M.	
	Hundreds of B.	H. of Th. of M.	
5th Period.	Trillions.	Billions.	3rd Period.
	Tens of T.	Tens of B.	
	Hundreds of T.	Hundreds of B.	
6th Period.	Quadrillions.	Thousands of B.	
	Tens of Q.	Tens of Th. of B.	
	Hundreds of Q.	H. of Th. of B.	

Though the invention of our numerical characters is commonly ascribed to the Arabs, there can be little doubt that they owe their origin to the philosophers of India. Arabian writers attribute the honor to the Indians, and though the opinion has been controverted by very learned men, no conclusive, nor even very weighty argument, has been brought forward against it. An attempt has been made to prove that the Arabs derived their characters from the Greeks, and transmitted them to the nations farther east; and, in support of this opinion, an alleged similarity between the figures themselves, and the letters used by the Greeks, to denote numbers, has been insisted on; but this resemblance is altogether imaginary, and would, probably, never have been discovered except by one, anxious to establish a favorite theory. There is a tradition, among the Indians, that their numbers, as well as their knowledge of the science of Geometry, w[illegible] [illegible]m a nation in the north of Tartary. We find nothing, [illegible] to corroborate this opinion, since no traces of an enlighten[illegible] are to be found in that part of the world. But whatever may [illegible] been *the origin* of numbers, there is no obscurity as to the channel, through which we have received them. About the middle of the 7th century, the Arabs overran Persia and Egypt, and very soon extended their conquests over all northern Africa. In the early part of the 8th century, they advanced into Spain, bringing with them the arts and learning, which they had acquired from conquered nations, and likewise, their own system of *numeration*. About the middle of the 11th century, the Arabic characters were introduced into England. On the continent, their use had already become very extensive; and long before the discovery of America, it had become general throughout the civilized world.

We have mentioned that the notation by tens, or the *decimal notation*, had its origin, most probably, in accident; and that numbers might have been made to increase in any other ratio. Near the beginning of the second century, and of course, long before the introduction of the Arabic characters into Europe, a different scheme was actually introduced. It is called the *sexagesimal notation*, from the fact, that its *Radix* is 60. It was introduced by Ptolemy, an Egyptian philosopher, and was probably derived, like the decimal notation, from the East, though the invention is commonly ascribed to him. The Indians and Chinese employ it at the present day

in reckoning time, using periods of 60 years, instead of centuries. Their smaller divisions are similar. We have remnants of it left in the division of time into *minutes* and *seconds*, and likewise, in the division of the circle, into *degrees*, *minutes*, and *seconds*, for which latter purpose, Ptolemy seems to have intended it. This division, in both cases, is often carried to *thirds*, *fourths*, &c. It is not, however, to be supposed, that 60 different *characters* were employed. The *common mode* of writing, that is, the Roman, or Greek, was used as far as sixty, and then the *same notation* was used *over again* for the next higher order, with an accent (′) drawn down at the right. For the third order, two accents were used (″). For the fourth, three, (‴) and so on. These accents answered the same purpose as the Arabic cypher; except that when different orders were written together, the accents were retained, where the cypher would be dropped. This was necessary, because several characters were employed within the same order. Without retaining the accents, we should not be able to determine the dividing point between a higher and a lower order. To illustrate by Arabic characters, 31′23 signifies 31 *sixties* and 23. For some purposes, this notation would be convenient, and for many, inconvenient. These cannot be explained here.

Of the modes of notation, employed before the introduction of the Arabic, all were more or less objectionable. The Roman combined more advantages than any other, and, as it has not yet entirely fallen into disuse, some knowledge of it is absolutely necessary. The following is a brief account of

THE ROMAN NOTATION.

We have mentioned that the decimal scale was probably suggested by the number of the human fingers. But some nations instead of counting by the fingers of *both hands*, only employed those of *one*, and, therefore, fell into the habit of reckoning by *fives*. In other words, they employed *five* as the Radix of their system of Numeration. This is still the case with some uncultivated tribes in Africa, and on this continent. The Roman Notation, though adapted both to the *quinary* and *decimal* scales, is most simply explained by the former, from which, indeed, it seems to have sprung.

Most nations have coincided in expressing *unity* or *one* by a single mark, thus, I. This indeed seems the most natural mode that could have been invented, and this was the mode employed in the Roman scale. This character resembles the letter I. That letter was, consequently, afterwards used in its stead. *Two* was expressed by two marks, II; *three* by three marks, III; and *four* by four marks, IIII. Five employs all the fingers on one hand to express it, and seems therefore to have been made the Radix of the system. A peculiar character was therefore given it, consisting of two lines, joined at one end, thus, V. For this, the letter V was afterwards employed. Six was expressed, of course, by this character and one mark, thus VI; seven, thus VII; eight, VIII; nine, VIIII. As ten is two fives, the character V was used twice to express it, and for convenience, these seem to have been joined, thus ╳. For this, the letter X was substituted. Eleven would then be XI; twelve, XII; thirteen, XIII; fourteen, XIIII; fifteen, XV; sixteen, XVI; seventeen, XVII; eighteen, XVIII; nineteen, XVIIII; twenty, being two tens, was of course, XX; twenty-one, XXI; and so on. Thirty was XXX, and forty XXXX. Then, as a particular character was written for *five*, *fifty* naturally received a particular character likewise. L (afterwards, the letter L,) was, therefore, put for fifty, LX for sixty, LXX for seventy, LXXX for eighty, and LXXXX for ninety. As one hundred is twice fifty, it was naturally expressed by L, used twice. These are placed most conveniently thus, [. This became rounded by writing, to C. Then, two hundred was CC, three hundred CCC, and four hundred CCCC. For the same reason that five and fifty received a particular character, five hundred would probably receive one. Accordingly we find that, as no more figures could be conveniently made with two marks, three were disposed thus, Δ, for five hundred. This afterwards became D by writing. Six hundred was then written DC, seven hundred DCC, &c. As two V's united signified ten, and two L's, one hundred, it was natural to unite two D's for one thousand. It was done thus ᗡD, or more compactly thus CD. This afterwards became M. These, then, are the characters employed by the Romans, for expressing numbers.

Numbers, therefore, came to be expressed, as they are even at the present day, as follows:

One	**I**	**Twenty-one, &c.**	**XXI &c.**
Two	**II**	**Thirty**	**XXX**
Three	**III**	**Forty**	**XL or XXXX**
Four	**IV or IIII**	**Fifty**	**L**
Five	**V**	**Sixty**	**LX**
Six	**VI**	**Seventy**	**LXX**
Seven	**VII**	**Eighty**	**LXXX**
Eight	**VIII**	**Ninety**	**XC or LXXXX**
Nine	**IX or VIIII**	**One hundred**	**C**
Ten	**X**	**Two hundred**	**CC**
Eleven	**XI**	**Three hundred**	**CCC**
Twelve	**XII**	**Four hundred**	**CCCC**
Thirteen	**XIII**	**Five hundred**	**D**
Fourteen	**XIV or XIIII**	**Six hundred**	**DC**
Fifteen	**XV**	**Seven hundred**	**DCC**
Sixteen	**XVI**	**Eight hundred**	**DCCC**
Seventeen	**XVII**	**Nine hundred**	**DCCCC**
Eighteen	**XVIII**	**One thousand**	**M**
Nineteen	**XIX or XVIIII**	**Two thousand**	**MM**
Twenty	**XX**	**1830**	**MDCCCXXX**

It will be observed that four is written IV or IIII. The former method was introduced at a late period, and is not convenient where arithmetical operations, such as Addition and Subtraction, are to be performed with Roman characters. In other cases, it is perhaps to be preferred, as occupying less room. The same may be said of IX, XL, and XC.

We have explained the scale as far as it is in common use at the present day, which is far enough for the ordinary student. It may be well, however, to mention that IↃ is sometimes used for D. When this is the case, additional Ↄs are sometimes annexed *at the right*, and each additional Ↄ *increases the number ten times.* Thus IↃ is five hundred; IↃↃ is five thousand; IↃↃↃ fifty thousand, &c. CIↃ is also put for M, and every additional C Ↄ *at each end*, in this case, *increases the number*, as before, *ten times.* Thus CIↃ is one thousand; CCIↃↃ ten thousand; CCCIↃↃↃ one hundred thousand, &c. A line drawn over a number *increases its value a thousand times.* Thus $\overline{\text{V}}$ is five thousand; $\overline{\text{X}}$ ten thousand; $\overline{\text{C}}$ one hundred thousand; $\overline{\text{M}}$ one million; $\overline{\text{MM}}$ two millions, &c.

Some have supposed that C is taken from the Latin *centum*, which means *one hundred*, and M from *mille*, which means *one thousand*, but, if this *be* true, there is no similar mode of accounting for the use of the other letters.

In the usual mode of explaining this Notation by the decimal scale, the V is supposed to have sprung from a divided X, and therefore to have been naturally employed in expressing half as great a number. So L is thought to have come from a divided C; and IↃ or D from a divided M or CIↃ. It is in favor of this explanation, that there exists no single character for twenty-five, which, on the quinary scale, we ought to have expected. On the whole, however, the Notation seems to have been commenced on the quinary, and continued on the decimal scale, thus forming a mixture of both. In like manner, we find that the Chinese at the present day, employ the *decimal* and *duodecimal* scales in reckoning their cycle, (or recurring period, by which they estimate time,) of 60 years. The same scales were likewise to some extent combined by the ancient Scandinavians.

◆

ADDITION.

MENTAL EXERCISES.

§ VII. 1. Two barrels stand in one place, and two in another. How many are there in all; 2 and 2 are how many?

2. A boy is carrying two pails, and three stand on a bench. How many pails in all? 2 and 3 are how many?

3. Two apples are falling from a tree, and four are lying on the ground. How many are there in all? 2 and 4 are how many?

4. Two pens are in an inkstand, and five lie on the table. How many are there in all? 2 and 5 are how many?

5. John made 3 marks on one leaf of his book, and 6 on another. How many marks did he make? 3 and 6 are how many?

6. His teacher punished him, for soiling the book, by giving him 4 blows on one hand, and 5 on the other. How many blows did he strike him? 4 and 5 are how many?

7. 7 boys laughed at him on one side of the house, when he was punished, and 2 on the other. How many boys laughed? 7 and 2 are how many?

8. One boy has 6 marbles and another has 7. How many have both? 6 and 7 are how many?

9. A boy bought a story book for 10 cents, and another for 6. How many cents did he give for both? 10 and 6 are how many?

10. 7 pins were on a pin-cushion, and a girl put on 9 pins more. How many were on it then? 7 and 9 are how many?

11. 8 boys were playing together, and 5 more came to play with them. How many boys were there in all? 8 and 5 are how many?

12. A man gave 9 dollars for a barrel of flour, and 10 dollars for a firkin of butter. How many dollars did he give for both? 9 and 10 are how many?

13. If an orange is worth 7 cents, and a lemon 4 cents, how many cents are both worth? 7 and 4 are how many?

14. If a pint of filberts is worth 6 cents, and half a pound of figs is worth 9 cents, how many cents are both worth? 6 and 9 are how many?

15. 11 ducks are on the shore of a pond, and 3 are swimming in it. How many ducks are there in all? 11 and 3 are how many?

16. How many are 11 and 5? 11 and 6? 11 and 4? 11 and 2? 11 and 7? 11 and 8? 11 and 10? 11 and 9? 11 and 12? 11 and 11?

17. 10 men are going to market, and 5 are going home. How many men are there in all? 10 and 5 are how many?

18. 10 and 7 are how many? 10 and 3? 10 and 4? 10 and 2? 10 and 6? 10 and 8? 10 and 9? 10 and 10? 10 and 11? 10 and 12? 10 and 13? 10 and 14?

19. How many are 5 and 2? 5 and 7? 5 and 6? 5 and 9? 5 and 12? 5 and 11? 5 and 13? 5 and 14? 5 and 15?

20. How many are 7 and 6? 7 and 7? 7 and 8? 7 and 10? 7 and 9? 7 and 5? 7 and 11? 7 and 12? 7 and 14? 7 and 15?

21. How many are 6 and 6? 6 and 7? 6 and 9? 6 and 10? 6 and 12? 6 and 5? 6 and 8? 6 and 11? 6 and 13? 6 and 15? 6 and 14?

22. How many are 8 and 8? 8 and 7? 8 and 5? 8 and 9? 8 and 6? 8 and 4? 8 and 10? 8 and 11? 8 and 13? 8 and 12? 8 and 14? 8 and 15?

23. How many are 9 and 7? 9 and 4? 9 and 3? 9 and 2? 9 and 5? 9 and 6? 9 and 8? 9 and 9? 9 and 10? 9 and 11? 9 and 12? 9 and 13? 9 and 15? 9 and 14?

24. How many are 4 and 1? 4 and 4? 4 and 3? 4 and 5? 4 and 2? 4 and 7? 4 and 6? 4 and 8? 4 and 10? 4 and 13? 4 and 15? 4 and 17? 4 and 19?

25. Recite the

ADDITION TABLE.

2 and	0 are	2	4 and	0 are	4	6 and	0 are	6	8 and	0 are	8
2	1	3	4	1	5	6	1	7	8	1	9
2	2	4	4	2	6	6	2	8	8	2	10
2	3	5	4	3	7	6	3	9	8	3	11
2	4	6	4	4	8	6	4	10	8	4	12
2	5	7	4	5	9	6	5	11	8	5	13
2	6	8	4	6	10	6	6	12	8	6	14
2	7	9	4	7	11	6	7	13	8	7	15
2	8	10	4	8	12	6	8	14	8	8	16
2	9	11	4	9	13	6	9	15	8	9	17

3 and	0 are	3	5 and	0 are	5	7 and	0 are	7	9 and	0 are	9
3	1	4	5	1	6	7	1	8	9	1	10
3	2	5	5	2	7	7	2	9	9	2	11
3	3	6	5	3	8	7	3	10	9	3	12
3	4	7	5	4	9	7	4	11	9	4	13
3	5	8	5	5	10	7	5	12	9	5	14
3	6	9	5	6	11	7	6	13	9	6	15
3	7	10	5	7	12	7	7	14	9	7	16
3	8	11	5	8	13	7	8	15	9	8	17
3	9	12	5	9	14	7	9	16	9	9	18

[The teacher should ask questions upon the table, promiscuously, after it has been recited, in order to render it necessary for the pupil to have each amount fixed in his mind, independently of every other. The pupil may recite the table *in order*, without deriving any benefit from it, merely from his knowledge of the regular succession of the numbers.]

MENTAL EXERCISES CONTINUED.

1. In one ladder there are 7 rounds, in another 8, and in another 6. How many rounds are there in all? 7 and 8 and 6 are how many?

2. On one end of a table are 5 apples, and on the other 4; and on the floor are 9. How many apples are there in all? 5 and 4 and 9 are how many?

3. James put all his marbles on a bench; 10 have fallen on the floor; 3 are falling; and 9 remain where he put them. How many marbles has James, in all? 10 and 3 and 9 are how many?

4. One boy has 11 cents, another 2, and another 6. How many have all together? 11 and 2 and 6 are how many?

5. A man bought a barrel of flour for 8 dollars, and a load of wood for 3 dollars, and a plough for 7 dollars. How much did he give for all? 8 and 3 and 7 are how many?

6. A boy gave to one of his companions 9 apples, to another 11, and to another 6. How many did he give away to all? 9 and 11 and 6 are how many?

7. William's father gave him 12 cents, his mother gave him 7, and his uncle gave him 14. How many cents had William? 12 and 7 and 14 are how many?

8. A boy gave 10 cents for a top, and 4 cents for a whistle, and 8 cents for some marbles. How many cents did he spend? 10 and 4 and 8 are how many?

9. 11 and 4 and 3 are how many? 5 and 8 and 2? 7 and 6 and 12? 12 and 9 and 2? 3 and 5 and 7? 6 and 9 and 4? 7 and 8 and 9? 5 and 3 and 10?

10. 8 and 9 and 10 are how many? 10 and 11 and 9? 9 and 12 and 11? 8 and 11 and 13? 7 and 10 and 14? 6 and 8 and 17? 12 and 13 and 14?

11. 5 and 15 and 20 are how many? 20 and 7 and 8? 25 and 5 and 10? 22 and 4 and 11? 19 and 20 and 4?

12. 20 and 30 and 40 are how many? 30 and 40 and 10? 20 and 50 and 30? 30 and 60 and 10? 10 and 70 and 20? 40 and 30 and 30?

13. One man owns 3 houses, another 4, another 5, and another 6. How many do they all own? 3 and 4 and 5 and 6 are how many?

14. John has 7 apples, and William 5, and Joseph 9, and James 11. How many have they all? 7 and 5 and 9 and 11 are how many?

15. Some boys went a fishing. One caught 6 fish, another 8, another 4, another 15, and another 12. How many fish did they all catch? 6 and 8 and 4 and 15 and 12 are how many?

16. Seven men went a hunting. One shot 10 birds, another 13, another 8, another 7, another 5, another 9, and another 11. How many did they all shoot? 10 and 13 and 8 and 7 and 5 and 9 and 11 are how many?

17. 9, and 8, and 7, and 6, and 5, and 4, and 3, and 2, and 1, are how many?

18. 10, and 9, and 8, and 7, and 6, and 5, and 4, and 3, and 2, are how many ?

19. 11, and 10, and 9, and 8, and 7, and 6, and 5, and 4, and 3, are how many ?

20. 12, and 11, and 10, and 9, and 8, and 7, and 6, and 5, and 4, and 3, and 2, and 1, are how many ?

21. Add each of these rows of figures, first from the bottom to the top, then from the top to the bottom, then from right to left, and then from left to right.

1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12
5	6	7	8	9	10	11	12	13
6	7	8	9	10	11	12	13	14
7	8	9	10	11	12	13	14	15
8	9	10	11	12	13	14	15	16

[This addition should be made aloud, as should every other calculation in the mental exercises.]

22. How many are 9 and 2 ? 19 and 2 ? 29 and 2 ? 39 and 2 ? 49 and 2 ? 59 and 2 ? 69 and 2 ? 79 and 2 ? 89 and 2 ? 99 and 2 ?

[The teacher should here take 9 and 3, 29 and 3, and so on; after which, he should begin 8 and 2, 18 and 2, &c.; 8 and 3, 18 and 3, &c.; 7 and 2, &c.; 7 and 3, &c., as far as 2 and 2, 12 and 2, 22 and 2, &c.; 1 and 2, &c., up to 91 and 9. This is intended for inexperienced pupils. A few questions are given below, for those in every stage of improvement.]

23. How many are 6 and 6 ? 16 and 6 ? 26 and 6 ? 36 and 6 ? 36 and 7 ? 46 and 7 ? 56 and 8 ? 76 and 8 ? 96 and 8 ? 86 and 9 ? 96 and 9 ?

24. How many are 25 and 9 ? 35 and 9 ? 45 and 8 ? 55 and 8 ? 65 and 7 ? 75 and 7 ? 85 and 4 ? 95 and 4 ?

25. How many are 3 and 4 ? 13 and 4 ? 23 and 4 ? 33 and 4 ? 43 and 5 ? 53 and 5 ? 63 and 6 ? 73 and 7 ? 83 and 8 ? 93 and 9 ? 104 and 10 ?

26. How many are 4 and 2 ? 14 and 2 ? 24 and 2 ? 34 and 2 ? 44 and 3 ? 54 and 4 ? 64 and 5 ? 74 and 6 ? 84 and 7 ? 94 and 8 ? 104 and 9 ?

27. How many are 7 and 4 ? 17 and 4 ? 27 and 5 ? 37 and 5 ? 47 and 6 ? 57 and 6 ? 67 and 7 ? 77 and 8 ? 87 and 9 ? 97 and 9 ?

28. How many are 8 and 2 ? 18 and 3 ? 28 and 4 ? 38 and 4 ? 48 and 5 ? 58 and 5 ? 68 and 7 ? 78 and 7 ? 88 and 9 ? 98 and 9 ? 108 and 8 ? 118 and 8 ?

29. Add these rows of figures, from left to right, and write down the amount of each row.

2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	9	9	9

2	2	3	3	4	4	5	5	6	6	7	7
3	3	3	5	5	5	7	7	7	9	9	9
1	1	1	1	2	2	2	2	8	8	8	8
5	5	5	5	5	7	7	7	7	7	4	4
6	6	6	6	4	4	4	4	2	3	4	5
9	1	8	2	7	3	6	4	5	5	5	5
2	3	1	5	6	9	2	5	5	4	8	9
3	5	2	7	6	3	9	5	4	8	6	2

◆

§ VIII. 1. Copy these rows of figures on your slate, add them up, and set down the amount in figures.

1	2	2	3	7	1	2	1	2	5	2	3
2	7	2	3	7	2	5	9	3	6	3	4
3	6	3	4	6	3	7	8	4	7	5	5
5	5	3	4	6	4	6	7	1	8	7	6
6	4	4	5	3	5	3	4	2	9	8	7
8	3	4	5	4	6	4	5	3	1	6	8

2. In the last example you added rows of units, and set down the amount. Add, now, these rows of tens, and hundreds, and thousands, &c., in the same manner.

10	100	3000	90000	500000	7000000
20	200	4000	90000	600000	8000000
30	300	5000	80000	700000	5000000
40	400	6000	80000	900000	4000000
50	500	7000	70000	500000	7000000
60	600	8000	50000	600000	6000000

3. Here are some simple examples, containing different orders.

	1	2	3	600	6000	900	8000	90000
	10	20	90	3	3	20	204	2000
Ans.	11	22						

4. The following examples are similar.

	70	400	70	8000	70000	60000	700000
	6	60	4	700	3046	4002	3040
	1	3	903	32	403	111	50304
Ans.	77	463					

5. **The following are larger, but are to be added in the same manner.**

50839601
13020134
23120020
13020244
____
Ans. 99999999

100000001
202020202
333333333
121212121
212121212

111111111111
222222222222
100100100100
101010101010
465556465556

1010202030304040
1313020214140303
2000000000000000
1212121201010101
1111000022220000
2222222222222222

12000000001200000000
4012340012345678
230300002100000000
61341122221302020211
3225085634110
10000000000

[The learner will readily understand and perform the above examples. If, however, he should hesitate, let him study, or let the teacher repeat to him, the following explanation.]

In the third row of examples above, 1 ten is added to 1 unit. The ten is a unit of the second order, and should therefore stand in the second place, and the unit is a unit of the first order, and ought to be written in the first place. We will take away, therefore, the cypher on the right of the ten, and in its place will put the unit.

All of the first row of examples are performed in this way, and the same explanation will apply to them.

In the fourth row of examples, 70 or 7 tens are added to 6 units, and 1 unit. The 6 units, and 1 unit, make 7 units, which are written, as before, in place of the 0 in 70. The others are all similar.

[The teacher will perceive, that, thus far in the examples containing several columns, no one column added up, has exceeded 9. He should require his pupils to explain the addition of every column, in the manner given above, using the name of the *order* he is adding. Thus, in the last example, he should begin 1 unit and 8 units are 9 units, which are to be written in the units' place, in the answer. 1 ten and 1 ten are two tens; 2 tens and 7 tens are 9 tens, to be written in the tens' place in the answer, &c.]

It is plainly of no consequence *on which side you begin* to add the examples above, since each column is added separately. It is better, however, to begin *on the right hand*, as will presently be seen. It is, also, very manifest, that as all the units are to be added together, all the tens together, &c., it is most convenient to write the numbers so that units shall stand under units, and tens under tens, &c.

The pupil should be very careful never to add the figures of *different orders* together; for 3 units and 2 tens, are neither 5 units, nor 5 tens. But 3 units and 2 units are 5 units; and 3 tens and 2 tens are 5 tens.

6. Write down 15 and 6, and add them.
15
6 First, add the units together. 5 units and 6 units are 11
— units; that is, 1 ten and 1 unit, written 11. Set this down.
11 There is one ten in the upper number. Bring it down
1 under the tens' place. Now add the tens. 1 ten and 1 ten
— are 2 tens, or twenty. Join this to the 1 unit, and it is 21.
Ans. 21

7. Write 17 and 26, and add them.
17
26 First, add the units. 7 units and 6 units are 13 units;
— equal to 1 ten and 3 units. Set this down. Now add
13 the tens. 1 ten and 2 tens are 3 tens. Set these under
3 the tens. Now add again. 3 tens and 1 ten are 4 tens;
— which, joined with the 3 units, make 43.
43

8. Let the following examples be performed and explained in the same way.

37	65	39	44	37	28	76	52	67	56
54	27	27	46	55	39	18	28	19	25
—	—	—	—	—	—	—	—	—	—
11	12								
8	8								
—	—								
91	92								

9. In like manner, add the following.

46	49	32	12	21	15	21	14	12	21	13
13	15	31	19	32	13	27	15	19	21	14
24	13	13	13	17	23	20	11	23	15	23
—	16	14	14	11	21	12	12	11	26	25
13	—	—	—	12	11	13	32	13	10	12
7				—	—	—	—	—	—	—
—										
Ans. 83										

The learner will see, that, when his units are more than 9, it is impossible to write them in the units' place; because *there are no characters to express them.* Every *even ten,* therefore, he is *obliged* to add with the tens. And if there are any units over, he sets them down in the units' place. Of course, after he has added his units column, a part of the amount goes to the units' place, and a part to the tens' place. In the examples above, these two parts are *both set down.* But this is inconvenient, because it renders, at least, two additions necessary. The following process is better.

10. Add 54 and 39.
54 9 units and 4 units are 13 units; that is, 1 ten and 3 units.
39 The 1, is the part that belongs to the tens' place. Therefore,
— *in mind,* add it with the tens' column. Thus, 1 ten and 3 tens
93 are four tens; 4 tens and 5 tens are 9 tens. The answer is 93.

In the same manner, we add 1 to the hundreds' column, for every ten tens, 1 to the thousands' column for every ten hundreds, and 1 *to any column, or order of figures, for every ten of the next lower order.*

THIS ADDITION OF ONE FOR EVERY TEN TO THE NEXT HIGHER ORDER IS CALLED CARRYING.

If we were to write down the whole amount of any column, whose sum is more than 9, as in the examples above, it would take at least two figures, and sometimes more. Of these, it is plain, that the one on the right hand is the one to be set down, and the other, or others, the number to be *carried*. Thus:

11. Add 375, and 463, and 999, and 888.

375
463
999
888
———
2725

The amount of the units is 25; of course, we set down the 5, and carry the 2. There are 32 tens. Therefore, we set down 2, and carry 3. There are 27 hundreds. We therefore set down 7, and carry 2. As there is nothing to add it to, we put it in the next place, by itself.

[The pupil should, frequently, be exercised in explaining operations in addition, in the manner of the preceding illustrations.].

The following are to be performed, in the manner of the preceding.

12. In an orchard, 19 trees bear cherries, 28 bear peaches, 8 bear plums, and 58 apples. How many trees in all? *Ans.* 113.

13. Four men purchased a field. The first gave 74 dollars; the second, 67; the third, 41; and the fourth, 27. How much did all give? *Ans.* 209.

14. How many times does the hammer of a clock strike in 24 hours, if it strikes regularly from 1 to 12, and then from 1 to 12 again? *Ans.* 156.

15. How many days in the Spring months, containing March, 31 days; April, 30; and May 31? *Ans.* 92.

16. How many in the Winter months, containing December, 31; January, 31; February, 28? *Ans.* 90.

17. How many in the Summer and Autumn months, containing June, 30; July, 31; August, 31; September, 30; October, 31; November, 30? *Ans.* 183.

18. Then, how many days in the year? *Ans.* 365.

19. If it require 650 men to man a 74 gun ship; 475, to man a 44; 350, to man a 36; 275, to man a 32; 200, to man a 20; and 180, to man an 18; how many men will it require to man the whole? *Ans.* 2,130.

20. A man spent 30 years in the United States, 5 years in France, 12 years in Italy, 7 years in Germany, 4 years in the Netherlands, and afterwards returned to the United States, where he lived 26 years. How old was he at his death? *Ans.* 84.

The pupil is now prepared to understand the following rule.

I. WRITE DOWN THE NUMBERS TO BE ADDED, SO THAT THE SAME ORDERS IN EACH, MAY STAND IN THE SAME COLUMN, UNDER EACH OTHER.

II. BEGIN WITH THE FIRST ORDER, ADD EACH COLUMN BY ITSELF, AND, IF THE AMOUNT BE NOT MORE THAN 9, SET IT DOWN UNDER THAT COLUMN.

III. IF THE AMOUNT BE MORE THAN 9, THINK WHAT FIGURES YOU WOULD USE TO WRITE THAT AMOUNT. OF THOSE FIGURES, SET DOWN THE RIGHT HAND ONE, UNDER THE ORDER ADDED. CARRY ALL TO THE LEFT OF THIS, TO THE NEXT HIGHER ORDER.

You have now been learning to perform ADDITION.

You find it is a process, which makes one number out of several. For brevity's sake,

THE SINGLE NUMBER, THUS FOUND, IS CALLED AMOUNT OR SUM.

Then,

ADDITION IS FINDING A SINGLE NUMBER, WHICH SHALL EXPRESS THE AMOUNT OF TWO OR MORE NUMBERS.

+ *between two numbers, or sets of numbers, signifies that they are to be added.*

This sign is commonly called *plus*, which is a Latin word, meaning *more*.

= *between two numbers, or sets of numbers, signifies that they are equal to each other.* Thus, $6+8=14$ signifies that 6 and 8 equal 14.

EXAMPLES FOR PRACTICE.

1. A man bought 4 pieces of cloth, containing, the first, 60 yards; the second, 97; the third, 82; and the fourth, 112. How many yards in all? *Ans.* 351.

2. For the cloth, he paid as follows: for the first piece, 235 dollars; for the second, 384; for the third, 327; and for the fourth, 486 dollars. How much did he give for all? *Ans.* 1,432.

3. A man had a sum of money to pay. He paid at one time, 693 dollars; at another, 75; at another, 1,421; at another, 1,652; and at another, 542. There were 964 dollars left unpaid. What was the whole sum? *Ans.* 5,347.

4. A merchant bought wine to the amount of 437 dollars, and sold it so as to gain 94 dollars. What did he sell it for? *Ans.* 531.

5. There are four bags, containing, the first, 275 dollars; the second, 178; the third, 356; and the fourth, 69. How many dollars in all? *Ans.* 878.

6. A merchant bought at one time, 543 barrels of beef; at another, 407; at another, 152; and at another, 414. How many did he buy in all? *Ans.* 1,516.

7. From the creation of the world to the flood, were 1,656 years; thence, to the building of Solomon's temple, 1,344 years; thence, to the birth of our Saviour, 1,004; thence, to the present time, 1,830. How old is the world? *Ans.* 5,834 years.

8. A man left his estate, at his death, to four children. They paid debts to the amount of 1,476 dollars, and then had 4,768 dollars ch. How much was the whole estate? *Ans.* 20,548 dollars.

9. A man paid debts, as follows: one of 427 dollars; another of 763 dollars; another of 654; another of 500; another of 325; and another of 3,250. How much did he pay? *Ans.* 5,919 dollars.

10. In 1820, the following was the population of the several New-England States. How many in all?

Maine,	298,335	
New-Hampshire,	244,161	
Vermont,	235,764	
Massachusetts,	523,287	
Rhode-Island,	83,059	
Connecticut,	275,248	*Ans.* 1,659,854.

11. How many in all the Middle States, containing severally,

New-York,	1,372,812	
New-Jersey,	277,575	
Pennsylvania,	1,049,398	
Delaware,	72,749	
Maryland,	407,350	*Ans.* 3,179,884.

12. The following States, called the Southern States, contained severally, the following numbers. How many in all?

Virginia,	1,065,366	
N. Carolina,	638,829	
S. Carolina,	490,309	
Georgia,	340,989	
Kentucky,	564,317	
Tennessee,	422,813	
Alabama,	127,901	
Mississippi,	75,448	
Louisiana,	153,407	*Ans.* 3,879,379.

13. How many inhabitants in the following States and Territories?

Ohio,	581,434	
Indiana,	147,178	
Illinois,	55,211	
Missouri,	66,586	
Arkansas Ter.	14,273	
Michigan Ter.	8,896	
Dist. of Columbia,	33,039	*Ans.* 906,617.

14. Then, how many inhabitants were there in the whole United States, in 1820? *Ans.* 9,625,734.

The best method of ascertaining whether you have performed your addition correctly, is, to

LOOK OVER THE EXAMPLE CAREFULLY A SECOND TIME, ADDING THE COLUMNS IN A REVERSED ORDER; THAT IS, DOWNWARDS, IF YOU FIRST ADDED THEM UPWARDS, AND UPWARDS, IF YOU FIRST ADDED THEM DOWNWARDS.

The ascertaining whether an operation in Arithmetic is correctly performed, is called *proving the operation*, and the process by which this is done, is called PROOF.

EXAMPLES FOR PRACTICE.

1. A man owns five farms. The first is worth 16,750 dollars; the second, 12,875; the third, 4,387; the fourth, 9,321; and the fifth, 6,223 dollars. What is the value of all?

2. A man possesses property to the amount of 11,764 dollars. He draws a prize in a lottery of 20,000; he obtains by marriage, 8,650; his uncle dies and leaves him 14,825; in a speculation he gains 5,346; and by his regular trade, he acquires 7,432 dollars. How much has he in all?

3. Add the following numbers: 1,827,554; 9,200,305; 723; 5; 68; 23; 537,452; 12,900,020; 15; 804.

4. Add 12; 37,853,000,503; 2; 91; 16,020,755; 81; 307; 205; 7,033; 15,000,333; 16,275,850,905,306.

5. Add 555,777,333,222,111; 888,222,666,444,999; 678,678,678, 678,678; 324,324,324,324,324; 123,123,123,123,123; 987,987,987, 987,987; 111,222,333,444,555.

6. Add 891,011,123; 12,765,892; 987,654; 321,234; 12,621; 13,789; 15,910; 64; 7,835,876; 2,867,955.

7. Add 272; 5,004; 10,000; 20,000,002; 59; 892,736,552; 11,111,222; 901; 87; 5,600,222.

8.

123456789
25792579
3412345678
987654
34343434
567891010

9.

1599513456
2349765001
330022000
889944
12345678
976428735

10.

31456789000
343994001
1000012345
23455500018
932418653032
70031

11.

813000234001219
300387543210
1216393876345 6
2345246
12343498363
9865634003031

12.

1314151617181921 22
2345670000127862 09
1357914812111222 34
8765498763000124 57
2726357943176291 85
7676989834013502 18

13.

123456789123456789123456789
100020003000400050006000789
22203330444055506660777000
1000200030000400050 00666
9878886565432314147989220
800900300400003 0085
3456287195432783463 9385
134614783689000023641
5555333 33
14189634197819

OBSERVATIONS ON ADDITION, FOR ADVANCED PUPILS.

§ IX. The rule directs, that units of the same order should be written under each other, in the same column, before the process of adding is commenced. But it is manifestly of little importance, what *arrangement* is adopted, if we only *add together units of the same orders*, and *avoid adding* those of *different orders* to each other. The arrangement prescribed by the rule, is adopted, because most convenient, since it brings as near together as possible, the numbers to be added to one another.

The rule directs to begin with the units' order. That this is not necessary, the following example will show.

Add
2357
8468
9116
———
19941

2357
8468
9116
———
12.
19...
21
8..
———
19941

Add, first the tens; then the thousands; then the units; and then the hundreds. Then add these amounts, as they stand.

It will be seen that the answer is the same, as when the numbers are added by the rule. The example, however, shows that it would be impossible to *carry*, *mentally*, unless the addition were to commence with the first order. Hence the advantage of following the rule.

The learner will perceive, that, in order to add with ease, it is necessary to commit to memory an Addition table as far as 9; that is, a table containing the sums of all the possible combinations of two digits. For, thus far, a separate character is used for every number. Numbers greater than 9, are expressed by more than one figure, and the addition is made in each order with as much simplicity as in that of units. Now if we could not add the number expressed by any single figure, *all at once*, we should be obliged to add unit after unit, until we arrived at the number of units, expressed by the figure. This is the manner in which children always add. For this reason, they often count on their fingers, in order to ascertain when they have added the proper number of units. All uneducated persons do the same.

These remarks go to show the advantage of our scheme of notation. If we had an independent name and character for every number, we should find it very difficult to perform examples, where large numbers were concerned, and we should either be obliged to commit to memory long and tedious addition tables, or we should be under the necessity of adding unit after unit, as children do. With our present scheme of notation, the largest numbers are managed as easily as units under nine. If the ratio of increase towards the left hand had been twelve fold, instead of ten fold, it would have been necessary to extend our addition tables as high as 11. If it had been thirty fold, it would have been necessary to continue them to 29, &c. Of course, we may extend them farther than necessity requires, if we choose. Our common tables extend to 12.

It only remains that we consider the mode of proof. No method can be devised, which is a perfect verification of the accuracy of the operation. By none of those, which have been proposed, will an error be *necessarily* detected, though there is a strong *probability* that it may.

The most natural mode of proving an operation, is to reverse the process, and go back, from the final result, to the starting point. Hence, the natural mode of proving Addition, is by Subtraction. As the student, who peruses these observations, is supposed to be acquainted, at least, with the ground rules, we will explain this method, a little more fully. If two numbers be added, we obtain a third equal to both together. If from this sum then, we take away, or *subtract*, one of the numbers added, the remainder ought to be the other. This affords us an easy proof, when but two numbers are added. But it usually happens, that several are to be united into one sum. It would be a tedious process, to subtract these, one by one, from the sum; and hence the proof by subtraction, is not used. We might, it is true, add the given numbers into *two sums*, and take one of these from the total. The remainder would be the other: But there are easier modes than this.

We will briefly mention all, that have been suggested. The first, and simplest, is, to add the numbers over again in the same manner as at first. But, if there was an error, in the first process, we shall be likely to fall into the same error again; because all the numbers recur in the same order of succession.

It is better, therefore, to reverse the order, as directed in the last section. Yet, in this case, we cannot *positively* assert, that we have made *no error*; though, if the two results agree, it is very probable that the addition has been correctly performed.

A third method is, to divide the example into parts, and to find the amount of each part, separately. Then, add these several amounts, and, if the first result was correct, and *this is correct likewise*, the two will agree. Thus,

Add	23586	
	5432	
	97630	
	358946	
		485594
	325	
	7806	
	39768	
		47899
	305	
	4007	
	389058	
		393370
	5786116	
	91234	
	3608	
	70365	
		5951323
	6,878,186	6,878,186

In this example, we made, by way of proof, four separate additions; dividing the numbers added, into as many different portions. Adding the results, thus obtained, we have a total, which agrees with that first found. Whence we conclude the operation right. This method of proof is the same, in principle, with that found in common Arithmetics; viz., cut off the upper, or lower line, add the other figures, and, to the result, add the line cut off.

It may be objected to this proof, as to the one first mentioned, that the recurrence of the numbers in the same order, renders it liable to error. This objection, however, is not very strong, when the first row is cut off, since, though the others recur in the same order, each figure is added to a *different number*, from that to which it was added in the first process. Thus, in the example above, if we begin at the bottom, 8 is added to 5, making 13; 4 to 13, making 17, &c. If in the proof, the first row be cut off, 4, which was before added to 13, is added to 8, making 12; 6, which was before added to 17, is added to 12, making 18, and so on; each figure, in the proof, being added to a different number, from that to which it was added in the first process. Thus, the chance of error, on account of the figures recurring in the same order, is very small. For, if we made a mistake in adding 4 to 13, it is very improbable that we shall make the *same* mistake in adding it to 8; and so, in the other cases.

Another method is, to make a second Addition, arranging the numbers differently; placing, for example, the number which was uppermost, in the middle, and that which was in the middle, at the top or bottom; so as to avoid adding them twice in the same order of arrangement. This is a very good method, but often long and tedious.

Another method is, to add each column, and set down its whole amount separately, and then to add the amounts as they stand, thus,

$$\begin{array}{r l}
9999 & \\
8888 & \\
7777 & \\
2345 & \\
\hline
\left.\begin{array}{r} 26\ldots \\ 27.. \\ 28. \\ \cdot 29 \end{array}\right\} & \text{Partial sums.} \\
\hline
29009 & \text{Total amount.}
\end{array}$$

But, after all, it is better to avoid errors by care and attention, than to detect them after they are made. If, however, any mode of proof be employed, there is one, which, for ingenuity, certainty, and expedition, is superior to any of those which have been mentioned. But, as it is somewhat complex in principle, it was thought proper to defer its insertion to this place. It is called, *the proof by casting out of nines*, or, more simply, *the proof by nines*. Before proceeding to it, however, we will attend to a few simple properties of numbers.

Art. i. 1. Two men start together, and travel the same way, one at the rate of 10, the other of 9 miles, an hour. At the end of one hour, the former will be 10, the latter, 9 miles from the starting point. Of course, they will be 1 mile apart. If they were, then, both to proceed at the rate of 9 miles an hour, they would always continue 1 mile apart. But, during the second hour, the forward man goes 10 miles, instead of 9, and, consequently, makes the distance between them a mile greater. So that in 2 hours, they are 2 miles apart. For the same reason, in 3 hours, they are 3 miles apart; in 4 hours they are 4 miles apart; and so on. Here we see, that each hour makes one mile more of distance between them. Therefore, *the number of miles, between them, will always be equal to the number of hours.*

2. I wish to measure a stick of timber. I use a pole 10 feet long, and make a mark at the end of every ten feet I measure. Another man follows me, using a pole 9 feet long. He, likewise, makes a mark at the end of every *nine* feet. As his pole is 1 foot shorter than mine, his first mark will be one foot behind mine. If he

should then take a 10 foot pole, every one of his marks would be one foot behind my marks. But, as he keeps on measuring with his 9 foot pole, his second mark will be 2 feet behind mine; his third, 3 feet; his fourth, 4 feet; and so on. Thus, at every measurement, his mark falls a foot farther behind mine. Therefore, *our marks will be always just as many feet apart, as the number of measurements.*

3. From this illustration it will be very clear, that as 9 is one less than 10, so two 9s are 2 less than two 10s; three 9s, 3 less than three 10s; and so on. For one 10 consists of a 9 and a unit. And two 10s are one 10 and one 10. Then, if each of these 10s contain a 9 and a unit, both of them must contain two nines and two units. So, likewise, three 10s contain three 9s and 3 units; four 10s, four 9s and 4 units; *and any number of 10s contains just as many 9s as there are 10s, and just as many units besides.*

4. Therefore, if from one 10 I take 1 unit, one 9 remains; if from two 10s I take away 2 units, two 9s remain; if from three 10s, 3 units, three 9s remain; and, *if from any number of 10s, I take the same number of units, the same number of 9s remains.* That part of this idea which we wish to have *distinctly remembered,* may be concisely expressed as follows:

If from ANY NUMBER OF TENS, *the* SAME *number of* UNITS *be taken away,* THE REMAINDER WILL CONSIST OF EVEN 9s.

ART. II. It appears (I. 4) that if from ten 10s, that is, from 100, I should take away 10 units, ten 9s would remain. Now these 10 units taken away, make one 10, which is equal to one 9 and 1 unit. Therefore, in the ten 10s or 100, there are ten 9s, and one 9 and 1 unit; that is, eleven 9s and 1 unit. Therefore, if from 100, I take away one unit, a number of even 9s, that is, eleven 9s, will remain. If, from 200 I should take 2 units, a number of even 9s would, likewise, remain. For 200 is 100 and 100; and if one unit be taken from each hundred, it will leave a number of even 9s in each case. But taking one unit from each, is taking 2 units from the whole. Therefore, if 2 units be taken from 200, a number of even 9s remains; if 3 units be taken from 300, a number of even 9s remains; and, *if any number of units be taken from the same number of hundreds, a number of even 9s remains.* Hence,

If from ANY NUMBER OF HUNDREDS, *the* SAME *number of* UNITS *be taken away,* THE REMAINDER WILL CONSIST OF EVEN 9s.

ART. III. 1. It appears (II.) that ten hundreds, or 1,000 consist of even 9s and 10 units. But these 10 units make one 10, which is equal to one 9 and 1 unit. Therefore, 1,000 consists of even 9s, and 1 unit. Hence, if from 1,000, I should take away 1 unit, a number of even 9s would remain. Then, as in hundreds, it may be shown, that, if 2 units be taken from 2,000, a number of even 9s will remain; if 3 units from 3,000; 4 units from 4,000; or, *if any number of units be taken from the same number of thousands, a number of even 9s will remain.* And,

If from ANY NUMBER OF THOUSANDS, *the* SAME *number of* UNITS *be taken away, the* REMAINDER WILL CONSIST OF EVEN 9s.

2. And, by the same method of proof it may be shown, that,

If from ANY NUMBER OF UNITS OF ANY ORDER, *the* SAME *number of* SIMPLE UNITS *be taken away*, THE REMAINDER WILL CONSIST OF EVEN 9s.

ART. IV. 1. Let there be given any number, as 324.

This consists of three hundreds = 300
And two tens = 20
And four units = 4

Now it has been shown, (III. 3.) that,

300 contains even 9s and 3 units,
And 20 contains even 9s and 2 units,
And 4 contains no 9s and 4 units.

The whole number, 324, then, consists of even 9s, and 3 units, and 2 units, and 4 units.

It is plain, then, that if 3 units and 2 units and 4 units, when added together, make 9, or any number of even 9s, the number 324, will consist of even 9s, with no remainder. 3 and 2 and 4 are 9. Therefore, 324 consists of even 9s, without remainder. But 3 and 2 and 4 are the figures which make 324. Therefore,

IF THE SUM OF THE FIGURES WHICH COMPOSE A NUMBER, CONSISTS OF EVEN 9s, THE NUMBER ITSELF CONSISTS OF EVEN 9s. Also,

IF THE SUM OF THE FIGURES WHICH COMPOSE A NUMBER, DOES NOT CONSIST OF EVEN 9s, THE NUMBER ITSELF DOES NOT CONSIST OF EVEN 9s, BUT LEAVES THE SAME REMAINDER AS THAT LEFT BY THE SUM OF ITS FIGURES.

Let there now be given an example in addition, as the following. Add

178	7
543	3
821	2
——	–
1542	3

After adding, draw a line up and down, on the right. Add the figures 1, 7 and 8, which compose the upper number. Their sum is 16=9+7. Reject the 9, and set down the 7, opposite the upper number, and outside the line. Add the figures of the second number, reject and set down, as before. Do the same with the third number. We know, therefore, that the upper number consists of even 9s and 7; the second, of even 9s and 3; and the third of even 9s and 2. Of course, if 7 and 3 and 2 added together, are even 9s, the sum of the numbers ought to consist of even 9s. But 7+3+2 =12=9+3. Therefore, the sum of the numbers ought to be even 9s, and 3 over. Set down this 3 opposite the amount, 1542.

It is plain, that if the example is correctly performed, this amount ought to contain just as many 9s as the numbers added, with the same excess, or remainder. What this remainder is, we shall find by adding the figures 1, 5, 4 and 2, which compose the amount, and rejecting the 9s from their sum. The sum is 12,=9+3. Reject the 9 and 3 remains. This remainder is exactly what it was shown above that it ought to be. Hence, we conclude, that the amount is correct, and that the addition has been correctly performed.

From what has been said, we derive the following rule for the proof of addition.

I. ADD THE FIGURES COMPOSING EACH OF THE GIVEN NUMBERS.

II. REJECT OR CAST OUT THE 9s FROM EACH SUM, AND SET DOWN THE EXCESSES OPPOSITE THE SEVERAL NUMBERS.

III. ADD THESE EXCESSES, REJECT THE 9s FROM THEIR SUM, AND PLACE THE REMAINDER OPPOSITE THE AMOUNT.

IV. ADD THE FIGURES COMPOSING THE AMOUNT, REJECT THE 9s, AND, IF THE REMAINDER, THUS FOUND, AGREES WITH THE LAST, THE OPERATION MAY BE CONSIDERED AS CORRECTLY PERFORMED.

NOTE. *It will be found most convenient and expeditious, to reject the* 9*s* WHILE ADDING, *that is, as often as you obtain an amount of* 9 *or more, reject the* 9, *and proceed with the remainder, if any, as before.*

This rule may, *possibly*, make a false result appear correct. For, if we make one figure of the amount a unit or two too large, and another, as much too small, the sum of the figures, and of course, its excess of 9s, will remain the same. Or if there be any number of errors made, such, that half their amount shall be on the side of excess, and the other half on the side of deficiency, the excess of 9s will remain unaltered, and, of course, the result will *seem* correct. But it is very improbable that errors, *exactly counterbalancing each other*, will be made, in the same operation; and hence, this mode of proof may, perhaps, be better depended upon than any other.

It is, perhaps, needless to remark that errors may sometimes be made in *proving*, as well as in performing the first operation; so that *no mode of proof* is a *certain* test of the accuracy of a result.

The property of the number 9 on which the above mode of proof depends, is not an *essential* property of the number; that is, it is not a property which belongs to it from its nature, but it is one which results from *its place in the scheme of notation*. It would belong to 11, if we were to reckon by twelves; and to 29, if we were to reckon by thirties. In other words, it always belongs to the number, which is one less than the *radix* of the system. The same property, also, belongs to the number 3; because 3 is contained in 9 an exact number of times. Of course, any number of 9s will be an exact number of 3s. We may, therefore, prove addition *by casting out of* 3*s*, in the same manner as by casting out of 9s.

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MULTIPLICATION.

MENTAL EXERCISES.

§ X. 1. A shoemaker can make one pair of shoes in a day. How many can he make in 2 days? 2 times 1, or twice 1 are how many?

2. A cabinet maker makes two tables in a day. How many can he make in two days? 2 times 2, or twice 2 are how many?

3. If one inkstand has 3 pens in it, how many pens have two inkstands? 2 times 3 are how many?

4. If one camel has two humps on his back, how many humps have 4 camels? 4 times 2 are how many?

5. One boy has 3 apples. How many apples have 5 boys? 5 times 3 are how many?

6. If one chair has 4 legs, how many legs have 6 chairs? 6 times 4 are how many?

7. A window has 5 rows of panes, and 7 panes in each row. How many panes of glass in the whole window? 5 times 7 are how many?

8. One boy has 6 marbles. How many marbles have 8 boys? 8 times 6 are how many?

9. Three boys have 7 cents apiece. How many cents have all together? 3 times 7 are how many?

10. One boy has 5 school books. How many books have 4 boys? 4 times 5 are how many?

11. In one peck are 8 quarts. How many quarts in 3 pecks? 3 times 8 are how many?

12. One tree produced 6 bushels of apples. How many bushels would 5 trees produce? 5 times 6 are how many?

13. Two oranges cost 6 cents apiece. What did both cost? 2 times 6, or twice 6 are how many?

14. At 12 cents a pound, how much will 3 pounds of raisins cost? 3 times 12 are how many?

15. A chess board has 8 rows of squares, and 8 squares in a row, How many squares on the board? 8 times 8 are how many?

16. Five men drank a glass of ale apiece, and paid 3 cents a glass. How much was paid in all? 3 times 5 are how many?

17. If one lemon cost 4 cents, what will 2 cost? What will 3 cost? What will 4? 5? 6? 7? 8? 9? 10?

18. At 3 dollars a barrel, what will 2 barrels of cider cost? What will 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? 13? 14? 15? 16?

19. At 5 cents apiece, what will 2 story books cost? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

20. How many are 3 times 5? 4 times 5? 6? 7? 8? 9?

21. Repeat the

70 60 90

MULTIPLICATION TABLE.

2	times	0	are	0	5	times	0	are	0	8	times	0	are	0	11	times	0	are	0
2	×	1	=	2	5	×	1	=	5	8	×	1	=	8	11	×	1	=	11
2		2		4	5		2		10	8		2		16	11		2		22
2		3		6	5		3		15	8		3		24	11		3		33
2		4		8	5		4		20	8		4		32	11		4		44
2		5		10	5		5		25	8		5		40	11		5		55
2		6		12	5		6		30	8		6		48	11		6		66
2		7		14	5		7		35	8		7		56	11		7		77
2		8		16	5		8		40	8		8		64	11		8		88
2		9		18	5		9		45	8		9		72	11		9		99
2		10		20	5		10		50	8		10		80	11		10		110
2		11		22	5		11		55	8		11		88	11		11		121
2		12		24	5		12		60	8		12		96	11		12		132

3	times	0	are	0	6	times	0	are	0	9	times	0	are	0	12	times	0	are	0
3	×	1	=	3	6	×	1	=	6	9	×	1	=	9	12	×	1	=	12
3		2		6	6		2		12	9		2		18	12		2		24
3		3		9	6		3		18	9		3		27	12		3		36
3		4		12	6		4		24	9		4		36	12		4		48
3		5		15	6		5		30	9		5		45	12		5		60
3		6		18	6		6		36	9		6		54	12		6		72
3		7		21	6		7		42	9		7		63	12		7		84
3		8		24	6		8		48	9		8		72	12		8		96
3		9		27	6		9		54	9		9		81	12		9		108
3		10		30	6		10		60	9		10		90	12		10		120
3		11		33	6		11		66	9		11		99	12		11		132
3		12		36	6		12		72	9		12		108	12		12		144

12

4	times	0	are	0	7	times	0	are	0	10	times	0	are	0	13	times	0	are	0
4	×	1	=	4	7	×	1	=	7	10	×	1	=	10	13	×	1	=	13
4		2		8	7		2		14	10		2		20	13		2		26
4		3		12	7		3		21	10		3		30	13		3		39
4		4		16	7		4		28	10		4		40	13		4		52
4		5		20	7		5		35	10		5		50	13		5		65
4		6		24	7		6		42	10		6		60	13		6		78
4		7		28	7		7		49	10		7		70	13		7		91
4		8		32	7		8		56	10		8		80	13		8		104
4		9		36	7		9		63	10		9		90	13		9		117
4		10		40	7		10		70	10		10		100	13		10		130
4		11		44	7		11		77	10		11		110	13		11		143
4		12		48	7		12		84	10		12		120	13		12		156

22. If the interest of one dollar be 6 cents a year, what will be the interest of 2 dollars, for the same time? of 3? of 4? of 5? of 6? of 7? of 8? of 9? of 10? of 11? of 12?

23. If the interest of a dollar be 6 cents for 1 year, what will it be for 2 years? for 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

24. At 6 dollars a week, what cost 3 weeks board? 5 weeks? 7 weeks? 9? 11? 13?

25. Two men start from Hartford towards New-Haven at the same moment. One goes 7 miles an hour, the other, 3 miles an hour. How far apart are they at the end of one hour? 2 hours? 3 hours? 6? 9? 8? 7? 11? 12?

26. Two men leave Hartford at the same moment. One goes north, and the other south. One travels 4 miles an hour, and the other 2 miles an hour. How far apart are they at an hour's end? How far in 2 hours? 3? 4? 5? 6? 7? 8? 9? 12? 11?

27. If 3 men can dig a ditch in 1 day, how many days would one man be doing it?

28. If 3 men can mow a field in 2 days, how many days would 1 man be doing it?

29. If 7 men can build a wall in one day, how many days would it take one man to do it?

30. If 4 men can make a fence in 3 days, how long would 1 man be making it?

31. If 5 men can do a piece of work in 7 days, in how many days could 1 man do it?

32. If a barrel of cider will last 10 men 4 weeks, how many weeks will it last 1 man?

33. If 8 men can do a piece of work in 5 days, how many men can do it in 1 day?

34. If 7 men can do a piece of work in 3 days, how many men can do it in 1 day?

35. If 9 men can live on a quantity of provision 6 weeks, how many men could live on the same provision 1 week?

36. A man purchased 8 sheep at 4 dollars apiece; 5 barrels of cider, at 2 dollars a barrel; 4 yards of cloth, at 1 dollar a yard; and 3 bushels of apples, at 3 dollars a bushel. How much did they all come to?

37. A lady bought 3 yards of ribbon for 6 cents a yard; 1 yard of silk for 50 cents; 6 needles at 1 cent apiece; 1 paper of pins at 12 cents; and 1 pair of scissors at 30 cents. How much did they all come to?

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§ XI. Let the pupil write the following exercises on his slate.

1. A man bought 5 pounds of beef at 6 cents a pound. How much did he give for the whole?

PROCESS.

6
6
6
6
6
—
30

Here we set down 6, the price of a pound, 5 times, and add up the numbers. The amount is 30. But we know by the table that 5 times 6 are 30. Therefore, if we put down the 6 *once*, with the 5 under it, we may write down the answer immediately. Thus,

6
5
—
30

The whole process is performed, by saying 5 times 6 are 30.

Thus we see, that examples of this kind may be performed by *addition*, and likewise by a shorter process.

Let the pupil perform, by both these methods, the following.

2. At the rate of 7 miles an hour, how far can I travel in 8 hours?
3. At 9 dollars a barrel, what will 7 barrels of flour cost?
4. At 3 dollars a ream, what will 6 reams of paper come at?
5. At 5 cents a pound, what cost 4 pounds of pork?

Thus far, we have been repeatedly adding numbers, consisting of *one figure* only. Let us now try an example in which larger numbers are concerned.

6. At 75 cents a bushel, what will 4 bushels of salt cost?

Here we have a number, consisting of 7 tens and 5 units, to be repeated 4 times.

By addition, the process would be as follows:

That is, 75 added 4 times to itself, makes 300.

75
75
75
75
—
300

The same may be performed as follows:

Here we say, 4 times 5 units are 20 units, and set down the 20. We then say, 4 times 7 tens are 28 tens, and set dow nthe 28, with the 8, of course, in the tens' place. We then add the two together, and the answer is 300, as before.

75
4
—
20
28
—
300

7. At 58 cents a yard, what will 6 yards of cloth cost? A. 348 *cts.*

The pupil will see that he may *carry*, as in Addition. Thus,

8. At 18 cents a pound, what cost 8 pounds of raisins? 8 times 8 units are 64 units, equal to 6 tens and 4 units. The 4 belongs to the *units' place*. Reserve the 6 tens, *in the mind*. 8 times 1 ten are 8 tens. To the 8 tens add the 6 tens reserved, and the amount is 14 tens, or 1 hundred, and 4 tens. The answer, then, is 1 hundred, 4 tens, and 4 units, or 144.

$$\begin{array}{r}18\\ 8\\ \hline 144\end{array}$$

[The pupil should frequently be required to explain his processes, in the manner of the above illustrations.]

9. In one hogshead are 63 gallons. How many gallons in 7 hogsheads?

$$\begin{array}{r}63\\ 7\\ \hline 441\end{array}\ \textit{Ans.}$$

10. A man has 7 lots, each containing 54 acres of ground. How many acres in all? *Ans.* 328.

11. A man employed 27 laborers, and gave them 4 dollars a week. How much did he give the whole a week? *Ans.* 108.

12. At 5 dollars a firkin, what cost 39 firkins of butter? A. 195.

You see that if we were not to employ this short process in performing these examples, it would be necessary to use Addition. We have, then, found a means of performing Addition more rapidly than by the ordinary mode. This is called MULTIPLICATION.

You see, also, that Multiplication consists in *repeating the* SAME *number*; that is, *in adding the* SAME NUMBER TO ITSELF. Then,

MULTIPLICATION IS A CONCISE MODE OF PERFORMING ADDITION, WHEN THE NUMBERS TO BE ADDED ARE EQUAL. *Or,*

MULTIPLICATION IS FINDING A NUMBER, EQUAL TO A GIVEN NUMBER OF REPETITIONS OF SOME OTHER NUMBER.

You see, further, that two numbers are always given you; one *to be repeated*, and another, which *shows how often it is to be repeated.*

THE NUMBER TO BE MULTIPLIED, OR REPEATED IS CALLED THE MULTIPLICAND.

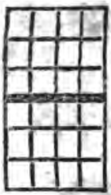
THE NUMBER WHICH SHOWS HOW MANY REPETITIONS ARE TO BE MADE, IS CALLED THE MULTIPLIER.

THE RESULT OBTAINED FROM MULTIPLICATION, IS CALLED THE PRODUCT.

Whenever two or more numbers are multiplied together, each of them is called a factor.

Then, in Multiplication, BOTH MULTIPLIER AND MULTIPLICAND ARE CALLED FACTORS.

× *between two numbers signifies that they are to be multiplied together.* Thus, 6×5=30, signifies that 6 multiplied by 5, equals 30.



Here is a window. A person wishes to discover how many panes of glass are contained in it. He counts the *upper row of panes*, and finds that there are 4 panes. He then counts the number of *rows*, and finds 6 rows. Then he says, 6 rows of panes, with 4 panes in each row, make up all the panes in the window. Therefore, there are 24, because 6 times 4 are 24. Another person counts the panes in *one row up and down*, and finds 6 panes. He counts the rows, and finds 4 rows. Then he says, 4 rows with 6 panes in a row, make up all the panes in the window. Therefore, there are 24 panes, because 4 times 6 are 24.

Four times 6, then, is equal to 6 times 4; because either is equal to all the panes in the window, that is, to 24.

Hence it appears that, IN PERFORMING MULTIPLICATION, EITHER FACTOR MAY BE MADE THE MULTIPLICAND, WITHOUT ALTERING THE PRODUCT.

For convenience, however, we usually place the larger number uppermost.

EXAMPLES FOR PRACTICE.

1. What will 2 wagons cost at 23 dollars apiece? *Ans.* 46 dolls.

2. What cost 3 head of cattle at 17 dollars a head? *Ans.* 51 dolls.

3. What will 18 pounds of pork cost at 4 cents a pound? *Ans.* 72 cts.

4. What cost 21 barrels of cider at 3 dollars a barrel? *Ans.* 63 dolls.

5. At 5 dollars a yard, what cost 27 yards of cloth? *Ans.* 135 dolls.

6. At 6 cents per pound, what cost 32 pounds of beef? *Ans.* 192 cts.

7. At 7 dollars a barrel, what cost 35 barrels of flour? *Ans.* 245 dolls.

8. At 8 dollars a hundred weight, what cost 43 hundred weight of sugar? *Ans.* 344 dolls.

9. At 9 dollars a barrel, what cost 653 barrels of flour? *Ans.* 5,877 dolls.

10. A man bought 23 kegs of brandy, each containing 9 gallons. How many gallons did he buy? *Ans.* 207 galls.

11. He paid for it at the rate of 8 cents a quart. How much did it come to, allowing 4 quarts to the gallon? *Ans.* 6,624 cts.

NOTE. As 100 cents make a dollar, the pupil will readily tell how many dollars are contained in any number of cents. For all the even hundreds will be dollars; and hence, if we cut off the two lower orders, viz. *tens* and *units*, they will be cents, and all to the left will be dollars. According to this principle, the answer in the 5th example is 1 dollar 92 cents; and in the last, 66 dollars 24 cents.

12. A man bought 17 barrels of ale at 9 dollars a barrel; and 25 barrels of flour at 7 dollars a barrel; and 9 pieces of broadcloth, each piece containing 47 yards, at 6 dollars a yard. What did it all come to? *Ans.* 2,866 dolls.

13. A man bought 15 pounds of beef at 7 cents a pound; 8 pounds of sugar at 18 cents a pound; 9 yards of cotton cloth at 22 cents a yard; and 3 pails at 62 cents apiece. What did the whole amount to? *Ans.* 6 dolls. 33 cts.

14.	365×3	*Ans.* 1,095	19.	74,635×3	*Ans.* 223,905
15.	273×8	2,184	20.	53,894×4	215,576
16.	786×3	2,358	21.	83,528×5	417,640
17.	1,338×5	6,690	22.	123,456×8	987,648
18.	2,487×9	22,383	23.	13,052,068×5	65,260,340
24.	12,345,006,789×7	*Ans.* 86,415,047,523			
25.	236,120,013×5	1,180,600,065			

Thus far, none of our multipliers have exceeded 12; and from observation, we derive the following rule.

I. PLACE THE MULTIPLIER UNDER THE MULTIPLICAND.

II. BEGINNING WITH UNITS, MULTIPLY EACH FIGURE OF THE MULTIPLICAND BY THE MULTIPLIER, AND CARRY AS IN ADDITION.

[When the pupil employs technical language, the teacher should frequently require him to explain its meaning. Thus, he should inquire the meaning of *multiplier, multiplicand, &c.*]

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§ XII. 1. At 255 cents per pound, what cost 24 pounds of Indigo?

Here our multiplier is more than 12. Therefore, we cannot proceed by the last rule.

But we know that 24 is twice as much as 12. Therefore, 24 *times any number* will be twice as much as 12 *times the same number.*

By the last rule, 12 times 255=3,060.

But 24 times 255=*twice as much*, or 2 times 3,060.

By the last rule again, 2 times 3,060=6,120.

Therefore, 24 times 255=6,120 *Ans.*

2. At 16 dollars a hogshead, what cost 472 hogsheads of molasses? 16 is 4 times 4. Therefore, 16 *times* any number will be 4 times as much as 4 *times the same number.*

By the last rule, 4 times 472=1,888.

But 16 times 472=4 *times as much*, as 4 times 1,888.

By the last rule, again, 4 times 1,888=7,552.

Therefore, 16 times 472=7,552 *Ans.*

Thus we multiply by a *larger number*, by multiplying *successively by two smaller ones.*

But the two smaller ones must be such that their product shall be just equal to the larger one; that is, *the two smaller numbers must be factors of the larger one.*

THE FINDING OF TWO OR MORE NUMBERS, WHOSE PRODUCT SHALL BE EQUAL TO A GIVEN NUMBER, IS CALLED RESOLVING THE GIVEN NUMBER INTO FACTORS.

After thoroughly understanding the foregoing, let the pupil perform the following

EXAMPLES FOR PRACTICE.

3. At 21 dollars a hogshead, what cost 284 hogsheads of molasses? *Ans.* 5,964 dolls.

4. What cost 28 bales of cotton at 75 dollars a bale? *Ans.* 2,100 dolls.

5. If one cask of brandy cost 15 dolls., what cost 321? *Ans.* 4,815 dolls.

6. If 1 pound of coffee cost 27 cents, what cost 1,234? *Ans.* 333 dolls. 18 cts.

7. If 1 barrel of sugar cost 35 dolls., what cost 11,962? *Ans.* 418,670 dolls.

8. At 42 dollars an acre, what cost 825 acres of ground? *Ans.* 34,650 dolls.

9. At 81 cents per pound, what cost 1,437 pounds of tea? *Ans.* 1,163 dolls. 97 cts.

10. At 63 cents per pound, what cost 127 pounds of tobacco? *Ans.* 80 dolls. 01 ct.

11.	275×18	*Ans.* 4,950	18.	9,876×49	*Ans.* 483,924
12.	164×45	7,380	19.	21,378×36	769,608
13.	40,073×54	2,163,942	20.	209,402×72	15,076,944
14.	764,131×48	36,678,288	21.	786,340×56	
15.	91,738×81	7,430,778	22.	92,810,333×48	
16.	3,002×28	84,056	23.	987,543,221×63	
17.	4,381×32	140,192	24.	9,895,461,343×49	

So far, our examples have been performed by dividing or *resolving* the multiplier into *two factors.*

But, we may resolve it into three, four, or *any number of factors*, and proceed in the same way. For example,

25. How many quarts of grain in 42 sacks, each sack containing 64 quarts?

We know that 3 times 7 are 21, and that 2 times 21 are 42. Therefore, 42 may be resolved into the factors 2, 3, and 7.

	64
First, multiply 64 by 7, thus,	7
	448
That product by 3, thus,	3
	1344
That product by 2, thus,	2
	2688 *Ans.*

This answer, viz. 2,688, is 42 times 64, because twice 64 multiplied by 3, (that is, taken 3 times,) is 6 times 64; and 6 times 64 multiplied by 7, (that is, taken 7 times,) is 42 times 64.

26. Find how many gallons are contained in 16 casks, each holding 58 gallons.
2 times 2 are 4; 2 times 4 are 8; and 2 times 8 are 16. We may, therefore, resolve 16 into the 4 factors, 2, 2, 2, and 2. A. 928.

When the above is thoroughly understood, let the pupil proceed to the following.

EXAMPLES FOR PRACTICE.

27. At 108 dollars an acre, what cost 34,462 acres of ground? 108=3×4×9. *Ans.* 3,721,896.

28. At 144 dollars a pipe, what cost 615,243 pipes of wine? 144=4×4×9. *Ans.* 88,594,992.

29. What cost 223,332 bales of cotton at 105 dollars a bale? 105=7×3×5. *Ans.* 23,449,860.

30. At 64 dolls. a ton, what cost 87,543 tons of iron? 64=4×4×4. *Ans.* 5,602,752.

31. At 72 cents per pound, what cost 507,832 pounds of aqua-fortis? 72=3×3×8. *Ans.* 365,639 dolls. 04 cts.

32. At 56 cts. per pound, what cost 342,516 pounds of snuff? 56=2×4×7. *Ans.* 191,808 dolls. 96 cts.

33. At 1 dollar and 40 cents, that is, 140 cts. per pound, what cost 41,364 pounds of tea? 140=2×2×7×5. *Ans.* 5,790,960 cts.

34. At 125 cents, or 1 dollar and 25 cts. per bushel, what cost 22,874 bushels of rye? 125=5×5×5. *Ans.* 2,859,250 cts.

35. Multiply 78,946 by 84.
36. 839,675×96.
37. 2,847,333×54.
38. 9,999,888×56.
39. 72,863,452×72.
40. 81.998,473×64.
41. 91,339,478×81.
42. 637,853,962×108.

The learner will soon find, that there are some numbers which *he cannot resolve into factors.* We shall teach him how to multiply by these hereafter.

A NUMBER WHICH CAN BE RESOLVED INTO FACTORS, (that is, one which is the product of two or more other numbers,) IS CALLED A COMPOUND OR COMPOSITE NUMBER.

The factors which produce the composite number are sometimes called its component parts.

Hence, to multiply by a composite number,

RESOLVE THE MULTIPLIER INTO FACTORS, AND MULTIPLY BY THESE FACTORS SUCCESSIVELY.

[It may here be suggested, once for all, that the teacher should always require an explanation of the language employed. If the pupil hesitates, he should be referred to that part of the book which explains it.]

◆

§ XIII. 1. What cost 13 tons of potash, at 139 dollars a ton?

Here 13, the multiplier, is more than 12, and is not a composite number.

First find, then, what 6 tons cost, thus,

```
139
  6
---
834
```

Then, what 7 tons cost, thus,

```
139
  7
---
973
```

Now 6 tons and 7 tons are 13 tons.

Therefore, add the two products together, thus,

```
 834
 973
----
1807
```

1,807 dollars, is, therefore, the price of 13 tons.

2. At 227 dollars an acre, what cost 23 acres of land? A. 5,221.

Then, when the multiplier is more than 12, and is not a composite number, we may

SEPARATE THE MULTIPLIER INTO SUCH PARTS THAT NONE OF THEM SHALL EXCEED 12, MULTIPLY BY THOSE PARTS SEPARATELY, AND ADD TOGETHER THE SEVERAL PRODUCTS.

EXAMPLES FOR PRACTICE.

3. What cost 123 barrels of flour at 17 dollars a barrel? *Ans.* 2,091 dolls.

4. At 13 cents per pound, what cost 175 pounds of sugar? *Ans.* 22 dolls. 75 cts.

5. At 19 cents a quart, what cost 245 quarts of gin? *Ans.* 46 dolls. 55 cts.

6. At 34 cents per pound, what cost 141 pounds of chocolate? *Ans.* 47 dolls. 94 cents.

7. 206×39	*Ans.* 8,034	13. 57,895×31
8. 205×23	4,715	14. 893,654×23
9. 876×29	25,404	15. 900,009×17
10. 9,893×31		16. 111,222×29
11. 7.640×37		17. 34,756,810×43
12. 63,904×41		18. 450,000,354×47

◆

§ XIV. The following mode, however, will be found more convenient.

1. At 134 dollars a pipe, what cost 31 pipes of brandy?

Here 31, the multiplier, is not a composite number; but 30 is, having the factors 3 and 10.

Now, if we multiply by 3 and 10 successively, we shall obtain the price of 30 pipes; to this we can add the price of 1 pipe, and we shall then have the price of 31 pipes, the answer required.

Multiply then, by 30, multiplying first by 3; thus,	134 3
	402
Then by 10; thus,	10
This is the price of 30 pipes; add the price of 1 more; thus,	4020 134
Therefore, 4,154 dollars is the price of 31 pipes.	4154

2. In 47 boxes, each containing 348 candles, how many candles?

Here 47, the multiplier, is not a composite number; but 45 is, having the factors 9 and 5. *Ans.* 16,356.

Hence, when the multiplier is not a composite number,

I. TAKE A CONVENIENT COMPOSITE NUMBER LESS THAN THE MULTIPLIER.

II. MULTIPLY BY IT ACCORDING TO THE RULE.

III. TO THE PRODUCT, ADD AS MANY TIMES THE MULTIPLICAND, AS THE COMPOSITE NUMBER CONTAINS UNITS LESS THAN THE MULTIPLIER.

EXAMPLES FOR PRACTICE.

3. If 1 acre of ground cost 23 dollars, what cost 62,123 acres ? *Ans.* 1,428,829 dollars.

4. If 1 pipe of wine contain 1,008 pints, what will 19 pipes contain ? *Ans.* 19,152.

5. If, in digging a canal, 35,432 cubic yards are excavated in 1 day, how much will be excavated in 29 days ? *Ans.* 1,027,528 yds.

6. If 33 dollars be paid a soldier, how much must be paid an army of 65,217 men ? *Ans.* 2,152,161.

7. Multiply 151×29	*Ans.* 4,379	13. 28,933×41
8. 623×19	11,837	14. 36,856×37
9. 7,512×17	127,704	15. 72,945×47
10. 2,931×23	67,413	16. 100,001×53
11. 2,004×31	62,124	17. 200,002×43
12. 19,070×13	247,910	18. 404,505×61

◆

§ XV. 1. At 10 cents a pound, what cost 53 pounds of butter ?

Here 10 is the multiplier.

By the first rule,

$$\begin{array}{r} 53 \\ 10 \\ \hline 530 \end{array}$$

Here the product is the same as the multiplicand, with a cypher annexed.

2. At 10 cents apiece, what will 25 oranges cost ?

Here 10 is the multiplier, again.

By the rule, as before,

$$\begin{array}{r} 25 \\ 10 \\ \hline 250 \end{array}$$

Here, again, the product is the same as the multiplicand, with a cypher annexed.

It seems, then, that *annexing a cypher at the right of any number, multiplies it by* 10.

The reason of this is very plain, when we consider that a figure in the tens' place, is ten times as great as the same figure in the units' place; because *numbers increase towards the left, in ten-fold proportion.* Also,

A figure in the hundreds' place, is ten times as great as the same figure in the tens' place. And, universally, *A figure in any place, is ten times as great as the same figure, in the next right hand place.*

If a figure be removed, then, from the units' to the tens' place, from the tens' to the hundreds', or from any place to the next higher, it will become *ten times as great* as before ; that is, *it will be multiplied by* 10.

Annexing a cypher to a number, removes every figure of that number a place higher. It therefore makes every figure ten times greater, and multiplies the whole number by 10.

Annexing two cyphers, removes every figure two places higher, and consequently multiplies the whole number by 100.

Annexing three cyphers, multiplies by 1,000, four cyphers, by 10,000, and so on.

When we use the word ANNEX, *we mean* PLACE AT THE RIGHT ; *when we use the word* PREFIX, *we mean* PLACE AT THE LEFT.

It will be seen that the number of cyphers, annexed, is just equal to the number of cyphers in the multiplier. Hence,

WHEN THE MULTIPLIER IS 1, WITH CYPHERS ANNEXED, MULTIPLICATION IS PERFORMED BY ANNEXING THE SAME NUMBER OF CYPHERS TO THE MULTIPLICAND.

EXAMPLES FOR PRACTICE.

2. At 10 dollars a barrel, what cost 357 bbls. of flour ? *Ans.* 3,570 dolls.

3. If I were to give 75 men 100 dollars apiece, what should I give the whole ? *Ans.* 7,500 dolls.

4. What would 275 pounds of beef come to, at 10 cts. pr. pound. *Ans.* 27 dolls. 50 cts.

The money of the UNITED STATES, is called FEDERAL MONEY. It is reckoned in EAGLES, DOLLARS, DIMES, CENTS and MILLS. These are called its *denominations.*

FEDERAL MONEY.

10	mills (marked m.)	make	1	cent, marked	ct.
10	cents	"	1	dime "	di.
10	dimes	"	1	dollar "	$.
10	dollars	"	1	eagle "	E.

---

* NOTE. The mill is merely a *nominal coin,* since there is no *real* coin in use of that value.

MENTAL EXERCISES.

1. How many cents in 2di? in 3di? in 5di? in 7di? in 12di?

2. How many dimes in 2 dolls? in 3 dolls? in 4 dolls? in 7 dolls? in 9 dolls?

3. How many mills in 2 cts? in 3 cts? in 4 cts? in 5 cts? in 9 cts? in 10 cts?

4. How many mills in 1 di.? in 2 di.? in 3 di.? in 4 di.? in 5 d.? in 7 d.? in 10 d.?

5. How many mills in 1 doll? in 2 dolls? in 3 dolls? in 4 dolls? in 10 dolls?

6. How many mills in 1 E.? in 2 E.? in 3 E.? in 7 E.? in 9 E.? in 10 E.?

Then, it seems that a dollar is 10 *dimes*, or 100 *cents*, or 1,000 mills. Hence,

DOLLARS ARE MADE DIMES BY ANNEXING ONE CYPHER; CENTS BY ANNEXING TWO CYPHERS; AND MILLS BY ANNEXING 3 CYPHERS.

If a sum be *cents* all the even *hundreds* will be dolls., of course; if it be *dimes* all the even *tens* will be dolls.; if it be *mills*, all the even *thousands* will be dolls. Hence,

TO CHANGE A SUM FROM DIMES TO DOLLARS, POINT OFF ONE FIGURE ON THE RIGHT; FROM CENTS TO DOLLARS POINT OFF TWO FIGURES; FROM MILLS TO DOLLARS, POINT OFF THREE FIGURES. ALL ON THE LEFT OF THE POINT WILL BE DOLLARS.

For this purpose a period (:) is the most convenient point. It is called the SEPARATRIX, because it is used to *separate* dollars from lower denominations. With the exception of this point the numbers follow one after another like common numbers. If there be but *one figure* after the point, the number may be read, *dollars and dimes*, or *the whole may be read as dimes.* If two figures follow the point, the number may be read *dollars dimes and cents*, or *dollars and cents*, or *all together as cents.* If three, *dollars, dimes, cents and mills*, or, *dolls. cents and mills*, or *all together as mills.* Many more variations in reading these numbers may be made, but it is of no use to repeat them, since they are not used in practice. The teacher, if he sees fit, may require them of the pupil, who will readily discover them for himself.

Accounts are kept in dollars and cents, the other denominations being disregarded. Thus instead of writing 3 E. 5 dolls. 4 di. 6 cts, we write 35 dolls. 46 cts. or, which more common $35.46. When the character $ is placed before a number, the names of the other denominations are omitted, and dollars are to be understood for the figures before the separatrix, cents for the first two after it, and mills for the third, if there be any. Hence, to read Federal Money,

CALL ALL THE FIGURES BEFORE THE SEPARATRIX, DOLLARS, AND ALL AFTER IT CENTS, OR CENTS AND MILLS.

To write Federal Money. **WRITE EAGLES AND DOLLARS, TOGETHER AS DOLLARS; AND PLACE THE SEPARATRIX: THEN WRITE DIMES AND CENTS TOGETHER AS CENTS, AND MILLS, IF THERE BE ANY, NEXT.**

In adding or subtracting Federal Money, the pupil must be careful to add or subtract the *same denominations* to or from each other; that is dollars, to or from dollars; cents, to or from cents, &c.

EXAMPLES FOR PRACTICE, CONTINUED.

5. How many cents are there in 700 dollars? *Ans.* 70,000.
6. How many mills are there in $965.13? *Ans.* 965,130.
7 What would 642 gallons of brandy cost at $1.00 pr. gal?
8. What would 987 barrels of flour cost at $10.00 pr. barrel? Give the several answers in dolls. cents and mills.
9. What cost 954 pounds of cheese at $0.10 pr. pound?

10.	Multiply	6,839	by	10	16.	1,349,864	10
11.	"	9,894	'	100	17.	2,957,863	100
12.	"	76,943	'	1000	18.	2,234,967	1,000
13.	"	194,972	'	100	19.	72,777,963	100,000
14.	"	279,864	'	10,000	20.	331,000,221	1,000,000
15.	"	1,943,867	'	100,000	21.	17,236	1,000,000,000

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§ XVI. 1. At 20 cents a pound, what will 78 pounds of sugar cost.

Here 20 is the multiplier.

But 20 is a composite number, because it is 2 tens or 2 times 1 ten.

Its factors, then, are 2 and 10.

By the rule for composite numbers, multiply first by 2,

thus, 78
2
———
156

By the last rule, multiply by 10, annexing thus, a cypher, 10
———
1560 *Ans.*

2. On one page of a book are 47 lines, how many on 60?
Here $60=6\times10$ is the multiplier. *Ans.* 2,820

3. At 48 dollars a hogshead, what cost 500 hogsheads of molasses?
Here $500=5\times100$ is the multiplier. *Ans.* 24,000

In all these examples, we multiply by the significant figures of the multiplier, and annex the cyphers to the product.

It is plain that if there are cyphers at the right of the multiplicand, these will, likewise, be found at the right of the product.

For example, multiply 30 by 3;
thus,

$$\begin{array}{r} 30 \\ 3 \\ \hline 90 \end{array}$$ *Ans.*

Multiply 500
by 9

$$\begin{array}{r} 500 \\ 9 \\ \hline 4500 \end{array}$$ *Ans.*

Here we have the same cyphers at the right of the products, which were at the right of the multiplicands. Hence,

WHEN THERE ARE CYPHERS AT THE RIGHT OF EITHER OR BOTH FACTORS, MULTIPLY BY THE OTHER FIGURES, AND ANNEX ALL THE CYPHERS TO THE PRODUCT.

NOTE. If the significant figures of the multiplier form a composite number, we may multiply by them according to the rule of composite numbers, and then annex cyphers. *In the same manner we may take advantge of any of the preceding rules.*

EXAMPLES FOR PRACTICE.

4. If 60 men can do a piece of work in 79 days, how many days will it take 4 men to do it? *Ans.* 4,740 days.

5. If 27 men can do a piece of work in 300 days, how many men will it take to do it in one day? *Ans.* 8,100 men.

6. If 500 labourers on a canal be paid 75 dollars a piece, how many cents is that for the whole? *Ans.* 3,750,000 cents.

7. If there are 275 pages in a book, how many pages are there in an edition of 7,000 books? *Ans.* 1,925,000.

8. If 742 pounds of provision will serve a garrison 1 day, how many will serve 300 days? *Ans.* 222,600.

9. If $250.00 will gain $15.00 interest in one year, what will it gain in 70 years? Give the answer in mills. *Ans.* 1,050,000 mills.

10. If a sum of money will gain $25.00 interest in one year, how much will it gain in 50 years? Give the answer in dimes.
Ans. 12,500 dimes.

11. If $900.00 gain a certain interest in a year, what sum will gain 320 times as much? Give the answer in cents.
Ans. 28,800,000 cts.

12. If 600 barrels of rum cost 50 dollars each, what cost the whole? Give the answer in mills. *Ans.* 30,000,000 mills.

13.	Mul.	765,000	by	270	16.	300,200	19
14.		500,700		18	17.	561,800	20
15.		200,000		17	18.	630,111	30

19.	894,220	90	24.	19,467,010	3,400
20.	76,000	300	25.	23,654,020	3,000
21.	70,620	800,000	26.	37,695,030	42,00
22.	51,963,000	810	27.	351,006,000	36,000
23.	27,334,000	9,000	28.	764,999,200	25,000

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§ XVII. We will now give a process, adapted to all cases in Multiplication.

1. A carpenter received 1,565 dollars for building a house. How much ought he to receive for building 17 houses at that rate?

Here 17 is the multiplier. First find how much he ought to receive for 7 houses; thus,

$$\begin{array}{r} 1565 \\ 7 \\ \hline \end{array}$$

Then find how much for 10 houses, by annexing a cypher to 1,565; thus,

$$\begin{array}{r} 10955 \\ 15650 \\ \hline \textit{Ans.}\ 26605 \end{array}$$

2. In a garrison, if the allowance of one man is 23 ounces of provision a day, how many ounces a day ought 634 men to have?

Here 23 is the multiplier.

First, find what it would amount to at 3 ounces per day; thus,

$$\begin{array}{r} 634 \\ 3 \\ \hline 1902 \end{array}$$

Then, at 20 ounces; thus,

$$\begin{array}{r} 634 \\ 20 \\ \hline \end{array}$$

But, 20 and 3 are 23. Therefore, add the two products; thus,

$$\begin{array}{r} 12680 \\ 1902 \\ \hline 14582\ \textit{Ans.} \end{array}$$

In these examples we found the answer from multiplying *by the units* in the multiplier, (according to § XI,) then, *by the tens*, (according to § XVI,) and, finally, adding the two products together.

We may do the same thing a little more concisely, thus:

3. How many yards in 29 pieces of cloth, each piece containing 132 yards.

Multiply first by the 9 units, and then by the 2 tens.

132
29
1188
2649

Then, add the two products, thus, 3828 *Ans.*

It will be seen that *the cypher need not be placed at the right of the tens' product*, if the *first figure* of that product be written *under the order of tens.* For it counts nothing in the addition.

We may proceed in the same manner, if the multiplier consists of more figures. For example:

4. What are 953 sheep worth, at $1.75 apiece?

Operation. 953
175
4765
6671
953
$1667.75

Here, we multiplied first by the 5 units, then, by the 7 tens, and then by the 1 hundred. We likewise placed *the right hand figures of the last two products, under their respective orders*, omitting to annex the cyphers, required by the rule in Sect. XVI, because, *in this case*, unnecessary.

Thus we have a product corresponding to each order of the multiplier. These products are sometimes called PARTIAL PRODUCTS; *their amount when added, the* TOTAL PRODUCT.

If any order in the multiplier is filled *by a cypher*, there will be, of course, *no partial product, corresponding to that order.* Thus,

5. A man gave 185 dollars a pipe, for 107 pipes of wine. What cost the whole?

Operation. 185
107
1295
185
$19795 *Ans.*

Here, the tens' order is filled by a cypher. It has, consequently, *no partial product.*

Hence, the general rule for Multiplication.

I. PLACE THE MULTIPLIER UNDER THE MULTIPLICAND SO THAT THE SAME ORDERS MAY STAND UNDER EACH OTHER.

II. MULTIPLY THE MULTIPLICAND BY EACH SIGNIFICANT FIGURE IN THE MULTIPLIER, PLACING THE FIRST FIGURE OF EACH PARTIAL PRODUCT UNDER THE ORDER OF ITS MULTIPLIER.

III. ADD THE SEVERAL PRODUCTS AS THEY STAND. THE AMOUNT WILL BE THE PRODUCT REQUIRED.

NOTE. *You will be sure to arrange your partial products correctly, if you place the right hand figure of each, exactly under the figure by which you are multiplying.*

EXAMPLES FOR PRACTICE.

6. What will 15 tons of hay come to, at $20.78 pr. ton? *Ans.* $311.70

7. What will 857 barrels of pork come to, at $18.93 pr. barrel? *Ans.* $16,223.01

8. What cost 144 bushels of wheat, at $3.51 pr. bushel? *Ans.* $505.44.

9. A man bought 52 oxen, at $37.00 a head. What did they all come to? *Ans.* $1,924.00.

10. At 15 cents pr. pound, what cost 600 pounds of sugar? *Ans.* $90.00.

11. What cost 33 bushels of rye, at 94 cts. pr. bushel? *Ans.* $31.02.

12. If a gallon of rum cost 58 cts., what cost 203 gallons? *Ans.* $117.74.

13. If 1 pound of snuff cost 47 cts., what cost 314? *Ans.* $147.58.

14. If a bushel of rye cost 75 cts., how much will 829 bushels cost? *Ans.* $621.75.

15. If an acre of land cost 37 dollars, what will 949 acres cost? *Ans.* $35,113.

16. 34,293×74 *Ans.* 2,537,682
17. 47,042×91 4,280,822
18. 37,864×209 7,913,576
19. 25,203×4,025 101,442,075
20. 269,181×4,629 1,246,038,849
21. 40,634×42,068 1,709,391,112
22. 134,092×87,362 11,714,545,304
23. 918,273,645×1,003,245 921,253,442,978,025

In § XI. it is shown that the product is the same, whichever of the factors is made the multiplier. Hence, *in order to prove multiplication,*

I. INVERT THE ARRANGEMENT OF THE FACTORS, AND MULTIPLY AGAIN.

But this is sometimes inconvenient; therefore, in such cases,

II. IF THE FIRST PRODUCT WAS OBTAINED BY THE RULE IN § XI, XV, XVI OR XVII, OBTAIN ANOTHER, BY THE RULE IN § XII, XIII OR XIV; AND IF THE FIRST WAS OBTAINED BY ONE OF THE LATTER RULES, OBTAIN ANOTHER, BY ONE OF THE FORMER.

It is hardly necessary to illustrate these modes of proof. We will, however, give one example of each.

I. At $1.25 a pound, what cost 173 pounds of tea?

FIRST OPERATION.	PROOF.
173	125
125	173
865	375
346	875
173	125
216.25 *Ans.*	216.25 *Ans. as before.*

II. At 56 dollars a barrel, what cost 129,874 barrels of brandy ?

	Operation by § XII.	*Proof by* § XVII.
	129874	129874
56 is a composite number, whose factors are 8 and 7.	8	56
	1038992	779244
	7	649370
	Ans. 7,272,944	7,272,944 *Ans. as before.*

EXAMPLES FOR PRACTICE.

1. At 27 dollars a piece, what cost 234 pieces of broadcloth ?
2. At 52 dollars a barrel, what cost 72 barrels of brandy ?
3. What will 763 hundred weight of sugar cost, at 15 dollars a hundred weight ?
4. At 35 dollars a ton, what cost 749 tons of iron ?
5. At 33 cts. pr. yard, what cost 16 pieces of cloth, each containing 29 yards ?
6. A ship sailed 26 days at the rate of 125 miles a day, and afterwards, 43 days at the rate of 98 miles a day. How far did she sail in all ?

7. 72,600×31,000.
8. 49,000×52,009.
9. 697,080×539,063.
10. 41,725×687,900
11. 38,603×950,950
12. 712,314×23,000,333
13. 37,969,868×98,647,564
14. 363,464,565×271,892,955
15. 437,920,213×234,678,956
16. 234,567,890,123×345,678,900,124

§ XVIII. We have seen in a preceding section, that money is reckoned in the United States in Dollars, dimes, cents, &c. These are the *denominations* of *Federal Money*. In other countries, it is reckoned differently. It is important that we should understand the English mode of reckoning money, because there are various *currencies* of money, as they are called, in this country, whose *denominations* are the same, though different in *value*. The following are the denominations of ENGLISH, or,

STERLING MONEY.

4 farthings (marked qr.)	make	1 penny,	(marked)	d.
12 pence	"	1 shilling	"	s.
20 shillings	"	1 pound	"	£
21 shillings	"	1 guinea.		

Note. £ is the mark for pounds. When it is placed *before* the pounds, the marks for the lower denominations are omitted, but when *after*, they must be employed. Thus, £3 ; 7 ; 9 ; 2 is written, instead of 3£. 7s. 9d. 2qrs. 1 farthing is sometimes written ¼d.; 2 farthings, ½d.; and 3 farthings ¾d.

EXAMPLES FOR PRACTICE.

1. How many farthings in 6 shillings? In 5 shillings? In 7?
2. How many shillings in 2 pounds? In 3 pounds? In 4 pounds?
3. How many pence in 2 shillings? In 3 shillings? In 4? In 5?
4. How many farthings in 1 shilling? In 2 shillings?
5. How many pence in 1 pound? In 2 pounds? *Ans.* 240. 480.
6. At 2 pence apiece, how many pence will 17 oranges cost?
7. How many pence in a guinea? In 2 guineas? *Ans.* 252. 504.
8. How many farthings in a pound? A. 960. In 1 guinea? A. 1,008.
9. How many shillings in 275 pounds? *Ans.* 5,500.
10. How many farthings in 341 guineas? *Ans.* 343,728.
11. How many pence in 3 pounds, 5 shillings, and 6 pence?

Note. First find the pence in 3 pounds, then in 5 shillings, then add these to 6 pence. *Ans.* 786.

12. How many shillings in 27 guineas, and 18 shillings?
13. How many farthings in 4 pounds, 9 shillings, and 5 pence?
14. How many farthings in 8£. 11s. 10d. 2qrs.? *Ans.* 8,250.
15. How many qrs. in 2,853£. 19s. 9d. 3 qrs.? *Ans.* 2,739,831.
16. How many qrs. in 78,364£. 15s. 7d. 1qr.?

We will now give some tables of Weights and Measures. The following are the *denominations* of

AVOIRDUPOIS WEIGHT.

16 drams (dr.)	make	1 ounce,	marked	oz.
16 ounces	"	1 pound	"	lb.
14 pounds	"	1 stone	"	sto.
28 pounds	"	1 quarter of a hundred weight,		qr.
4 qrs. or 112 lbs.		1 hundred weight,		cwt.
20 hundred weight		1 ton		T.

Note. By this weight are weighed all coarse and drossy articles, which are liable to waste, as groceries, flour, hay, tallow, &c.; and likewise all metals, except those usually called the *precious metals;* viz. gold and silver. It will be observed that 112 lbs. make 1 cwt. But merchants in our large towns, particularly in seaports, allow only 100 lbs. to the cwt.

Merchants are accustomed to make allowances upon goods, bought and sold by Avoirdupois Weight, for the weight of the box, cask, or bag, which contains them; and also for waste, dust, &c. The weight, *before these allowances are deducted*, is called *gross weight; after they are deducted*, it is called *net weight.* In some Arithmetics, rules are given for deducting these allowances, under the head of Tare and Trett, which are the names, given to certain deductions. These rules, however, are superfluous, for the student, familiar with *principles*, will easily make them for himself.

17. How many qrs. in 5 cwt.? How many in 8? In 6? In 12?
18. How many oz. in 2 lb.? In 3? In 4? In 5?
19. How many lbs. in 5 cwt.? *Ans.* 560.

20. How many drs. in 3 lbs. 2 oz.? *Ans.* 800.

NOTE. First find the drs. in 3 lbs., then in 2 oz. and add them together.

21. How many lbs. in 7 T. 19 cwt.? *Ans.* 17,808.
22. How many oz. in 13 T. 15 cwt. 3 qrs. 26 lbs.? *Ans.* 494,560.
23. How many drs. in 25 T. 17 cwt. 2 qrs. 13 lbs. 14 oz.? *Ans.* 14,841,312.
24. How many qrs. in 32 T. 11 cwt. 1 qr. 2 lbs. 1 oz. 2 drs.?
25. How many qrs. in 1,827 T. 19 cwt. 3 qrs. 15 lbs. 15 oz. 15 drs.?

The following are the *denominations* of

TROY WEIGHT.

24	grains (gr.) make	1	penny-weight,	dwt.
20	penny-weights	1	ounce	oz.
12	ounces	1	pound	lb.

NOTE. By this weight are weighed jewels, the precious metals, coin, bread and liquors. The quality, fineness, or purity of gold, is estimated in carats, by which term is meant the 24th part of any thing. Pure gold is said to be 24 carats fine; but pure gold has been found to be too soft for the purposes of coinage, since it wears away too rapidly. The law, therefore, allows and *requires* that there should be mingled in all gold coined, two carats, that is two 24th parts, of *alloy*; by which is meant some baser metal, usually copper. This is *standard gold*, and is, of course, 22 carats fine. *Standard silver* for coinage, is not estimated by carats. Pure silver is said to be 12 oz. fine; 18 dwts. of alloy are directed to be mixed with 11 oz. 2 dwts. of pure silver, so that the compound weighs a pound.

The fineness of silver and gold is tried in the fire. If they lose nothing in the trial, they are pure; the gold is 24 carats, and the silver 12 oz. fine. If any thing is lost, it indicates that there was alloy in the metal, and the loss must be deducted, in order to ascertain the fineness.

26. How many grains, in 2 dwts.? *Ans.* 48.
27. How many dwts. in 3 oz.? *Ans.* 60.
28. How many dwts. in 1 lb.? *Ans.* 240.
29. How many grs. in 1 lb.? *Ans.* 5,760. In 2 lbs.? *Ans.* 11,520.
30. How many dwts. in 7 lb. 15 dwts.? *Ans.* 1,695.
31. How many grs. in 19 dwts. 23 grs.? *Ans.* 479.
32. How many grs. in 37 lbs. 12 dwts. 13 grs.? *Ans.* 213,421.
33. How many grs. in 273 lbs. 5 oz. 3 dwts. 6 grs.?
34. How many grs. in 1,236 lbs. 11 oz. 19 dwts. 23 grs.?
35. How many oz. in 2,325,856 lbs.? How many dwts.? How many grs.?
36. How many grs. in 75,863,294 lbs, 3 oz. 5 dwts. 17 grs.? *Ans.* 436,972,575,017.

The following are the *denominations* of

APOTHECARIES' WEIGHT.

20	grains (gr.) make	1	scruple,	marked	sc.	or	℈
3	scruples	"	1	dram	"	dr. or	ʒ
8	drams	"	1	ounce	"	oz. or	℥
12	ounces	"	1	pound	"		℔

NOTE. This weight is used in *compounding* medicines. Drugs and medicines, however, are *bought and sold* by Avoirdupois weight. The pound and ounce of this weight are the same as those of Troy Weight. The *subdivisions* or *smaller* divisions are different.

37. How many grains in 8 ℥. 3ʒ. 2 ℈. ? *Ans.* 4,060.
38. How many ʒ. in 1 ℔. ? *Ans.* 96.
39. How many ℈. in 1℥. ? *Ans.* 24.
40. How many grs. in 7 ℔. ? *Ans.* 40,320.
41. How many grs. in 57 ℔. 9℥. 7ʒ. 2℈. 17 grs. ? *Ans.* 333,117.
42. How many grs. in 275 ℔. 11℥. 5ʒ. 1℈. 8 grs. ?
43. How many grs. in 2,134 ℔. 9℥. 6ʒ. 2℈. 19 grs. ?

The following are the *denominations* of

DRY MEASURE.

2	pints (pt.)	make	1	quart,	marked	qt.
2	quarts	"	1	pottle	"	pot.
2	pottles	"	1	gallon	"	gal.
8	quarts	"	1	peck	"	pk.
4	pecks	"	1	bushel	"	bu.
4	bushels	"	1	coomb	"	coomb.
2 8	coombs or bushels	" "	1	quarter	"	qr.
36	bushels	"	1	chaldron	"	ch.
5	quarters	"	1	load	"	load.

NOTE. This measure is used to measure grain, salt, seeds, oysters, coal, roots, lead ore, and all dry goods of this kind.

44. How many pints in a peck? *Ans.* 16.
45. How many qts. in 13 bu.? *Ans.* 416.
46. How many bu. in 58 ch.? *Ans.* 2,088.
47. How many pts. in 36 bu. 3 pks. 5 qts. 1 pt.? *Ans.* 2,363.
48. How many pks. in 6,254 bu.? How many qts.? How many pts.?
49. How many pts. in 21,857 bu. 3 pks. 7 qts. 1 pt.?
50. How many pts. in 578,656 bu. 2 pks. 1 qt. 1 pt.? *Ans.* 37,034,019.
51. How many bu. in 629 ch.?
52 How many bu. in 235,079 ch.? *Ans.* 8,462,844.
53. How many pks. in 21,953,205 qrs.?

The following are the *denominations* of

ALE OR BEER MEASURE.

2	pints (pt.)	make	1	quart,	marked	qt.
4	quarts	"	1	gallon	"	gal.
8	gallons	"	1	firkin of ale		fir.
9	gallons	"	1	firkin of beer		fir.
2	firkins	"	1	kilderkin	"	kil.
2	kilderkins	"	1	barrel	"	bar.
3	kilderkins	"	1	hogshead	"	hhd.
3	barrels	"	1	butt (of ale)		butt.

Note. This measure is used in measuring ale, beer and milk. In London the Ale firkin contains only 8 gallons, while the Beer firkin contains 9. Of course the higher measures differ in the same proportion.

54. How many gallons in the ale hogshead? *Ans.* 48.
55. How many in the beer hogshead? *Ans.* 54.
56. How many kils. in 15 butts? *Ans.* 90.
57. How many qts. in 13 Ale hogsheads? *Ans.* 2,496.
58. How many pts. in 16 butts, 7 gal. 3 qts. of Ale? *Ans.* 12,350.
59. How many pts. in 527 hhds. 6 gal. 2 qts. 1 pt. of Beer?
60. How many pts. in 325 bar. 1 kil. 1 fir. 5 gal. 3 qts. 1 pt. of Beer?

The following are the *denominations* of

TIME.

60	seconds (sec.)	make	1	minute,	marked	m.
60	minutes	"	1	hour	"	h.
24	hours	"	1	day	"	d.
7	days	"	1	week	"	w.
4	weeks	"	1	month	"	mo.
13 12	months, 1 day, 6 hours, or calendar months,		1	common or Julian year		yr.
100	years	"	1	century.		

Note. The year is called in the table the *Julian* year, because the calendar was regulated by Julius Caesar, Emperor of Rome. The true solar year contains 365 days, 5 hours, 48 minutes, and 50 seconds, instead of 365 days, 6 hours, according to the table. The calendar months, that is, the months, by which we reckon the year in the Almanac, or calendar, are unequal in length, but the number of days in each may easily be remembered by the following stanza.

"Thirty days hath September,
April, June, and November;
February, twenty eight alone,
All the rest have thirty one,
Except in leap year; then in fine,
February's days are twenty nine."

But, besides the even days, we have seen that the Julian calendar allows 6 hours to the year. 6 hours is the quarter of 24 hours, or of one day, and therefore, in four years, this excess will amount to another day. An allowance is made for this, by giving February 29 days, instead of 28, every fourth year, called Bissextile, or leap year. But we have seen, also, that the true excess is 5 hours, 48 minutes, 50 seconds, which is 11 minutes, 10 seconds, less than 6 hours. The Julian calendar, therefore, made the year 11 minutes, 10 seconds, too long. This would make 1 day in about 130 years, or nearly 3 days in 400 years. Pope Gregory XIII observed this error and rectified it, by ordering three leap years in every 400 years to be considered as common years, and no additional day to be annexed to February. As every centurial year, (that is, year on which a century is completed, as 1700, 1800, &c.,) is a leap year, it was ordered, that three successive centurial years should be reckoned as common years, and the fourth centurial year as a leap year. There is a slight error still, but it will hardly amount to a day in 4,000 years. Pope Gregory found, likewise, that, in consequence of the error in the Julian Calendar, the months had fallen back from their true places in the year. That is, since the year was made longer than it ought to have been, January, and of course, the other months, began later than they ought, on each successive year; so that, in his time, the first day of January came where the 10th should be. He, therefore, determined to restore the months to their places, by leaving 10 days out of the calendar, all at once. This was done in the month of October, the day after the 4th being counted the 15th, instead of the fifth. Catholic countries, of course, instantly adopted this reformation, but it was long before it was adopted in those states, which did not acknowledge the authority of the Pope. It was not adopted in England, until the error had become a day greater, being eleven days instead of ten. By act of Parliament, however, in 1752, 11 days were suppressed after the 2d of September, and, by the same act, the beginning of the year was transferred from the 25th of March to the 1st of January. The reckoning of dates by the old method is called OLD STYLE, by the new, NEW STYLE. To any date reckoned by *Old Style*, therefore, we must add 11 days, to obtain the same date according to the *New Style*; and, if the date, in *Old Style*, be between the 1st of January and the 25th of March, we must reckon the *year*, 1 *greater*: because, carrying back the beginning of the year, from the latter of those days, to the former, threw *all the intermediate days into the next following year*. Thus, the first of March, 1750, *Old Style*, is the same as the 11th of March, 1751, *New Style*. Russia has not yet adopted the *New Style*, and the difference is now 12 days. This should be remembered in dates coming directly from that country, and allowance made. The pupil has, no doubt, often observed in the Almanacks, that a letter is put for *Sunday*, while the other days of the week are denoted by the figures 2, 3, 4, &c., or by the first two letters of their names. This letter is called the *Dominical* letter. The first seven letters of the alphabet were used by the Primitive Christians, to stand for the days of the week in the calendar. They called the 1st of January, A, the second, B, the third, C, and so on, to the 7th, which was G. They then repeated the *same letters* again, calling the 8th, A; the 9th, B, and so on. The letter, which fell upon Sunday, was called the *Dominical* letter. At the present day, these letters are entirely disused in many calendars. Some, however, still retain the *Dominical* letter. In the following couplet, the initials, or first letters, of the several words, are the letters, which fall on the first days of the months in the year.

Jan. Feb. Mar. Apr. May. June.
"At Dover Dwells George Brown, Esquire,
July. Aug. Sept. Oct. Nov. Dec.
Good Carlos Finch And David Fryer."

Of course, if I know the Dominical letter, I can easily tell on what day of the *week, any day of any month* will happen. Thus, if I wish to know on what day of the week the 4th of March will happen, in 1830, when the Dominical letter is C, I observe that the word, Dwells, answers to March, which, therefore, comes in on Monday. The 4th day, then, will be Thursday. In leap year, on account of the additional day in February, two Dominical letters are used, one for January and February, and the other for the rest of the year. The latter of these, is the letter, which precedes the former in the calendar. Thus, if the Dominical letter, with which leap year begins, be C, it will be changed to B, after the last day of February. In the same manner, B will be changed to A, A to G, G to F, &c.

Years are reckoned, in all Christian countries, from the birth of our Saviour, which is called the *Christian Era*.

61. How many days in 9 (not calendar) mo.? *Ans.* 252.

62. How many hours in a year of 365 d. 6 h.? How many minutes? How many seconds?
Ans. 8,766 h. 525,960 m. 31,557,600 sec.

63. How many seconds in a year of 365 d. 5 h. 48 m. 50 sec.?
Ans. 31,556,930.

64. How many seconds in a man's age, who is 52 yrs. old; allowing 365 d. 6 h. to the year? How many, allowing 365 d. 5 h. 48 m. 50 s.? *Ans.* 1,640,995,200 and 1,640,960,360.

Note. For the answers to this example, the pupil may multiply the number of seconds in one year, (as found in ex. 62 and 63,) by 52, the number of years.

65. How many seconds from the commencement of the Christian Era, to the end of the year 1,830; allowing for the length of the year as in ex. 62? How many, allowing as in ex. 63?

66. How many seconds old is the world at the end of 1,830, 4,004 years having elapsed before the Christian era; reckoning by each mode of allowance?

In the preceding examples, you have been changing numbers from one *denomination* to another. Changing numbers, in this way, is commonly called *reducing* them, and the process, by which they are reduced, is called Reduction. Then,

Reduction is the process of changing numbers from one denomination to another, without altering their value.

In the examples above, it has been required to reduce *larger*, or *higher* denominations, to *smaller*, or *lower*.

This is called Reduction Descending, and is performed, as we have seen by Multiplication.

Reduction Ascending is the reverse of this, and is the process of changing numbers from *lower* denominations to *higher*. It is performed, as we shall see, by Division. The general rule for Reduction Descending, may be expressed as follows.

I. Multiply the highest denomination given, by the number required of the next lower, to make one of the denomination multiplied, and add the units of the next lower, if any, to the product.

II. Multiply this sum, by the number, required of the next lower denomination still, and add in, as before.

III. Proceed thus, till you arrive at the denomination required.

EXAMPLES FOR PRACTICE.

67. If an ingot of silver weigh 3 lb. 6 oz. 13 dwt. what is it worth at 4 cts. pr dwt.? *Ans.* 34.12.

68. In 11 lb. 6 oz. 19 dwt. 9 gr. how many grains? *Ans.* 66,705.

69. What is the value of a silver cup, weighing 9 oz. 4 dwt. 16 gr. at 3 mills pr. gr.? *Ans.* $13,296.

70. At 9 cts. a pound, what cost 3 cwt. 2 qrs. 16 ℔. of sugar? *Ans.* $36.72.

71. In 9℔. 8℥. 1ʒ. 2℈. 19 gr. how many gr.? *Ans.* 55,799.

72. At 27 cts. pr. nail, what cost 2 yds. 1 qr. 3 nls. of cloth? *Ans.* $10.53.

73. How many sec. in 4 years, allowing 13 mo. 1 d. 6 h. to the year? *Ans.* 126,230,400.

74. How many minutes are there, from the 13th July, at 43 minutes past 9 in the morning to the 5th Nov., at 19 m. past 3 in the afternoon. *Ans.* 165,936.

75. How many sec. in 8 yrs. 3 mo. 2 wks. 2 d. 19 h. 43 m. 57 sec., allowing 13 mo. 1 d. 6 h. to the year? *Ans.* 261,171,837.

76. How many seconds old are you?

77. In 156 T. 1 cwt. 0 qr. 2 lb. 6 oz. how many oz.? *Ans.* 5,592,870.

78. In 19 lb. 13 oz. 7 dr. how many dr.? *Ans.* 5,079.

79. In 90£. 17s. 9d. 1 qr. how many qrs.? *Ans.* 87,253.

80. In 34 lbs. 5 oz. 19 dwts. 10 gr. how many gr.? A. 198,706.

81. In 27 ℔. 9℥. 2ʒ. 1℈. 16 gr. how many gr.?

82. At 12 cts a grain, what must be paid for a gold cup, weighing 1 lb. 3 oz. 13 dwt. 17 gr.?

83. At 6 cts. a pint, what cost 27 bu. 3 pks. 7 qt. 1 pt. of wheat?

84. At 5 cts. a pint, what cost 16 butts, 2 bar. 1 kil. 1 fir. 7 gal. 3 qts. 1 pt. of ale?

85. In 16 yrs. 24 d how many sec. allowing 365 d. to the year? *Ans.* 506,649,600.

86. A merchant sold 84 yds. 3 qrs. 2 na. of broadcloth at $0.50 a nail. In return he received 25 boxes of raisins at $4.00 a box, and the rest in flour at $11.00 a barrel. How much flour did he receive?

87. A farmer sold 6 loads of wood, at $3.00 a load; and 3 cwt. of butter, at 9 cts. a pound; and 70 bushels of wheat, at $2.125 a bushel; and 93 bushels of rye, at 75 cts. a bushel. In return he received 8 ploughs, at $12.00 apiece; 1 yoke of oxen, $60.00; 1 pr. of horses, at 75 dollars apiece; and the rest in money. How much money did he receive? *Ans.* $20.74.

NOTE. The last two questions belong to the rule, commonly called BARTER.

◆

OBSERVATIONS ON MULTIPLICATION, FOR ADVANCED PUPILS.

§ XIX. The order of writing numbers for multiplication, required by the rule, is only adopted for the sake of convenience. We need some certain method, and this is as good as any.

It is manifestly of no consequence which of the figures of the multiplier is first used, since if the partial products are set down correctly, the answer will be the same, whether we begin on the left, on the right, or somewhere in the middle.

In order to multiply with ease, it is necessary that *the several products of all the possible combinations of two digits, should be fixed in the memory;* exactly as in Addition, it was found necessary to retain the *sums* of the same combinations in the mind. A little reflection will show, that it is even more necessary to be acquainted with a multiplication table, than with a table for addition. For in adding, we might add numbers *unit by unit*, as children do; but in multiplying, this process would become very tedious, and liable to error. The following table, generally supposed to have been invented by Pythagoras, a Greek philosopher, contains all the products which are *absolutely necessary* to be learned. It is convenient to know the products of numbers still higher.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

In order to find the product of two numbers in this table; *look for one of the numbers in the left hand column, and for the other in the upper row. Then, in a line with the former, and directly under the latter, you will find the product sought.* Thus, opposite 9, and under 6, will be found 54, the product of 9 and 6.

The construction of this table is very simple. It may be entirely formed by Addition. In the upper row, let the nine digits be written down in order. Then, let every figure in this row be added to itself, and the amount placed directly under the figure thus added. By this process, the second row is found. For the third, add each number in the second, to the one above it in the first. For the fourth, add, in the same manner, the third and first; or add the second to itself. For the fifth, add the fourth and first; or the third and second. For the sixth, add the fifth and first; or the fourth and second; or add the third to itself, and so on.

The former of the two modes of proof, given in the last section, is that found in common Arithmetics. The latter embraces several varieties, which the inquisitive student will discover for himself. The natural mode of proof, by reversing the operation, would be to *divide the product by one of the factors.* The quotient, of course, ought to be the other factor.

We will give an illustration of this method.

OPERATION.	PROOF.	PROOF.
Multiply 68	68)2516(37	37)2516(68
by 37	204	222
476	476	296
204	476	296
2516	00	00

Multiplication may also be proved by the *casting out of nines*, as follows:

Multiply	672	6
by	11	2
		–12
	7392	3

The proof may be explained thus:
672 is even 9s and 6.
11 is even 9s and 2.

To multiply 672 by 9 and 2, then, is the same, in effect, as to multiply it by 11. 692×9, is, of course, even 9s. In multiplying 672 by 2, multiply first its even 9s by 2. This product, is, likewise, even 9s. Then, multiply the 6, (excess,) by 2. If this product be even 9s, the whole is plainly even 9s. If not, the whole will have the same excess, which the product of 6 and 2 has. 6×2=12=9+3. Then, 672×11, ought to have 3 for its excess of 9s. Casting out 9s from 7,392, the product obtained above, 3 remains. Hence, we conclude the work right.

From this explanation, we derive the following rule:

I. REJECT THE 9S FROM THE MULTIPLIER AND MULTIPLICAND.

II. MULTIPLY THE EXCESSES TOGETHER, AND REJECT THE 9S FROM THEIR PRODUCT.

III. REJECT THE 9S FROM THE PRODUCT, AND IF THE EXCESS, THUS FOUND, AGREE WITH THE LAST, THE OPERATION MAY BE CONSIDERED AS CORRECTLY PERFORMED.

We may prove Multiplication in the same manner, by casting out 3s, for the reasons mentioned under Addition.

We have seen that Multiplication is an abbreviation of Addition. It may seem singular that addition should be employed to abbreviate multiplication. It is a fact, however, that by means of what are called *Logarithms*, the nature of which we cannot explain here, tedious multiplications may be performed by the mere addition of two numbers.

We will here insert some useful hints, not found in the ordinary Arithmetics. They may be classed under the head of

ABBREVIATED MULTIPLICATION.

ART. I. It is easier, always, to multiply by small figures, than by large; because we have *smaller numbers to carry.*

Let us take the following example.

1275
93
3825
11475
118575

Here, we shall find it most convenient, after obtaining a product by 3, to multiply that product by 3 again, instead of multiplying the multiplicand by 9. The result will be the same in either case—for 3 times 3 are 9.

It will be observed that this operation is the same, in principle, with the rule for composite numbers. Take another example.

8136 Here, after multiplying by 8, we multiply the product thus ob-
248 tained, by 3; which is equivalent to multiplying the multiplicand
—— by 24, the remaining part of the multiplier. *In this case, we obtain*
65088 *a single product, where the common rule would make two necessary.*
195264 Hence, WE MAY OFTEN OBTAIN A PARTIAL PRODUCT, FROM
—— ONE ALREADY OBTAINED, BY MULTIPLYING ACCORDING TO
2017728 THE RULE FOR COMPOSITE NUMBERS.

When the larger number comes first, we may often find a second partial product by division.

ART. II. We have seen, (§ xv.) that annexing [illegible]her to any number, multiplies that number by 10. Now 5 is half of ten; and 5 times any number, is half of 10 times the same number. If, then, we annex a cypher to any number, and then divide *that number, with the cypher annexed,* by 2; we shall obtain a result, equal to 5 times the original number.

Thus, to multiply 724 by 5; annex first a cypher, thus, 7,240. Then, take half of this, and it is 3,620, which is 5 times 724.

To multiply 531 by 5. Annex a cypher, thus, 5,310. Divide by 2, and the result is 2,655, which is 5 times 531.

Hence, *To multiply by* 5, ANNEX A CYPHER TO THE MULTIPLICAND, AND DIVIDE BY 2.

ART. III. 15 is 10 and 5. Therefore, if we multiply a number by 10, and then by 5, and add the two products, their sum will be 15 times the number multiplied. But, (§ xv.) *annexing a cypher* is multiplying by 10; and (II.) *half the product by* 10, *is the product by* 5.

Then to multiply 337 by 15.

Multiply first by 10: thus,	3370
Divide by 2: thus,	1685
Add these results: thus,	5055=15 times 337.

Hence, *to multiply by* 15, ANNEX A CYPHER TO THE MULTIPLICAND, AND ADD THIS RESULT TO HALF ITSELF.

ART. IV. To multiply by 100, (§ xv.) we annex two cyphers. But 25 is a *quarter of a hundred.* Therefore, 25 *times a number,* is a *quarter as much* as 100 *times* the *same number.* Of course, if we multiply by 100, and divide the result by 4, we shall obtain a result equal to 25 times the number multiplied.

Thus, multiply 452 by 25.

It is multiplied by 100 by annexing two cyphers; thus, 45,200.

Divide this result by 4, and we obtain 11,300=25 times 452.

Hence, *To multiply by* 25, ANNEX TWO CYPHERS TO THE MULTIPLICAND, AND DIVIDE BY 4.

ART. V. 9 is 1 less than 10. Therefore, 9 *times a number* is *once that number less* than 10 *times* the *same number.* Now annexing a cypher, (§ xv.) multiplies by 10. If, then, after multiplying by 10, we take away *once the number,* we have 9 *times* the number left.

Thus, to multiply 87 by 9.	870
First annex a cypher, which multiplies by 10, and then take away once 87.	87
	783=9 times 87.

So, because 99 is 1 less than 100, we can multiply by 99, by annexing *two cyphers,* and subtracting *once the multiplicand.* And, to multiply by 999, we may annex *three cyphers* and subtract *once the multiplicand,* and so on. Hence,

To multiply by 9, *or by a number consisting of several* 9*s,* ANNEX TO THE MULTIPLICAND AS MANY CYPHERS AS THERE ARE 9S IN THE MULTIPLIER, AND SUBTRACT ONCE THE MULTIPLICAND.

ART. VI. 98 is two less than 100. We may, therefore, multiply by 98, by annexing *two cyphers,* and subtracting *twice the multiplicand.* 97 is *three* less than 100. We may, therefore, multiply by 97, by annexing *two cyphers,* and subtracting *three times* the multiplicand. 998 is *two* less than 1,000. We may, therefore, multiply by 998, by annexing *three cyphers*, and subtracting *twice* the multiplicand, and so on. Hence,

When the multiplier consists of 9 or 9s, with other figures following, ANNEX AS MANY CYPHERS AS THERE ARE FIGURES, AND SUBTRACT AS MANY TIMES THE MULTIPLICAND, AS THE MULTIPLIER FALLS SHORT OF A UNIT OF THE NEXT HIGHER ORDER.

The above abbreviations are the most important of those which may be made in multiplication, without the use of Logarithms. Many others might be given; but the thorough student will be able to derive them from the rules in the preceding sections, combined with the instances noticed here. Such are rules for multiplying by 19, 29, 39, 49, &c.; and also for multiplying by 55, 155, 95, 995, 955, 555, 50, 550, &c. Indeed, the practised scholar in multiplication will find few cases in which he will not be able to introduce some mode of operation, shorter than that prescribed by the general rule for multiplication, in § XVII.

◆

SUBTRACTION.

MENTAL EXERCISES.

§ XX. 1. 3 boys were skating. One fell down. How many did not fall? 1 from 3 leave how many?

2. 5 chairs are standing together, but 2 have broken backs. How many are whole? 2 from 5 leave how many?

3. 6 tumblers stand on a table, and 4 are full of cider. How many are empty? 4 from 6 leave how many?

4. 8 bottles are on a shelf, and 3 have their noses broken off. How many are whole? 3 from 8 leave how many?

5. George put 9 marbles on a table, and 3 rolled off. How many were left? 3 from 9 leave how many?

6. 7 lamps stand on a table, and 4 are burning. How many are not burning? 4 from 7 leave how many?

7. A boy had 9 quills. He made pens of 4. How many were left? 4 from 9 leave how many?

8. 8 birds were on a tree. A man shot 4. How many were left? 4 from 8 leave how many?

9. 7 penknives are on a card. 5 are open. How many are shut? 5 from 7 leave how many?

10. 6 chickens are together. 2 are drinking out of a basin. How many are not drinking? 2 from 6 leave how many?

11. 12 boys were playing together, and 3 went home. How many were left? 3 from 12 leave how many?

12. A man bought several sheep for 19 dollars. He sold them for 14 dollars. How many dollars did he lose? 14 from 19 leave how many?

13. A boy had 16 marbles, and gave away 11. How many had he left? 11 from 16 leave how many?

14. In a school, consisting of 20 boys, 13 learn different studies, and the rest learn grammar. How many learn grammar? 13 from 20 leave how many?

15. A man bought some goods worth 18 dollars, and to pay for them, gave a barrel of flour worth 8 dollars, and the rest in money. How much money did he give? 8 from 18 leave how many?

16. A farmer had 21 sheep, and a dog killed 5. How many had he left? 5 from 21 leave how many?

17. In an orchard, consisting of 27 trees, 12 bear apples, and the rest bear pears. How many bear pears? 12 from 27 leave how many?

18. George had 26 cents, and spent 11. How many had he left? 11 from 26 leave how many?

19. Samuel had 12 books, and lost 3. How many had he left? 3 from 12 leave how many? 5 from 12 leave how many? 6 from 12? 8 from 12? 11 from 12? 2 from 12? 7 from 12? 10 from 12?

20. 4 from 15 leave how many? 10 from 15? 9 from 15? 8 from 15? 3 from 15? 12 from 15? 2 from 15? 13 from 15? 14 from 15? 7 from 15? 5 from 15? 6 from 15? 11 from 15?

21. 6 from 17 leave how many? 5 from 17? 7 from 17? 4 from 17? 8 from 17? 9 from 17? 3 from 17? 2 from 17? 11 from 17? 10 from 17? 12 from 17?

22. 16 men were in a boat. The boat upset and 12 were drowned. How many were left? 12 from 16 leave how many? 5 from 16? 4 from 16? 3 from 16? 7 from 16? 8 from 16? 13 from 16? 15 from 16? 9 from 16? 10 from 16? 6 from 16?

23. 5 from 13 leave how many? 4 from 13? 10 from 13? 7 from 13? 9 from 13? 12 from 13? 8 from 13? 6 from 13? 11 from 13?

24. 8 from 14 leave how many? 6 from 14? 4 from 14? 10 from 14? 7 from 14? 2 from 14? 12 from 14? 11 from 14? 3 from 14? 5 from 14? 9 from 14?

25. 6 from 18 leave how many? 7 from 18? 8 from 18? 10 from 18? 9 from 18? 5 from 18? 4 from 18? 3 from 18? 2 from 18? 11 from 18? 12 from 18? 13 from 18? 14 from 18? 15 from 18? 16 from 18?

26. 7 from 19 leave how many? 8 from 19? 6 from 19? 5 from 19? 4 from 19? 3 from 19? 2 from 19? 9 from 19? 10 from 19? 11 from 19? 12 from 19? 13 from 19? 14 from 19? 15 from 19?

27. Repeat the

SUBTRACTION TABLE.

1 from	1 leaves	0	4 from	4 leaves	0	7 from	7 leaves	0
1	2	1	4	5	1	7	8	1
1	3	2	4	6	2	7	9	2
1	4	3	4	7	3	7	10	3
1	5	4	4	8	4	7	11	4
1	6	5	4	9	5	7	12	5
1	7	6	4	10	6	7	13	6
1	8	7	4	11	7	7	14	7
1	9	8	4	12	8	7	15	8
1	10	9	4	13	9	7	16	9
2 from	2 leaves	0	5 from	5 leaves	0	8 from	8 leaves	0
2	3	1	5	6	1	8	9	1
2	4	2	5	7	2	8	10	2
2	5	3	5	8	3	8	11	3
2	6	4	5	9	4	8	12	4
2	7	5	5	10	5	8	13	5
2	8	6	5	11	6	8	14	6
2	9	7	5	12	7	8	15	7
2	10	8	5	13	8	8	16	8
2	11	9	5	14	9	8	17	9
3 from	3 leaves	0	6 from	6 leaves	0	9 from	9 leaves	0
3	4	1	6	7	1	9	10	1
3	5	2	6	8	2	9	11	2
3	6	3	6	9	3	9	12	3
3	7	4	6	10	4	9	13	4
3	8	5	6	11	5	9	14	5
3	9	6	6	12	6	9	15	6
3	10	7	6	13	7	9	16	7
3	11	8	6	14	8	9	17	8
3	12	9	6	15	9	9	18	9

28. 2 from 9 are how many? 2 from 19? 2 from 29? 2 from 39? 2 from 49? 2 from 59? 2 from 69? 2 from 79? 2 from 89? 2 from 99? 2 from 109?

[Let the teacher here take 3 from 9, 3 from 19, and so on, until he has gone through all the combinations, that can be made in this way, in the same manner as directed in Addition, §VII.]

29. 4 from 7 leave how many? 4 from 17? 4 from 27? 4 from 37? 4 from 57? 4 from 87? 4 from 47? 4 from 67? 4 from 77? 4 from 97? 4 from 107?

30. 8 from 16 leave how many? 8 from 26? 8 from 36? 8 from 46? 8 from 56? 8 from 66? 8 from 76? 8 from 86? 8 from 96? 8 from 106?

31. 7 from 15 leave how many? 7 from 25? 7 from 35? 7 from 45? 7 from 55? 7 from 65? 7 from 85? 7 from 95? 7 from 105?

32. 9 from 9 leave how many? 9 from 19? 9 from 29? 9 from 39? 9 from 49? 9 from 59? 9 from 69? 9 from 79? 9 from 89? 9 from 109?

33. 6 from 15 leave how many? 6 from 25? 6 from 45? 6 from 35? 6 from 55? 6 from 65? 6 from 75? 6 from 85? 6 from 95? 6 from 105?

34. 5 from 7 leave how many? 5 from 6? 5 from 9? 5 from 8? 5 from 11? 5 from 10? 5 from 12?

◆

§ XXI. 1. Write the following numbers on your slate, take away, or *subtract* the lower one from the upper, and set down the difference in figures.

7	8	6	5	8	9	4	5	7	9	8	5
3	5	2	3	4	7	3	2	5	6	6	4

2. Take an example of some higher orders.

90	80	60	40	70	50	30	90	200	500	600
30	70	20	30	50	20	10	60	100	200	300

800	3000	8000	90000	400000	7000000	90000000
500	1000	6000	70000	200000	6000000	40000000

3. Take an example where several significant figures are used.

20800	38009	570308	60805	47023	580321469
10400	21003	140205	30201	21012	210110357
10400	17006				

Hence, it is evident, that, *in finding the differences of numbers, units of the same orders must be taken from each other, and their differences put in the places of the orders employed.*

[The pupil should be taught to explain operations in Subtraction by repeating the process aloud as in Addition.]

16. Here are some further examples.

93570224	76583424	56249879	45987688	5795786314
82350113	14372312	33126663	32744352	2133332202
——	——	——	——	——

29763022	67896789	23232323	78978978	592800394267
12341011	12341234	12111211	23562467	220000013002

The pupil will observe, that it matters not *on which side* he begins to perform the above examples. He will find it best, however, to accustom himself to commence on the right, as this will be *necessary* in most of the examples, which follow. Neither is it of any consequence how the given numbers are set down, nor which is written uppermost, if units are always taken from units, tens from tens, &c. But the mode which we have used above, is altogether the most convenient; viz., to place the larger number above the other, in such a manner that *the same orders may stand under each other;* that is, units under units, tens under tens, &c.

We must caution the pupil against subtracting units of *different orders* from one another, since 6 tens from 9 hundreds are neither 3 tens nor 3 hundreds. But 6 tens from 9 tens are 3 tens, and 6 hundreds from 9 hundreds are 3 hundreds.

19. A man, having 26 acres of land, sold 9. How many had he left?

9 units cannot be taken from 6 units.

26
9
——
17 *Ans.*

But 9 units can be taken from 26 units, and the remainder is 17.

20. A merchant had 53 yards of cloth, and sold 16 yards; how many had he left?

6 units cannot be taken from 3 units. But 6 units may be taken from 53 units, leaving 47 units, or 4 tens and 7 units. From this, take the 1 ten, and there remain 3 tens, and 7 units, or 37.

53
16
——
37 *Ans.*

21. Let the following examples be performed in this way.

72	91	34	76	321	465	317	645	239
34	62	15	68	113	219	222	173	164
——	——	——	——	——	——	——	——	——
38	29							

1542	2187	58632	32984	53890	3124
1213	1234	13472	15763	41763	1216

We might perform all examples in Subtraction in this manner: but an easier method is the following.

22. In a cistern there were 73 gallons of water, and 25 were used out. How many were left?

Instead of proceeding as before, I first separate the 73, which is 7 tens and 3 units, into 6 tens, and 1 ten and 3 units; or 6 tens and 13 units, thus,

I then separate the 25 into 2 tens, and 5 units, thus,

Then 5 units from 13 units, leave 8 units; and 2 tens from 6 tens, leave 4 tens. The answer therefore is 4 tens and 8 units, or 48.

$$\begin{array}{rcrcr} 60 & + & 13 & = & 73 \\ 20 & + & 5 & = & 25 \\ \hline 40 & + & 8 & = & 48 \end{array}$$

This process is nothing more than taking one of the tens, and writing it with the units, for the sake of convenience. This is called BORROWING, and is the reverse of *carrying* in Addition.

23. The United States declared themselves independent on the 4th of July, 1776. How many years from that time, to the 4th of July, 1829?

Here, the tens in the less number are more than those of the greater. We must, therefore, *borrow* one of the hundreds. Separating then as before, we have

$$\begin{array}{l} 1000+700+120+9=1829 \\ 1000+700+\ 70+6=1776 \\ \hline \end{array}$$

And,

Therefore, the answer is 0+ 0+ 50+3= 53 years.

Now, this process may be performed *in the mind*, without any *actual separation* of the parts. For, we see, *that the figure from which we borrow, becomes one less, and, the number to which we unite what we borrowed, becomes* 10 *greater, because it is of a lower order.*

24. A man raised 763 bushels of wheat, one year, and 851, the next. How much more did he raise the second year than the first?

Here, we increase the 1 unit by 10, making it 11 units, from which we subtract the 3 units, leaving 8. We then diminish the 5 tens by 1 ten, leaving 4 tens. But we cannot take 6 tens from this. Therefore, we borrow again in like manner.

$$\begin{array}{r} 851 \\ 763 \\ \hline 88 \end{array}$$ *Ans.*

In the same manner, perform the following.

25. From a garrison containing 9,563 men, there were sent away 2,764. How many were left? *Ans.* 6,769

26. In 1,800 there were 903 post-offices in the United States, and in 1,828 there were 7,530. How much had their number increased. *Ans.* 6,627

27. During the whole American revolution, the continental troops employed were 231,971, and the militia 56,163. How many more continental troops, than militia? *Ans.* 175,808

You have now been learning to perform SUBTRACTION. Then,

SUBTRACTION IS FINDING THE DIFFERENCE BETWEEN TWO NUMBERS.

Of course, *it is exactly the reverse of Addition.*

THE DIFFERENCE FOUND IS CALLED THE REMAINDER,

THE NUMBER, SUBTRACTED, IS CALLED THE SUBTRAHEND,

THE NUMBER, FROM WHICH THE SUBTRACTION IS MADE, IS CALLED THE MINUEND.

— between two numbers, signifies that the latter is to be subtracted from the former; thus, 5—2=3 signifies that 5 less 2 equals 3. This sign is called *minus*, which is a Latin word meaning *less*.

From the preceding examples, we derive the following rule.

PLACE THE SUBTRAHEND UNDER THE MINUEND, SO THAT THE SAME ORDERS MAY STAND UNDER EACH OTHER. BEGIN AT THE RIGHT, AND TAKE EACH FIGURE IN THE SUBTRAHEND FROM THE ONE ABOVE IT. IF THE FIGURE IN THE MINUEND BE TOO SMALL, INCREASE IT BY 10, AND DIMINISH THE NEXT HIGHER ORDER OF THE MINUEND BY 1.

◆

§ XXII. 1. A man bought a farm for 7,608 dollars, and being in want of money, sold it for 6,139 dollars. What did he lose?

7000+500+90+18	7608
6000+100+30+ 9	6139
1000+400+60+ 9	1469 *Ans.*

Here, we are obliged to borrow in the units place. But there are no tens in the upper number. Now, as we must borrow a *ten*, and there are no *tens* to take it from, we must take it from the *hundreds*. We have separated the orders, to show how this is done. We take 1 hundred from the 6 hundreds, leaving 5 hundreds. This 1 hundred contains *ten tens*, of which we borrow *one ten*, and set down the remaining *nine tens* in the *tens' place*. We then subtract each order of the lower separately. It is plain that, without separating the orders, we should have obtained the same answer, if after borrowing, we had called the 6 hundreds, 5 hundreds, and the 0 tens, 9 tens.

2. A man owns stock in a bank, to the amount of 5,003 dollars, but the bank failing, he sold it for 1,328 dollars. What did he lose?

4000+900+90+13	5003
1000+300+20+ 8	1328
3000+600+70+ 5	3675 *Ans.*

The result is here, evidently the same as it would have been, if, after borrowing, we had called the 5 thousands, 4 thousands, and the 0 hundreds and the 0 tens, 9 hundreds and 9 tens.

3. A man, having 60,000 dollars, lent 13,846. How many had he left? *Ans.* $46,154.

4. In a storm, the crew of a vessel were obliged to throw overboard goods to the amount of $69,893. The whole cargo was worth $200,000. How much was left? *Ans.* $130,107.

5. A man had $30,000 of which he invested $7,600 in bank stock. How much was left? *Ans.* $22,400.

6. A person owed $9,000.00 and paid $6,872.32. How much did he then owe? *Ans.* $2,127.68.

7. Take 936,859 from 3,000,331

8. " 72,869,993 " 230,470,000

9. Take 87,964,321 from 230,449,000

10. " 230,112,780 " 1,275,003,006

Hence, IF A CYPHER OR CYPHERS OCCUR WHERE THE RULE REQUIRES A FIGURE OF THE MINUEND TO BE DIMINISHED, CALL EACH CYPHER A 9, AND DIMINISH THE NEXT SIGNIFICANT FIGURE BY 1.

EXAMPLES FOR PRACTICE.

11. A person had property to the amount of $62,007, of which he lost by a fire, $12,148. What had he left? *Ans.* $49,859.

12. A wine merchant bought 721 pipes of wine for $90,846 and sold 543 pipes for 86,049 dollars. How many pipes has he left, and what are they worth to him? *Ans.* 178 pipes—worth to him $4,797.

13. From 200,000 take 99,999. *Ans.* 100,001.

14. From 100,000 take 55,555. *Ans.* 44,445.

15. From 360,418 take 293,752. *Ans.* 66,666.

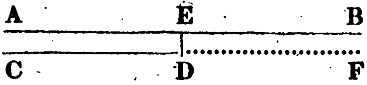
16. From 54,026 take 9,254. *Ans.* 44,772.

After *borrowing*, the rule in most Arithmetics requires to *increase* the next figure of the *subtrahend*, instead of *diminishing* the next figure of the *minuend* by 1. The effect upon the *answer* is the same in both cases. We have given that which is simplest in principle, and easiest in practice. We will illustrate the other method, however, by an example, and leave the student to choose for himself.

From 32
Take 16
—
16 *Ans.*

Here we borrow, and, by our rule, must diminish the 3 tens by 1 ten, leaving 2 tens. From this, we take the 1 ten, and the remainder is 1 ten.

The other rule does not diminish the 3 tens, but increases the 1 ten, by 1 ten; making 2 tens. This taken from 3 tens, leaves 1 ten remainder, as before.



We have here two lines, marked A B, and C D. If we should take from A B, a portion equal to C D, (viz. A E) there would remain E B.

Then A B may be called a minuend, C D, a subtrahend, and E B, the remainder.

If to C D, I should add a portion (D F) equal to the remainder E B, the whole, C F, would be just as long as the remainder A B. Hence, to prove Subtraction,

I. ADD THE REMAINDER AND THE SUBTRAHEND. THEIR SUM OUGHT TO EQUAL THE MINUEND.

Also, if from the minuend, A B, I take the remainder, E B, a portion, A E, will be left, just as long as the subtrahend, C D. Hence, to prove Subtraction,

II. TAKE THE REMAINDER FROM THE MINUEND. THE DIFFERENCE OUGHT TO BE EQUAL TO THE SUBTRAHEND.

EXAMPLES FOR PRACTICE.

21. Take one from one million.

22. A man having $365.90 lays out $168.99. What has he left?

23. In Boston, in 1,810, there were 33,250 inhabitants, and in 1,820, 43,278. What was the increase in 10 years?

24. In 1,820, in Boston there were 43,278 inhabitants, and in New-York 123,706. What was the difference in numbers?

25. A man's income is $2,963.47 and he spends $1,596.89. What does he save?

26. A gentleman bought a house and garden for $12,963.31. The house alone was worth $10,876.69. What was the garden worth, and what was the house worth, more than the garden?

27. 7,800 — 4,390
28. 40,809 — 13,963
29. 334,657 — 211,761
30. 289,367 — 189,367
31. 573,842 — 473,841
32. 27,635,231,594,333 — 1,999,998,764

OBSERVATIONS ON SUBTRACTION FOR ADVANCED PUPILS.

§ XXIII. The pupil will perceive that Subtraction is a process exactly the *reverse of Addition*. For, as Addition teaches to bring several numbers together into *one sum*, so Subtraction teaches to separate a *single number* into *two others*. He will also see that it is not only reverse *in principle*, but also in all its practical operations. For, while, in Addition, we often *carry something forward* to the next higher order; in Subtraction, we *take back*, or *borrow*, from that order. Hence, also, it will be evident, that, in performing an operation, in which it is necessary to *borrow*, we must always begin on the right. For, in *borrowing*, we take a unit from the *next higher order*. Therefore, we must proceed from right to left, and of course begin at the right hand.

The order of writing, prescribed by the rule, is only adopted for convenience. It brings the individual figures between which the subtractions are to take place, near each other. Which number should stand uppermost, is a matter of indifference. The arrangement, prescribed by the rule is the one usually adopted.

Besides the modes of proof given in the last section, Subtraction may be proved by casting out 9s. As the Remainder and Subtrahend, added together, are equal to the minuend, the rule is the same, in principle, with that for proof of Addition. The excess of 9s in the Minuend, ought to equal the excess, obtained from the Remainder and Subtrahend. Hence, to prove Subtraction,

I. CAST OUT THE 9s FROM THE REMAINDER AND SUBTRAHEND, AND SET DOWN THE EXCESSES OPPOSITE.

II. CAST OUT THE 9s FROM THESE EXCESSES, AND PLACE THE RESULTING EXCESS OPPOSITE THE MINUEND.

III. CAST OUT THE 9s FROM THE MINUEND, AND, IF THE OPERATION HAS BEEN CORRECTLY PERFORMED, THE EXCESS WILL AGREE WITH THAT LAST FOUND.

DIVISION.

MENTAL EXERCISES.

§ XXIV. 1. A boy divided 4 apples equally between 2 of his companions. How many did he give each? If you divide 4 into 2 equal parts, how many will there be in each part?

2. A gentleman bought 6 little books for 2 of his children. How many did he give each? If you divide 6 into 2 equal parts, how many will there be in each part?

3. A boy, having 12 peaches, divided them equally among 3 of his companions. How many did he give each? If you divide 12 into 3 equal parts, how many will there be in each part?

4. A man placed 16 sacks of grain in 4 equal heaps. How many sacks in each heap? If you divide 16 into 4 equal parts, how many will there be in each part?

5. George arranged 18 marbles in 3 equal rows. How many marbles in each row? If you divide 18 into 3 equal parts, how many will there be in each part?

6. Lucy put 24 pins on a square pin-cushion, so that there was an equal number on each side. How many pins on each side? If you divide 24 into 4 equal parts, how many will there be in each part?

7. George made a five cornered figure with 30 pencils, putting an equal number of pencils on each side of the figure. How many pencils on each side? If you divide 30 into 5 equal parts, how many will there be in each part?

8. In a window there are 35 panes of glass in 7 rows. How many panes in each row? If you divide 35 into 7 equal parts, how many will there be in each part?

9. A man put 36 logs in 6 equal piles. How many logs in each pile? If you divide 36 into 6 equal parts, how many will there be in each part?

10. 5 boys found a purse containing 25 silver dollars, and agreed to share the money equally. How many dollars had each boy? If you divide 25 into 5 equal parts, how many will there be in each part?

11. 20 boys chose sides to play ball. There were 2 sides. How many on a side? If you divide 20 into 2 equal parts, how many will there be in each part?

12. If you travel 7 miles an hour, how many hours will it take you to travel 42 miles? If you divide 42 into 7 equal parts, how many will there be in each part?

13. If you divide 24 into 6 equal parts, how many will there be in each part? How many, if you divide 18? How many, if 36? If 42? If 48? If 54? If 60?

14. If you divide 14 into 7 equal parts, how many in each part? How many, if you divide 21? How many, if 28? If 35? If 42? If 49? If 63? If 56?

15. If you divide 64 into 8 equal parts, how many in each part? If 32? If 16? If 24? If 40? If 48? If 72? If 56?

16. If you divide 33 into 11 equal parts, how many in each part? How many, if 66? How many, if 99? If 22? If 44? If 55? If 77? If 88?

17. If you divide 48 into 12 equal parts, how many in each part? If 24? If 96? If 108? If 72? If 84? If 36? If 60?

18. A man having 8 oxen, yoked them in pairs. How many pairs had he? How many *twos* in 8?

19. A boy having 12 oranges, found he could give 3 to each of his companions. How many companions had he? How many *threes* in 12?

20. A man divided 15 cents among his children, giving them 5 cents apiece. How many children had he? How many *fives* in 15?

21. A merchant had 18 tea-cups, and he told his clerk to place them in piles of 6 cups each. How many piles were there? How many *sixes* in 18?

22. I wish to divide 16 boys into classes of 4 boys each. How many classes can I have? How many *fours* in 16?

23. If a hundred dollars gain 6 dollars interest in 1 year, in how many years will it gain 48 dollars? How many 6s in 48?

24. How many times 3 in 9? In 6? In 12? In 36? In 33? In 27? In 24? In 15? In 18?

25. How many times 5 in 25? In 35? In 45? In 55? In 10? In 15? In 20? In 30? In 40? In 50? In 60?

26. How many times 4 in 8? In 32? In 16? In 24? In 40? In 36? In 44? In 48? In 52?

27. Repeat the

DIVISION TABLE.

2	in	2	1	times.	6	in	6	1	times.	10	in	10	1	times.
2		4	2		6		12	2		10		20	2	
2		6	3		6		18	3		10		30	3	
2		8	4		6		24	4		10		40	4	
2		10	5		6		30	5		10		50	5	
2		12	6		6		36	6		10		60	6	
2		14	7		6		42	7		10		70	7	
2		16	8		6		48	8		10		80	8	
2		18	9		6		54	9		10		90	9	

3	in	3	1	times.	7	in	7	1	times.	11	in	11	1	times.
3		6	2		7		14	2		11		22	2	
3		9	3		7		21	3		11		33	3	
3		12	4		7		28	4		11		44	4	
3		15	5		7		35	5		11		55	5	
3		18	6		7		42	6		11		66	6	
3		21	7		7		49	7		11		77	7	
3		24	8		7		56	8		11		88	8	
3		27	9		7		63	9		11		99	9	

4	in	4	1	times.	8	in	8	1	times.	12	in	12	1	times.
4		8	2		8		16	2		12		24	2	
4		12	3		8		24	3		12		36	3	
4		16	4		8		32	4		12		48	4	
4		20	5		8		40	5		12		60	5	
4		24	6		8		48	6		12		72	6	
4		28	7		8		56	7		12		84	7	
4		32	8		8		64	8		12		96	8	
4		36	9		8		72	9		12		108	9	

5	in	5	1	times.	9	in	9	1	times.	13	in	13	1	times.
5		10	2		9		18	2		13		26	2	
5		15	3		9		27	3		13		39	3	
5		20	4		9		36	4		13		52	4	
5		25	5		9		45	5		13		65	5	
5		30	6		9		54	6		13		78	6	
5		35	7		9		63	7		13		91	7	
5		40	8		9		72	8		13		104	8	
5		45	9		9		81	9		13		117	9	

28. 3 boys had 15 cents apiece, and they agreed to divide the whole among 5 poor persons. How many cents did they give each?

29. 9 boys had 12 marbles apiece, and they put them into 3 equal parcels. How many were in each place?

30. A man bought 5 yards of cloth at 12 shillings a yard. How many dollars did it come to, allowing 6 shillings to a dollar?

31. A man bought 4 loads of wood, at 3 shillings a load. How many dollars did it come to?

32. How many times 3, in 6 times 2? in 9 times 2? in 7 times 12?

33. How many times 7, in 3 times 14? in 2 times 21?

34. How many times 2, in 3 times 8? in 7 times 4? in 9 times 6?

35. 18 *twos* are how many 3s.? how many 4s.? how many 9s.?

36. 12 *threes* are how many 9s.? how many 6s.? how 4s.?

37. 16 *fours* are how many 8s.? how many 32s.? how many 2s.?

38. 10 *threes* are how many 5s.? how many 15s.? how many 6s.?

39. 8 *nines* are how many 3s.? how many 4s.? how many 12s.?

40. 6 *fives* are how many 15s.? how many 10s.? how many 3s.?

41. 4 *nines* are how many 12s.? how many 2s.? how many 3s.?

42. 9 *tens* are how many 5s.? how many 3s.? how many 6s.?

43. 3 *sixteens* are how many 4s.? how many 12s.? how many 8s.?

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§ XXV. The learner will observe that, in the last section, the examples have required him either,

I. TO SEPARATE A NUMBER INTO SEVERAL EQUAL PARTS, or

II. TO FIND HOW OFTEN ONE NUMBER IS CONTAINED IN ANOTHER.

The following examples are illustrations of these two operations. The same numbers are employed in both.

I. A man had 12 acres of land, which he wished to fence off into 4 equal lots. How many acres might he have in each lot?

First, take one acre for each lot. This will make 4 acres. Subtract the 4 from 12, and 8 acres are left. Then take one more acre for each, making 4 more, and subtract as before. 4 are left. Take 1 acre for each again, making 4 more, subtract again, and nothing is left. We have now distributed all the land into 4 equal lots.

12
4
—
8
4
—
4
4
—
0

This we did, by taking one acre, for each lot, 3 times successively. But one acre, taken *three times* is 3 acres. Therefore, each lot will contain 3 acres.

II. A man had 12 acres of land, which he wished to divide into lots of 4 acres each. How many lots had he?

12
4
—
8
4
—
4
4
—
0

First, take 4 acres for one lot, and 8 are left.

Then, 4 acres for another, and 4 are left.

Then, 4 acres for another, and nothing is left.

We have now taken all the lots of 4 acres, which it is possible to take from 12 acres, and nothing is left. On looking back, we see that we have taken 4 acres away 3 times. Therefore there are 3 lots.

From these illustrations, we see, that, *in both these cases, the practical operation is the same; and consists, simply, in finding how often one number is contained in another.*

We see, moreover, that, *we may find how often one number is contained in another, by repeated Subtraction.* This however, would be, often, a long and tedious process. If, for instance, it were required to find how often 3 was contained in 3,729, we should be obliged to subtract 3, *one thousand, two hundred and forty three times,* in order to obtain the answer. For this reason, a shorter method has been contrived, called DIVISION. Then,

DIVISION IS A CONCISE METHOD OF PERFORMING MANY SUBTRACTIONS OF THE SAME NUMBER. Of course, *it is exactly the reverse of Multiplication.*

You see that *two numbers are always given;* one *to be repeatedly subtracted:* and another, *from which the repeated subtractions are to be made:* or, in other words, one to *divide by,* and another, *to be divided.*

The number, given, to divide by, is called the DIVISOR.
The number, given, to be divided, is called the DIVIDEND.
The result, obtained by division, is called the QUOTIENT.

÷ *between two numbers signifies that the former is to be divided by the latter.* Thus 8÷4=2 signifies that 8 divided by 4, equals 2.

But this sign is not often used. The most proper mode of indicating Division, is, to *write the Divisor, immediately under the Dividend, with a line between them.* As the whole dividend ought to be divided, in order to render the Division complete, if there be any thing left, we must write it *over the Divisor, and annex it to the Quotient;* thereby *indicating the Division of this part,* because it cannot be actually *performed.* Thus 23+7 is written $\frac{23}{7}$ or the Division is performed thus $3\frac{2}{7}$. This will be treated of more fully, hereafter.

In Subtraction, we have seen, that the difference, left, after taking the Subtrahend from the Minuend, is called the Remainder. So, likewise, in Division, which is a concise Subtraction, if there be any thing left, after taking away the Divisor as often as possible, it receives the same name. This excess must, of course, be less than the Divisor; for, were it not, the Divisor might be taken away again; which would be impossible, were the Division completely performed. Then,

The excess sometimes left, after performing Division, is called the REMAINDER, *which must always be less than the Divisor.*

Note. *It will be seen, that, the Division cannot be considered complete, until the Remainder is annexed to the quotient, as explained above.*

In order to understand division, we must observe, that, as the quotient shows, of how many times the divisor, the dividend consists, the quotient multiplied into the divisor will equal the dividend.

The Dividend, therefore, corresponds to the Product, in Multiplication; and the Divisor and Quotient, to the two factors. This gives us a new definition.

Division is finding a factor, which, multiplied by the Divisor, will produce the Dividend.

EXAMPLES.

1. A man bought 3 barrels of cider for 9 dollars. What did he give a barrel?

Set down the dividend, 9, with the divisor, 3, on the left of it, and a line between; thus. Then, think what number you must multiply the divisor, 3, by, to produce the dividend, 9. This is 3, which place on the right of the dividend, for a quotient. Multiply the divisor by it, and place the product, 9, under the dividend, 9. Subtract it from the dividend, and nothing remains. Therefore 3 is the answer. In like manner perform the following.

```
3) 9 (3
   9
   —
   0
```

2. A farmer paid 16 dollars for ploughs, giving 8 dollars apiece. How many ploughs did he buy?

3. If you can travel 40 miles in 8 hours, how far can you travel in 1 hour?

4. With 63 dollars, how many yards of cloth can you buy at 7 dollars a yard?

5. One man is 24 days performing a piece of work. How long would 4 men be about it?

6. Divide	18	by	6	Divide	28	by	7
"	16	"	8	"	33	"	3
"	25	"	5	"	45	"	9

If the dividends in the last example were to have each a cypher on the right, it is plain, we should have the same answers, with a cypher on the right of each.

7. Divide	180	by	6	Divide	280	by	7
"	160	"	8	"	330	"	3
"	250	"	5	"	450	"	9

8. Examples of dividends with cyphers on the right.

Divide	8,000	by	4	Divide	35,000	by	5
"	6,600	"	6	"	64,000	"	8
"	4,800	"	8	"	56,000	"	7

It is plain, however, that if the dividend consist of many significant figures, we cannot find the quotient in this manner. But, as in Multiplication, we multiplied each order separately, so, in Division, we may *divide each order separately, and unite the several results.* Thus,

9. A man travelled 369 miles, at the rate of 3 miles an hour. How many hours was he travelling? 369 consists of 300+60+9.

Divide	300	by	3	Quotient	100
"	60	"	3	"	20
	9	"	3	"	3
					——
Unite the results					123 TRUE QUO'NT.

3) 369 (123
    3
   ——
    6
    6
   ——
    9
    9
   ——

The same is thus performed, without separating the orders. The quotient is found for each order, and then multiplied by the divisor, and subtracted from the dividend. On the right, cyphers are annexed to the quotients and products, to show their real value. And, on the left, these cyphers are rejected, as unnecessary in the operation. The quotient figures are likewise written, one after another, in the order of their values, instead of separately. In subtracting, also, instead of bringing down all the figures of the dividend, which remain, we bring down only that, which is next to be divided.

3) 369 (100
   300   20
   ——   3
    69 ——
    60 123
   ——
     9
     9
   ——

Of course, *if any order of the dividend is a cypher, the divisor is contained in it,* NO *times, and a cypher must* *be placed in the quotient,* so that the quotient figures, to the left of it, may stand in their proper places.

10. A man paid 802 dollars for land, at 2 dollars an acre. How many acres did he buy? *Ans.* 401.

11. I buy flour to the amount of 884 dollars, giving 4 dollars a barrel. How many barrels do I buy? *Ans.* 221.

12. Divide 777 by 7 Divide 8,642 by 2
" 606 " 3 " 6,006 " 6
" 936 " 3 " 9,999 " 9
" 482 " 2 " 3,906 " 3

[Thus far each order has been divisible by the divisor, without any remainder. The pupil should recite the process of division, in these and the following examples, aloud, giving the names of the several orders, in succession.]

In the preceding examples, it is unimportant *on which side we begin* to divide, if the quotient figures are correctly arranged. Hereafter, we shall see, that it will be necessary to begin on the *left*. It may happen that the figures of some of the orders will leave a remainder, when divided. For example,

13. At $4.00, a box, how many boxes of oranges can I buy for $72.00?

4) 72 (18
4
—
32
32
—

Here the figure in the highest order is 7. 4 is in 7, only once. Therefore, 1 is the greatest quotient figure I can obtain for that order. There is a remainder of 3. I join this 3 (tens,) that is, 30 units, to the 2 units, making 32 units, and divide both together. 4 is in 32, 8 times. Therefore 18 is the answer.

14. In 3,134 pints, how many quarts? *Ans.* 1,567.

15. In 9,836 gills, how many pints? (4 gills make a pint.) *Ans.* 2,459.

16. In 9,872 lbs. of cheese, how many cheeses of 8 pounds each? *Ans.* 1,234.

17. I put 816 lbs. of butter in boxes of 6 lbs. each. How many boxes are there? *Ans.* 136.

18. In 783,954 barley-corns, how many inches? (3 barley-corns make 1 inch.) *Ans.* 261,318.

19. At 7 dollars a barrel, how many barrels of flour can I buy for 525 dolls.?

20. A merchant sold 7 cwt. of rice for $22.75. What was that a cwt.?

21. Divide 125 by 5. 3,375 by 9. 5,274 by 9. 83,136 by 6.

Hence, *when there is a remainder left, on dividing any order, consider it as so many tens of the next lower order, to which prefix it, and divide the whole together.* This is the opposite of *carrying*, and may be called *carrying downwards.*

If it should happen, that any order of the dividend is too small to contain the divisor, a cypher must be placed in the quotient, for that order; and the order itself must be treated exactly like a remainder, in the preceding cases: that is, *it must be considered as so many tens of the next lower order*, and prefixed accordingly.

In case it should be the highest order, we must *divide the first two figures of the dividend, in order to obtain the first quotient figure.*

22. In 136 quarts, how many gallons? *Ans.* 34.
23. In 216 pints, how many quarts? *Ans.* 108.
24. In 824 quarts, how many pecks? *Ans.* 103.
25. In 8,412 quarters, how many cwt.? *Ans.* 2,103.

The process will be shorter, if, instead of *writing down* each product in order to subtract it, we perform the subtraction *in the mind:* and if, when there is a remainder after dividing any order, we prefix it, as directed above, to the next lower order, *without writing it down.* Thus,

26. In 48 pecks, how many bushels? 4 | 48
12 Quotient.

We must divide by 4. 4 is contained in 4, 1 time, and in 8, 2 times. *In this case we may set the quotient immediately under the dividend.*

27. In 56 pecks; how many bushels? *Ans.* 14.

28. Divide 7,984 by 4.		32. Divide 66,672 by 9.
29. " 8,763 " 3.		33. " 33,222 " 6.
30. " 91,888 " 8.		34. " 882,924 " 6.
31. " 47,332 " 4.		35. " 9,999,333 " 9.

This kind of mental Division is called Short Division. *When each product is actually written down and subtracted, the process is called* Long Division. *Short Division* is commonly employed, *when the Divisor is less than* 12.

Thus far every divisor has been a single figure, and the following is the process of Division, *Short Division* being employed.

I. Place the Divisor at the left of the Dividend.

II. Begin at the left, divide each order of the Dividend by the Divisor, and write the result in its proper place in the Quotient.

III. If for any order of the Dividend, a significant Quotient figure cannot be obtained, write a cypher in the Quotient; and if any thing remains after dividing any order, prefix it to the next lower order.

EXAMPLES FOR PRACTICE.

36. Divide 18,945 by 5. *Ans.* 3,789.
37. Divide 14,756 by 7. *Ans.* 2,108.
38. Divide 27,954 by 9. *Ans.* 3,106.
39. Divide 108,423 by 9. *Ans.* 12,047.

40. Divide 259,362 by 6. *Ans.* 43,227.
41. Divide 271,835 by 5. *Ans.* 54,367.
42. Divide 409,876,960 by 5. *Ans.* 81,975,392.
43. Divide 290,874,224 by 8. *Ans.* 36,359,278.
44. Divide 321,942,201 by 3. *Ans.* 107,314,067.
45. Divide 225,631,832 by 4. *Ans.* 56,407,958.
46. Divide 200,000,000,007 by 9. 800,000,202 by 6. 400,001,000 by 8. 372,984 by 4. 361,060,707 by 7. 18,325,429,002 by 2. 987,654,321,005 by 5.

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MENTAL EXERCISES.

§ XXVI. 1. George had a top, but by accident he split it into 2 equal parts. What part of the whole top would one of those pieces be called? *Ans.* ONE HALF.

Then if any thing, or number, be divided into 2 equal parts, each of those parts is called ONE HALF *of that thing.*

We have no *single* character to express a half. We therefore employ a different mode of Notation, from that used for whole numbers. As there is one thing divided, and this one thing is divided into two parts; we take a 1, to signify the one thing, and write a 2 under it, to signify that it divided into two parts; thus, $\frac{1}{2}$.

2. How many halves make a whole one? *Ans.* Two halves, written $\frac{2}{2}$.

3. A man divided an acre of ground into 3 equal parts. What part of the whole acre was one of the portions? *Ans.* ONE THIRD. What part were 2 of the portions? *Ans.* TWO THIRDS.

Then if a thing or number be divided into three equal parts, one of those parts is called ONE THIRD, *and two of those parts* TWO THIRDS.

To write *one third* we take a 1, because there is one thing divided, and write a 3 under it, because the one thing is divided into 3 equal parts. Thus *one third is written* $\frac{1}{3}$. Two thirds are twice as much as one third: we therefore take a 2, instead of a 1, and write a 3 under it; thus, $\frac{2}{3}$.

Or, it is plain, that if 2 things be divided into 3 equal parts, each part will be twice as great as if only *one* thing were divided. Now, if 1 thing be divided into 3 equal parts, each part will contain *one third* of that thing. Of course, if 2 things be divided, each part will contain *two thirds*. In order, therefore, to write two thirds, we take a 2, because 2 things are supposed to be divided, and write under it a 3, because the 2 things are supposed to be divided into 3 equal parts. Thus, *two thirds are written* $\frac{2}{3}$.

4. How many thirds make a whole one? *Ans.* Three thirds, written $\frac{3}{3}$.

5. A man cut a stick of timber into 4 equal parts. What part of the whole stick was one of the pieces? *Ans.* ONE FOURTH, OR ONE QUARTER. What part were two of them? What part were 3? Then, *if a thing or number, be divided*, *&c.*

To write *one fourth*, we take 1, &c. [Let the reasons be given.] Then, *one fourth is written* $\frac{1}{4}$, *two fourths*, $\frac{2}{4}$, *and three fourths*, $\frac{3}{4}$.

6. How many fourths make a whole one? How written?

7. If a yard of cloth be divided into five equal parts, what part of the whole, will one of the portions be? *Ans.* ONE FIFTH. What part will 2? 3? 4?

Then, *if a thing or number be divided*, *&c.*

One fifth is written $\frac{1}{5}$; two fifths, $\frac{2}{5}$; three fifths, $\frac{3}{5}$; and four fifths, $\frac{4}{5}$.

8. How many fifths make a whole one? How written?

9. A box, capable of holding a bushel, is divided by partitions into 6 equal parts. What part of a bushel will one of these divisions hold? What part will 2? 3? 4? 5?

Then, *if a thing or number be divided*, *&c.*

One sixth is written $\frac{1}{6}$; two sixths, $\frac{2}{6}$; three sixths, $\frac{3}{6}$; four sixths, $\frac{4}{6}$; five sixths, $\frac{5}{6}$.

10. How many sixths make a whole one? How written?

11. There are seven days in a week. What part of a week is 1 day? What part are 2 days? 3 days? 4? 5? 6?

Then, *if a thing or number be divided*, *&c.*

12. How is one seventh written? *Ans.* $\frac{1}{7}$. Why? How are two sevenths written? Why? Three sevenths? Why? Four sevenths? Why? Five sevenths? Why? Six sevenths? Why?

13. How many sevenths make a whole one? How written?

14. In a mile are 8 furlongs. What part of a mile is one furlong? 2 furlongs? 3? 4? 5? 6? 7?

Then, *if a thing or number be divided*, *&c.*

15. How is one eighth written? *Ans.* $\frac{1}{8}$. Why? How are two eighths written? Why? Three eighths? Four eighths? Five eighths? Six eighths? Seven eighths? Why?

16. How many eighths make a whole one? How written?

17. In one square yard are 9 square feet. What part of a square yard is a square foot? 2 square feet? 3 square feet? 4? 5? 6? 7? 8?

Then, *if a thing or number be divided, &c.*

18. How is one ninth written? *Ans.* $\frac{1}{9}$ Why? Two ninths? Three ninths? Four ninths? Five ninths? Six ninths? Seven ninths? Eight ninths? Why?

19. How many ninths make a whole one? How written?

20. In one cent, are 10 mills. What part of a cent is a mill? 2 mills? 3 mills? 4? 5? 6? 7? 8? 9?

21. How is one tenth written? *Ans.* $\frac{1}{10}$. Why? Two tenths? Three tenths? Four tenths? Five tenths? Six tenths? Seven tenths? Eight tenths? Nine tenths? Why?

22. How many tenths make a whole one? How written?

We have now been learning the Notation, (that is the *manner of writing*) for *part of numbers*, as far as tenths. It is unnecessary to go farther, for, by observing the above, the pupil will be able to write, for himself, any part, or number of parts of a unit, or any single part of a higher number. For he will observe, that two numbers are always employed, with a line between them; and that the number, from which the *name* of the part is derived, is always written below this line. If, then, he wishes to take one such part of *a unit*, he writes a 1 above the line; if he wishes to take 2 such parts, he writes a 2 above the line; if 3, he writes a 3; if 4, a 4; if 5, a 5; if 723, he writes 723 above the line. Or, if he wishes to take one such part of *two* units; he writes a 2 above the line; if of 3, a 3; if of 4, a 4; if of 347, he writes 347 above the line.

EXAMPLES FOR PRACTICE.

1. Write six twentieths. *Ans.* $\frac{6}{20}$. Write one twentieth of 6. *Ans.* $\frac{6}{20}$.

NOTE. We see that the two answers are the same, that is, that *one twentieth of six, is equal to six twentieths of* 1.

2. Write 15 forty sevenths. *Ans.* $\frac{15}{47}$. One forty seventh of 15. *Ans.* $\frac{15}{47}$.

3. Write 12 twenty sixths. 13 four hundred and fifty eighths.

4. Write 25 eighty fourths. 3 fifty ninths. 2 fortieths.

PROMISCUOUS EXAMPLES FOR MENTAL EXERCISE.

1. What do you understand by one half of any thing? *Ans.* If any thing, or number be divided, &c.

2. What do you understand by one twentieth of any thing? 6 twentieths?

3. What do you understand by 4 fifths of any thing? 3 fifths?

[The teacher should continue to ask questions similar to these, until the pupil answers without hesitation.]

4. 2 is $\frac{1}{2}$ of what number? 3? 4? 5? 6? 7? 8? 9? 10? 11?

5. 2 is $\frac{1}{3}$ of what number? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

6. 2 is $\frac{1}{4}$ of what number? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

7. 2 is $\frac{1}{5}$ of what number? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

8. 2 of $\frac{1}{6}$ of what number? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

9. 1 is $\frac{1}{2}$ of what number? 1 is $\frac{1}{3}$ of what number? 2 is $\frac{2}{3}$ of what number?

10. 1 is $\frac{1}{4}$ of what number? 2 is $\frac{2}{4}$ of what number? 3 is $\frac{3}{4}$ of what number?

11. 1 is $\frac{1}{5}$ of what number? 2 is $\frac{2}{5}$ of what number? 3 is $\frac{3}{5}$ of what number?

12. 2 is $\frac{1}{7}$ of what number? 4 is $\frac{2}{7}$ of what number? 6 is $\frac{3}{7}$ of what number?

13. 4 is $\frac{2}{3}$ of what number? 6 is $\frac{2}{3}$ of what number? 12 is $\frac{2}{3}$ of what number?

14. 6 is $\frac{3}{4}$ of what number? 9 is $\frac{3}{4}$ of what number? 12 is $\frac{3}{4}$ of what number?

15. 8 is $\frac{4}{9}$ of what number? 12 is $\frac{4}{9}$ of what number? 16 is $\frac{4}{9}$ of what number?

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§ XXVII. Expressions like the above are called FRACTIONS. Then,

FRACTIONS ARE EXPRESSIONS FOR PARTS OF NUMBERS.

They are called *fractions* from a Latin word which means *broken;* because they stand for numbers divided or *broken* into parts.

The term *integer*, a Latin word, signifying *whole*, is applied to the one whole thing or unit, of which fractions are broken parts.

THE LOWER NUMBER IN A FRACTION IS CALLED THE DENOMINATOR; because from this, the Fraction receives its *name*, or *denomination.*

THE UPPER NUMBER IN A FRACTION IS CALLED THE NUMERATOR; because from this, we know the *number of parts*, for which the Fraction stands. The Numerator is, usually, *less* than the Denominator. When this is the case, *the Fraction is called a* PROPER FRACTION. Sometimes, however, the Numerator is *not less* than the Denominator, but is equal to it, or greater. When this is the case, *the Fraction is called an* IMPROPER FRACTION.

It has been seen, that, when the Numerator and Denominator are equal, the Fraction is equal to a whole one, or a unit, Thus, $\frac{2}{2}$, $\frac{3}{3}$,

$\frac{4}{4}$, $\frac{9}{9}$, $\frac{20}{20}$, &c. are each equal to 1. Then, as the Numerator of a Proper Fraction is never as great as its Denominator, A PROPER FRACTION IS ALWAYS LESS THAN A UNIT. Of course, AN IMPROPER FRACTION IS NEVER LESS THAN A UNIT.

$\frac{5}{4}$ is an Improper Fraction. It may be separated into the two Fractions $\frac{4}{4}$ and $\frac{1}{4}$. $\frac{4}{4}$, as we have seen, is equal to 1. Therefore $\frac{5}{4}$ is equal to 1 and $\frac{1}{4}$, or, as it is commonly written, $1\frac{1}{4}$. A whole number and a Fraction, written thus, are together called a MIXED NUMBER. $\frac{10}{5}$ is an Improper Fraction, and may be separated into $\frac{5}{5}$ and $\frac{5}{5}$. But each of these is equal to 1. Therefore, they are together equal to 2. $\frac{11}{5}$ is $\frac{1}{5}$ more than $\frac{10}{5}$. Hence, $\frac{11}{5}=2\frac{1}{5}$.

Hence it appears, that, *as often as the Denominator of an Improper Fraction is contained in the Numerator, so many whole ones, or Integers are contained in the Fraction; and, that if the Denominator will not divide the Numerator exactly, a Proper Fraction will remain.* Hence, also, any Improper Fraction may be changed, or *reduced* to a whole or mixed number; and in order thus to *reduce* it, we must,

DIVIDE THE NUMERATOR BY THE DENOMINATOR; WRITE THE REMAINDER, IF THERE BE ANY, OVER THE DENOMINATOR, AND ANNEX THE FRACTION, THUS FORMED, TO THE QUOTIENT.

EXAMPLES FOR PRACTICE.

1. Reduce $\frac{39}{4}$ to a whole or mixed number. *Ans.* $9\frac{3}{4}$.
2. Reduce $\frac{57}{6}$. *Ans.* $9\frac{3}{6}$. $\frac{85}{9}$. *Ans.* $9\frac{4}{9}$. $\frac{76}{5}$. *Ans.* $15\frac{1}{5}$. $\frac{19}{8}$. *Ans.* $2\frac{3}{8}$.
3. Reduce $\frac{158}{3}$. *Ans.* $52\frac{2}{3}$. $\frac{2825}{5}$. *Ans.* 565. $\frac{21825}{9}$. *Ans.* 2425.
4. Reduce $\frac{8121}{3}$. $\frac{6276}{6}$. $\frac{518432}{8}$. $\frac{915873}{7}$. $\frac{1325965}{5}$.
5. Reduce $\frac{987654321}{3}$. $\frac{700070007}{7}$. $\frac{600304002}{6}$.
6. Reduce $\frac{7112345499}{9}$. $\frac{4956350217}{7}$. $\frac{3322211136}{6}$. $\frac{592483221768}{8}$.

In the following examples, this process is reversed.

1. How many 4ths. in 1? How many in $1\frac{1}{4}$? In $1\frac{2}{4}$? In $1\frac{3}{4}$?
2. How many 5ths. in 1? In 5? In $1\frac{1}{5}$? In $1\frac{3}{5}$? In $7\frac{4}{5}$?
3. How many 7ths. in 7? In 8? In 12? In $7\frac{3}{7}$? In $5\frac{6}{7}$?
4. How many 12ths. in $9\frac{5}{12}$? In $7\frac{6}{12}$? In $3\frac{4}{12}$? In $5\frac{5}{12}$? In $8\frac{7}{12}$?
5. How many 6ths. in 3? In 4? In $5\frac{2}{6}$? In $7\frac{5}{6}$? In 8? In $9\frac{4}{6}$? In 12?
6. How many 27ths. in 3? In 2? In $5\frac{9}{27}$? *Ans.* $\frac{81}{27}$. $\frac{54}{27}$. $\frac{144}{27}$.
7. How many 19ths. in 15? In $13\frac{3}{19}$? In $17\frac{18}{19}$? *Ans.* $\frac{285}{19}$? $\frac{259}{19}$. $\frac{341}{19}$.

Here we multiply the whole number by the number expressing the parts, and add in the additional parts, if there be any. By this process, we obtain an Improper Fraction. Hence, the process is called, *reducing a whole or mixed number to an Improper Fraction.* The operation, as performed above, is as follows:

Multiply the whole number, by the number expressing the parts, and add the Numerator of the Fraction, if there be any, to the product. The sum will be the Numerator and the Multiplier, the Denominator of the resulting Improper Fraction.

◆

§ XXVIII. From the nature of Fractions, it is evident that *Division may be expressed by a Fraction.* For, in Division, the dividend is to be separated into a number of parts, denoted by the divisor. Hence,

Division may be indicated by writing the Divisor under the Dividend in the form of a Fraction.

This mode of *indicating* Division was given § xxv.

Hence, also the general principle,

All Fractions are instances of Division, in which the Numerator is the Dividend, and the Denominator the Divisor. Of course,

The value of a Fraction is the Quotient, which arises from dividing the Numerator by the Denominator.

When the dividend is not less than the divisor, division may be both indicated and performed. When it *is* less however, Division can only be indicated. We may therefore give a brief rule, for the case when the dividend is less than the divisor. We are the rather inclined to give it, though the case is very simple, because we have often seen pupils much perplexed by it.

If the dividend be less than the divisor, write the divisor under the dividend, and the fraction formed will be the answer.

EXAMPLES FOR PRACTICE.

Divide 37 by 45. 81 by 83. 97 by 120. 15 by 31. 17 by 19. 27 by 245. 383 by 384. 2 by 231. 3 by 756. 16 by 165. 4 by 71. 326 by 525. 482 by 491. 374 by 1,693.

The pupil will observe, that where a less number is divided by a greater, *the fractional quotient found, shows what part the dividend is of the divisor.* Thus, 1 is the seventh part of 7, and $1\div7=\frac{1}{7}$. 2 is two sevenths of 7 and $2\div7=\frac{2}{7}$, and so on.

It is common, likewise to extend the term *part,* to numbers, *greater than those, of which they are said to be parts;* thus. When the question is asked what part of 3 is 1, the answer is, one third, $=\frac{1}{3}$. When it is asked what part of 3 is 2, we say two thirds, $=\frac{2}{3}$. In like manner, when it is asked, what part of 3 is 4, the answer is four thirds, $\frac{4}{3}$. When it is asked what part of 3 is 5, the answer is five thirds, $\frac{5}{3}$, and so on. Hence, *Every Fraction shows what part the Numerator is of the Denominator.* Of course, INDICATING DIVISION IS FINDING WHAT PART OF THE DIVISOR IS EQUAL TO THE DIVIDEND. Therefore, *to find what part one number is of another,*

MAKE THE NUMBER CALLED THE PART THE NUMERATOR OF A FRACTION, AND THE OTHER NUMBER THE DENOMINATOR.

NOTE. This rule must be strictly followed, without regarding which number is the greater. This is sometimes called, finding the RATIO of one number to another. (§ xc.) The Fraction obtained expresses the RATIO of the denominator to the numerator.

EXAMPLES OF FINDING PARTS, OR RATIOS.

1. What part of 5 is 1 ? A. $\frac{1}{5}$. What part is 12 of 5 ? A. $\frac{12}{5}$
2. What part of 9 is 6 ? A. $\frac{6}{9}$. Is 3 ? is 15 ? is 19 ?
3. What part is 16 of 25 ? What part of 25 is 16 ?
4. What part of 3,684 is 27,942 ? What part is 3,684 of 27,942 ?

◆

§ XXIX. 1. A Gentleman paid 6,372 dollars, for 36 acres of land. How much did he give an acre ?

Here our divisor consists of two figures. We cannot therefore, divide by the last rule. But we know that 4 times 9 are 36 ; and that 4 times 9 acres are 36 acres. If we divide, then, by 4, we obtain the price of 9 acres. We can then divide this price by 9 and obtain the price of 1 acre.

```
   4)6372
   ------
   9)1593
   ------
A.   177
```

2. If 56 hogsheads of molasses cost 2,016 dollars, what cost 1 hogshead ? 2×4×7=56. Therefore, if we divide by 2, we shall obtain the price of half as many hogsheads, that is, of 28 hogsheads. If we divide this result by 4, we shall obtain the price of 6 hogsheads. If we divide this result by 7, we shall obtain the price of 1 hogshead, which is required.

```
   2)2016
   ------
   4)1008
   ------
    7)252
   ------
A.    36
```

Hence, *to divide by a composite number,* RESOLVE THE DIVISOR INTO FACTORS, AND DIVIDE BY THOSE FACTORS SUCCESSIVELY.

EXAMPLES FOR PRACTICE.

3. At 15 dollars a hhd., how many hhds. of sugar can be bought for $4,500 ? A. 300.

4. At $18 a ton, how many tons of iron can be bought for $2,250 ? A. 125.

5. If an acre of ground cost $25, what number of acres will $9,375 buy ? A. 375.

6. If a hhd. of molasses cost $27, what number of hhds. will $5,940 buy ? A. 220.

7. Divide 6,894 by 18
8. Divide 57,960 by 36
9. " 39,942 " 18
10. " 265,824 " 24
11. " 333,225 " 25
12. Divide 273,045 by 15
13. Divide 714,357 by 21
14. " 2,295,495 " 45
15. " 3,575,635 " 35
16. " 27,966,232 " 56

It is to be observed, that, in many instances, we have remainders when the division is completed. The pupil may not readily discover, in dividing by composite numbers, how to find the true remainder. We will endeavour to explain the manner, by an example.

17. How many cubic yards in 369 cubic feet?

There are 27 cubic feet in a cubic yard. Therefore, we must divide by 27=9×3. Divide first by 9 thus,

There is no remainder—Then by 3; thus,

9)369
3)41
13+2 Rem.

Here we have 2 remainder—But this does not signify that there are only 2 cubic feet left. For 41, the first quotient shows that there are 41 9's in 369. Then each unit of the quotient 41, is 9 units of the dividend, 369. Now, in dividing this 41, there is left a remainder of 2 units. But if 1 of these units is 9 units of the dividend, 2 of them will be twice as much; that is, 18 of the dividend. Of course there are 18 cubic feet left. Then, *when there is a remainder on the last Division, we must multiply that remainder, by the first divisor, to find the true remainder.*

18. At 36 dollars a ton, how many tons of iron can I buy with $762? 35=7×5 Divide by 7, thus, 7)762

Then by 5, thus, 5)108+6

21+3

Multiply the last remainder 3 by the first divisor 7, as directed by rule. 7×3=21. Then if there had been no remainder on the first Division, 21 would be the true remainder. But the first division left 6, which must, therefore, be added to the 21, making 27 the true remainder. Then when the divisor is resolved into two factors, in order to find the true remainder, we must *multiply the last remainder by the first divisor and add in the first remainder.*

For similar reasons, when the divisor is resolved into several factors, *multiply each remainder, arising after the first division, by all the preceding divisors, and add the products to the first remainder.*

19. At $25 a barrel, how many barrels of brandy can I buy for $263? A. $10\frac{13}{25}$.

20. At $27 an acre, how many acres of land can I buy for $988? A. $36\frac{16}{27}$.

21. Divide 853 by 54. A. $15\frac{43}{54}$.

22. Divide 971 by 63. A. $15\frac{26}{63}$.

23. Divide 4,761 by 45
24. " 9,893 " 72
25. Divide 333,222 by 49
26. " 641,641 " 36

◆

§ XXX. 1. At 10 cents a pound, how many lbs. of raisins can I buy for 80 cents?

We saw in Multiplication, that, if a figure were removed *one place to the left*, it was *increased in a ten-fold proportion;* that is, it was multiplied by 10.

So in Division, which is the reverse of Multiplication, if a figure be removed *one place to the right*, it is *diminished in a ten-fold proportion;* that is, *it is divided* by 10.

To divide 80 by 10, then, take away the cypher on the right, and the 8 will be one place lower; that is, it will be in the units' place instead of the tens': of course its value will be diminished ten-fold. The quotient, therefore, is 8.

Now if the dividend had been 89, instead of 80, taking away the 9 would have changed the 8 tens, to 8 units, as before; that is, would have diminished them ten-fold, or divided them by 10, and as 9 is not great enough to make another ten, it would have been the remainder. Then,

REMOVING THE RIGHT HAND FIGURE OF ANY NUMBER DIVIDES IT BY 10, AND THE FIGURE SO REMOVED IS THE REMAINDER.

The same reasoning will show *that removing two figures from the right of a number divides it by 100; and that the two figures so removed are the Remainder.* Also that *removing three, four, five figures, and so on, from the right of a number, divides it by* 1,000, 10,000, *and so on, and that the figures, so removed, are the Remainders.* Hence, to divide by a number, consisting of 1, with cyphers annexed,

REMOVE AS MANY FIGURES FROM THE RIGHT OF THE DIVIDEND, AS THE DIVISOR CONTAINS CYPHERS. THE FIGURES SO REMOVED, WILL BE THE REMAINDER; THOSE NOT REMOVED, THE QUOTIENT.

NOTE. It will not be necessary, actually to take the figures away; but merely to place a point, to separate the quotient and remainder.

EXAMPLES FOR PRACTICE.

2. How many dollars in 7,963 cents? A. $79.63.
3. How many dollars in 87,555 mills? A. $87.555.
4. 100 men were to share equally a prize of $97,543. How much was each man's share? A. $975.43.
5. In an army of 100,000 men, an amount of pay of $2,775,000 was distributed, each man sharing an equal sum with the rest. What was each man's share? A. $27.75.

6. Divide 33 by 10
7. " 45 " 10
8. " 360 " 10
9. " 945 " 100
10. Divide 81,960 by 100
11. " 230,893 " 100
12. " 679,821 " 1,000
13. " 350,325 " 10,000

§ XXXI. 1. At $40.00 a hogshead, how many hogsheads of molasses can I buy for $320.00 ?

40 is a composite number, made up of the factors 4 and 10. First divide by 10 according to the rule ; thus, 1,0)32,0

Then by 4 ; thus, 4)32

8 hogsheads.

2. Paid 80 laborers $570.00 distributing it equally. How many dollars had each ?

80=8×10 Divide by 10, 1,0)57,0

Then by 8, 8)57

7+1 Rem.

This remainder, of course, is one 10, or 10 dollars by the rule in § XXIX.

3. The paymaster of a garrison, distributed 5,845 dollars equally among 700 men. How many dollars did each receive ?

700=7×100 Divide by 100, 1,00)58,45

Then by 7, 7)58

8+2 Rem.

The 2, remainder, is of course, 2 hundreds=200, to be added to the 45, first remainder. The true remainder, is therefore 245, and is found by writing the last remainder 2, before the first remainder 45 ; since this brings the 2 to the hundreds' place where it belongs.

4. In an army of 63,474 men, how many regiments of 4,800 ? 4,800=6×8×100. A. 13$\frac{1074}{4800}$.

Hence, when there are cyphers at the right of the divisor,

I. REMOVE THE CYPHERS, AND LIKEWISE AS MANY FIGURES FROM THE RIGHT OF THE DIVIDEND.

II. DIVIDE THE REMAINING FIGURES OF THE DIVIDEND, BY THE REMAINING FIGURES OF THE DIVISOR.

III. PREFIX THE REMAINDER FOUND BY THIS DIVISION, TO THE FIGURES REMOVED FROM THE DIVIDEND, FOR THE TRUE REMAINDER.

EXAMPLES FOR PRACTICE.

5. Divide 7,861 by 30. A. 262$\frac{1}{30}$.

6. Divide 21,564 by 20. A. 1,078$\frac{4}{20}$.

7. Divide 31,943 by 300. A. 106$\frac{143}{300}$.

8. Divide 1,151 by 20 ; 2,873 by 30 ; 9,999 by 90 ; 2,864 by 80 ; 71,843 by 200 ; 59,995 by 500 ; 77,724 by 9,000 ; 325,963 by 70,000 ; 2,541,861 by 80,000.

◆

§ XXXII. But there are many cases of Division, for which the preceding rules are insufficient. For instance,

1. If 23 yards of cloth cost $322, what cost 1 yard?

```
23)322(14
   23
   --
    92
    92
    --
```

Our divisor, 23, consists of two figures. Of course, it cannot be contained in the first figure of the dividend. We must therefore, take the *first two* figures. Thus, we must find how often 23 is contained in 32. The remainder must be prefixed to the next lower order, as in the preceding examples.

Then the only difference between dividing by a single figure, and by several, consists in this; that in order to obtain the first quotient figure, *we must take as many figures of the dividend, as there are places in the divisor*. Or, if the divisor be *larger* than the same number of figures in the dividend, we must take *one more* figure in the dividend.

2. If a man's income be 1,248 dollars a year what is that per week, allowing 52 weeks to the year? A. $24.

3. A privateer took a prize of $7,735. It was equally divided among 65 men. What amount had each? A. $119.

4. A man bought 529 head of cattle for $15,341. What did he give a head? *Ans.* $29.

5. If a man's income be $49,640 a year, what is that a day allowing 365 days to the year? *Ans.* $136.

6. For $36.56 how many books can I buy at $4.57 each? *Ans.* 8.

It will be observed, that, we make a separate division for each quotient figure. The numbers, thus successively divided, are sometimes called the PARTIAL DIVIDENDS.

7. Divide 9,391 by 32. *Ans.* 293 $\frac{15}{32}$.

8. Divide 28,609 by 51. *Ans.* 569 $\frac{49}{51}$.

Hence, we obtain a *rule for division, in any case.*

I. PLACE THE DIVISOR ON THE LEFT OF THE DIVIDEND.

II. FOR THE FIRST QUOTIENT FIGURE, DIVIDE AS MANY PLACES ON THE LEFT OF THE DIVIDEND AS THERE ARE PLACES IN THE DIVISOR; OR IF THESE BE NOT SUFFICIENT, TAKE ONE MORE.

III. MULTIPLY THE DIVISOR BY THIS QUOTIENT FIGURE, PLACE THE PRODUCT UNDER THE PARTIAL DIVIDEND, FROM WHICH SUBTRACT IT, AND, TO THE REMAINDER, ANNEX THE NEXT FIGURE OF THE DIVIDEND. THIS WILL BE THE SECOND PARTIAL DIVIDEND WHICH DIVIDE AS BEFORE.

IV. PROCEED IN THIS MANNER, TILL ALL THE FIGURES OF THE DIVIDEND ARE EMPLOYED, AND IF ANY PARTIAL DIVIDEND BE TOO SMALL TO CONTAIN THE DIVISOR, WRITE A CYPHER IN THE QUOTIENT, AND TREAT IT AS A REMAINDER.

EXAMPLES FOR PRACTICE.

9. Divide 251,104 by 472. *Ans.* 532.

10. Divide 863,256 by 736. *Ans.* 1,172 $\frac{664}{736}$.

11. Divide 1,893,312 by 912. *Ans.* 2,076.

12. Divide 47,254,149 by 4,674. *Ans.* 10,110 $\frac{9}{4674}$.

13. Divide 761,858,465 by 8,465. *Ans.* 90,001.

14. Divide 119,184,693 by 38,473. A. 3,097 $\frac{33812}{38473}$.

15. Divide 280,208,122,081 by 912,314.
Ans. 307,140 $\frac{121}{912314}$.

16. Divide 7,328,946,264,418,232 by 814,313,515,623,303. *Ans.* 9 $\frac{124623808505}{814313515623303}$.

As the quotient shows how *many times* the divisor is contained in the dividend, and the remainder what is left; it is plain, that *to prove Division*, we must

MULTIPLY THE DIVISOR AND QUOTIENT TOGETHER, AND ADD THE REMAINDER TO THE PRODUCT. THE SUM OUGHT TO BE EQUAL TO THE DIVIDEND.

17.	Divide	7,154	by	14		24.	Divide	29,993	by	25
18.	"	8,961	"	29		25.	"	230,031	"	39
19.	"	13,243	"	37		26.	"	2,881,943	"	46
20.	"	9,001	"	16		27.	"	4,960,002	"	201
21.	"	11,804	"	15		28.	"	13,801,804	"	315
22.	"	17,020	"	41		29.	"	175,990,806	"	7,864
23.	"	35,700	"	26		30.	"	27,341,020,003	"	91,992

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§ XXXIII. We have given (§XVIII.) some of the tables of weights, measures, &c. The following are some that remain to be given. The first table contains the denominations of

WINE MEASURE.

4	gills, (gi.)	make	1	pint, marked	pt.
2	pints	"	1	quart "	qt.
4	quarts	"	1	gallon "	gal.
10	gallons	"	1	anker "	ank.
18	gallons	"	1	runlet "	run.
31½	gallons	"	1	barrel "	bl.
42	gallons	"	1	tierce "	tier.
63	gallons	"	1	hogshead "	hhd.
84	gallons	"	1	puncheon "	pun.
2	hogsheads	"	1	pipe or butt "	p. or b.
2	pipes	"	1	tun.	

NOTE. This measure is used for wine, brandy, spirits, mead, vinegar, honey, perry, cider oils, &c.

EAXMPLES FOR PRACTICE.

1. How many runlets in 925 gals.? A. 51 run. 7 gals.

How many tierces in 824 gals.? A. 19 tier. 26 gals.

3. How many puncheons in 976 gallons ? In 1,823 ?
A. 11 pun. 52 gals. & 21 pun. 59 gals.

4. How many hhds. gals. qts. pts. and gi. in 11,934 gi. ?
A. 5 hhds. 57 gal. 3 qts. 1 pt. 2. gi.

NOTE. Begin by dividing by 4, because 4 gi. make 1 pt. Then divide by 2, because 2 pts. make 1 qt., and so on.

5. How many tier. gal. qts. pts. and gi. in 38,254 gi. ?
A. 28 tier. 19 gal. 1 qt. 1 pt. 2 gi.

6. How many run. gal. qts. pts. and gi. in 38,254 gi. ?
A. 66 run. 7 gals. 1 qt. 1 pt. 2 gi.

7. How many gal. qts. and pts. in 218,363 pts. ?
A. 27,295 gal. 1 qt. 1 pt.

8. Change 3,834,579 gi. to hhds. gal. qts. &c.

9. How many bls. in 3,826 gal ? A. 121$\frac{29}{63}$.

The pupil will probably be at a loss how to divide by 31½. But he can find how many *half-gallons* there are in 31½, and also how many *half-gallons* there are in 3,826. Then he can divide the half-gallons in 3,826 by the half-gallons in 31½.

10. How many bl. in 15,835 gal. ?

11. How many bl. gal. &c. in 17,936 gi. ?

12. How many bl. gal. &c. in 29,853,729 gi. ?

CLOTH MEASURE.

2¼ inches, (in.)	make	1 nail,	marked	nl.
4 nails	"	1 quarter	"	qr.
4 quarters	"	1 yard	"	yd.
3 quarters	"	1 Ell Flemish	"	E. Fl.
4⅔ quarters	"	1 Ell French or aune	"	E. Fr.
5 quarters	"	1 Ell English	"	E. E.

NOTE. This measure is used for cloths, and all goods sold by th[illegible]ard or ell.

13. How many yds. in 275 nls. ? A. 17 yds[illegible] qr. 3 nls.

14. How many E. Fl. qrs. &c. in 2,753 in. ?
A. 101 E. Fl. 2 qrs. 3 nls. 1¼ in.

NOTE. The pupil must reduce the 2¼, and the 2,753 to *quarters of an inch*, before dividing, as, in example 9, he reduced his numbers to halves.

15. How many E. E. qrs. &c. in 7,286 in. ?
A. 161 E. E. 4 qrs. 2 nls. $\frac{2}{4}$ in.

16. How many yds. qrs. &c. in 27,854 in. ?

17. How many aunes in 754 qrs. ?

18. How many aunes, qrs. &c. in 5,876 in. ?

19. How many aunes, qrs. &c. in 47,854 in. ?

20. How many yds. qrs. &c. in 123,456,789 in. ?

CIRCULAR MEASURE, OR MOTION.

60 seconds, (")	make	1 minute,	marked	′
60 minutes	"	1 degree,	"	°
30 degrees	"	1 sign	"	S.

12 signs or 360°, The whole circle of the Zodiac.

NOTE. By Circular MOTION, is meant the motion of the earth and planets round the sun. Circular MEASURE is used for reckoning latitudes and longitudes on the earth. Every circle, whether great or small, is divided into 360 degrees; those degrees into 60 minutes each; these minutes into 60 seconds. Seconds are sometimes subdivided into thirds, thirds into fourths, &c. The sun *seems* to pass entirely round the earth in a circle, once in 24 hours. Of course, his apparent motion is 360 degrees, from noon to noon. *One minute* on a great circle of the earth, is a GEOGRAPHICAL MILE.

21. How many degrees in 3,600 minutes? A. 60°.

22. How many degrees in 3,600 seconds? A. 1°.

23. How many S. degrees, &c. in 217,554″? A. 2 S 0° 25′ 54″.

24. How many degrees does the sun pass over in an hour? A. 15°

25. What part of a degree does the sun pass over in a minute? How many minutes *motion* has the sun in one minute of *time?*
Last Ans. 15′.

26. BOSTON is about 75° west of LONDON. As the sun seems to come from the east to the west, it will be *past noon* at LONDON, when it is *noon* at BOSTON. How many hours difference of time between the two places? and what time will it be at LONDON when it is noon at BOSTON? A. 5 hours dif.—5 o'clock, P. M. at LONDON.

27. A gentleman in England and his friend in America agreed to look at a bright star every evening, at the same time. 9 o'clock was the hour fixed on, but as they were 75° apart, one was probably asleep, while the other was looking at the star. What was the dif. of time, and which was probably asleep?

NOTE. From these examples, we see that time is *later* in all places *east* of us, and *earlier* in all *west;* and that the difference is *one hour* for every 15° of longitude, *one minute* for every 15′ of longitude, and *one second* for every 15″ of longitude.

28. How many hours difference of time between us and the part of the earth exactly opposite to us, there being 180 degrees dif. of lon.? A[illegible]2.

NOTE. Of course, when it is noon to us, it is midnight at that place; and when it is midnight here, it is noon there.

29. Suppose a meteor so high in the heavens, as to be visible, at the same moment, at BOSTON, longitude 71° 2′ W.; at WASHINGTON, longitude 77° 43′ W.; and at the SANDWICH ISLANDS, longitude 155° W.: and suppose the time of appearance at WASHINGTON to be 5 minutes past 10, P. M. What time is it by the clocks at the other places?

LONG MEASURE.

3 barley corns, b. c.	make	1 inch,	marked	in.
12 inches	“	1 foot,	“	ft.
3 feet	“	1 yard,	“	yd.
5½ yards, or 16½ feet	“	1 rod, perch, or pole,		rd. or p.
40 rods	“	1 furlong	“	fur.
8 furlongs	“	1 statute mile,		ml.
3 miles	“	1 league,	“	l.

69½ miles (statute) or }
60 miles (geographical) } 1 degree on the earth.
360 degrees, the circumference of the earth.

NOTE. This measure is used for lengths only, without regard to breadth. [illegible] &c. are measured by it. *Leagues* are principally used in measuring [illegible] Besides the above table, [illegible]

4 inches make 1 hand; used in measuring the height of horses.
5 feet " 1 geometrical pace; used for distances.
6 feet " 1 fathom; used in measuring depths at sea.
[illegible] " 1 [illegible] } used in measuring the lengths of pendulum
[illegible] " 1 [illegible] } rods for time keepers.

30. How many miles in 3,820 rods?
A. 11 300/320, or 11 mls. 7 fur. 20 rods.

31. How many furlongs in 78,230 rods?
A. 1,955 30/40, or 1,955 fur. 30 rods.

32. How many degrees in 12,472,09[illegible] ft.?
A. 34° 9 mls. 2 fur. 28 rds. 4 yds.

33. How many mls. fur. rods, &c., in 218,300,654 in.?
A. 3,445 mls. 3 fur. 8 rds. 3 yds. 2 in.

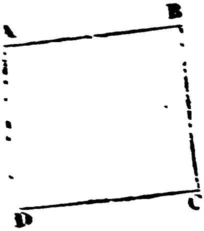
34. How many deg. in 27,999,666,333 b. c.?

35. How many hands high is a horse, that is 5 ft. in height?

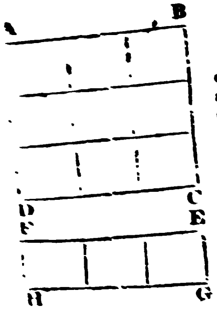
36. How many mls. in 7,[illegible]7 geometrical paces?

37. How many fathoms in 2,987 in.?

38. How many [illegible] in [illegible]576 b. c.?



A figure which is 1 inch in length, and 1 inch in breadth, and has all its corners or angles equal, is called a *square inch*. If each side is a foot long, it is called a square *foot*, &c. The form of a square is familiar to every one. A B C D is a square. In measuring surfaces, such as the floor of a room, the size of a door, the size or surface of planks or boards, portions of land, &c., we generally express the amount of surface in square inches, square feet, square yards, square rods, square miles, &c.



An ACRE, used often in measuring land, contains 160 square rods. A square yard is a square figure, one yard on each side. A B C D. represents a square yard. But a yard in length contains 3 feet; so that a square yard is a square figure, 3 feet long, and 3 feet wide. A figure 3 feet long and 1 foot wide, as F E G H, contains 3 square feet. Therefore, a figure 3 feet long, and 3 feet wide, (that is, a square yard,) contains 3 times as many, that is 9 square feet.

1. In a figure 7 inches long and 1 inch wide, how many square inches? *Ans.* 7.

2. In a figure 7 inches long, and 7 inches wide, how many square inches? *Ans.* 49.

3. Then what is the difference between 7 square inches, and a figure 7 inches square? *Ans.* 42.

4. In a figure 12 inches long and 1 inch wide, how many square inches?

5. In a figure 12 inches long and 12 inches wide, how mar
square inches?

6. Then what is the difference between 12 square inc
inches square?

7. In a figure, 9 inches long, and 1 inch
inches?

8. In a figure 9 inches long, and 9
inches?

9. Then, what is the differen
inches square?

10. How many square
inch wide?

11. How mar
inches wide

12. H
inche

1

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4 roods	"	**1** acre	"	acre.
640 acres, or **102,400** square rods,	"	**1** square mile,	"	sq.ml.

NOTE. In measuring land, a chain, called Gunter's chain, 4 rods in length, is used. It is divided into 100 links. Of course, 25 links make a rod, and 25 times 25=625 square links make a square rod. In 4 rods, there are 792 inches. Of course, 1 link is 7 $\frac{92}{100}$ inches.

As 100 links make a chain, it is easy to reduce any number of links to chains.
he *hundreds* will be chains, and the lower orders links. If we point off,
res from the right of any number of links, the remaining figures
is like reducing cents to dollars. (§ XV.)
4 times 4=16 square rods. An acre contains 160
refore $\frac{1}{16}$ of an acre. Therefore, square
off one figure on the right. A square
square links. In the same manner,
e chains by pointing off four fig-
ay be further reduced to acres
nks may be reduced to acres;

inks, how many

43

we make it *three* feet high, it will contain *three times as many* cubic feet, that is, 27 cubic feet. But if it be 3 feet high, it will be a yard high, and of course, the figure will be a *cubic yard*. Therefore, a figure three feet long, three feet wide, and three feet high, that is, a cubic yard, contains 27 cubic feet.

1. In a solid 7 inches long, 1 inch wide, and 1 inch high, how many solid inches?

2. In a solid 7 inches long, 7 inches wide, and 1 inch high, how many solid inches? *Ans.* 49.

3. In a solid 7 inches long, 7 inches wide, and 7 inches high, how many solid inches? *Ans.* 343.

4. Then what is the difference between 7 cubic inches and 7 inches cube, or cubed. *Ans.* 336.

5. In a solid 9 inches long, 1 inch wide, and 1 inch high, how many solid inches?

6. In a solid 9 inches long, 9 inches wide, and 1 inch high, how many solid inches?

7. What is the difference between 9 cubic inches, and 9 inches cube?

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49. How many solid inches in a cube, 31 feet on each side?

50. How many cubic feet in a cellar, 29 feet long, 18 feet wide, [illegible]d 10 feet deep?

51. How many cubic yards in a ditch 279 feet long, 9 feet wide, and 3 feet deep?

52. A room is 11 feet high, 21 feet long, and 16 feet wide. How many cubic feet of space in it?

53. How many cubic inches in a solid, 1 foot long, 1 foot wide, and 1 foot high?

SOLID OR CUBIC MEASURE.

1,728 cubic inches, (cu. in.) make 1 cubic foot, marked cu. ft.
27 cubic feet, " 1 cubic yard, " cu. yd.
166⅜ cubic yards, or } " { 1 cubic rod,
4,492⅛ cubic feet, } " { perch or pole, cu. rd. or cu. p.
32,768,000 cubic rods, " 1 cubic mile, cu. ml.
40 feet of round timber, or } 1 ton, or load.
50 feet of hewn timber, }
128 solid feet, or a solid 8 feet long, 4 feet high, 4 feet wide, } 1 cord of wood.

NOTE. A cord of wood contains what are called 8 FEET OF WOOD, or 8 CORD FEET. Then, as a cord of wood is 4 feet high, 4 feet wide, and *eight* feet long, a *foot of wood*, or a *cord foot*, is a solid 4 feet high, 4 feet wide, and *one* foot long. A *foot of wood*, therefore, contains *sixteen solid*, or *cubic* feet.

54. How many *cubic* feet in a pile of wood, 23 feet long, 5 feet wide, and 3 feet high? How many *feet of wood in* the same pile? How many cords?

Ans. 345 *cu. ft.*—21 $\frac{9}{16}$ *feet of wood*,—2 cords, 5 $\frac{9}{16}$ feet of wood, or 2 cords, 89 cu. ft.

55. In 468,374 cubic inches, how many cubic feet? *Ans.* $271\frac{86}{1728}$

56. In 86,400 solid inches, how many tons of hewn timber? *Ans.* 1.

57. In 221,184 solid inches how many cords? *Ans.* 1.

TABLE OF PARTICULARS.

12 single things,	make	1 dozen, marked	doz.
12 dozen or 144 single things,		1 gross	gro.
12 gro., 144 doz., 1,728 single things,		1 great gro.	grt.gro.
20 single things,	"	1 score,	sco.
24 sheets of paper,	"	1 quire.	
20 quires,	"	1 ream.	
200 pounds	"	1 barrel of beef or pork	
100 pounds,	"	1 quintal of fish.	
$21\frac{1}{2}$ stone,	"	1 pig of lead.	
8 pigs of lead,	"	1 fother.	

58. A man bought beef to the amount of $96,00, at 4 cents a pound. How many barrels did he buy?

59. How many grt. gro., gro., doz. and single things, in 689,[illegible] single things?

60. How many reams, quires, and sheets, in 598,476 sheets [illegible] paper?

Here we have been bringing lower denominantions to higher. The pupil has been told that this is REDUCTION ASCENDING. He will perceive that we have proceeded according to the following rule.

DIVIDE BY THE NUMBER REQUIRED OF THE DENOMINATION DIVIDED, TO MAKE ONE OF THE NEXT HIGHER. DIVIDE THE QUOTIENT BY THE NUMBER REQUIRED TO MAKE ONE OF THE NEXT HIGHER STILL, AND SO ON, TILL YOU ARRIVE AT THE DENOMINATION REQUIRED. THE SEVERAL REMAINDERS BELONG TO THE ANSWER, AND ARE OF THE SAME DENOMINATIONS AS THE DIVIDENDS FROM WHICH THEY ARE DERIVED.

EXAMPLES TO EXERCISE REDUCTION.

61. How many gallons in 586 barrels?

NOTE. $31\frac{1}{2}$ gallons make a barrel. The pupil will readily multiply by 31, but he may not be able to do so with $\frac{1}{2}$. But he should remember, that to multiply 586 by 3, we take 586 3 *times*; to multiply it by 2, we take it *twice*, or two times; to multiply it by one, we take it *once*, or *one time;* and, therefore, to multiply it by $\frac{1}{2}$, we must take it $\frac{1}{2}$ *a time*; that is, we must take *half of it*, or *divide it by* 2. So, to multiply by $\frac{1}{3}$, we must divide by 3, to multiply by $\frac{1}{4}$, we must divide by 4, &c. As $\frac{2}{3}$ is twice as much as $\frac{1}{3}$, in order to multiply by $\frac{2}{3}$, we must divide by 3 and multiply by 2; or first multiply by 2, and then divide by 3, since the result will be the same in either case. So, likewise, to multiply by $\frac{3}{4}$, we must divide by 4 and multiply by 3; or first multiply by 3, and then divide by 4, &c. Hence, *to multiply a number by a fraction*, MULTIPLY THE NUMBER BY THE NUMERATOR OF THE FRACTION, AND DIVIDE THE PRODUCT BY THE DENOMINATOR. The above example, then, is performed as follows:

$586 \times 31 = 18,166$, and $586 \times \frac{1}{2} = 293$. Then, $18,166 + 293 = 18,459$, *Ans.*

62. In 1 mile how many b. c.? A. 190,080.
63. How many geographical, and statute miles round the earth? A. 21,600 geog. mls.—24,912 stat. mls.
64. How many inches in 27 miles? A. 1,710,720.
65. How many feet round the earth? A. 131,535,360.
66. How many inches round the earth? A. 1,578,424,320.
67. How many b. c. round the earth? A. 4,735,272,960.
68. How many sq. yds. in 1 sq. ml. 570 acres, and 8 rods? A. 5,856,642.
69. How many sq. inches on the surface of the earth, there being 198,000,000 square miles? A. 794,[illegible]0,800,000,000.
70. In 201,283,905 sq. in. how many [illegible] A. 32 acres, 14 rds. 8 yds. 1 ft. 5 in.
71. In 224,765 farthings, how many £. &c. A. £234; 2; 7; 1.
72. In 243,648 farthings, how many dollars, at 6s.? A. $846.00.
73. In 32 hhds. how many pts.? How many gi.?
74. In 17,833 in. how many E. E. qrs. &c.
75. Reduce 129,302 sq. in. to sq. mls.
76. Reduce [illegible],257,864,001,000,000 solid inches, to cu. mls.
77. Reduce [illegible] cubic miles to cu. in.
78. Reduce 27 centuries to seconds.
79. Reduce 37,681,001,002,302,900,000 seconds to centuries.
80. Reduce 879 lb. 9 oz. 19 dwt. 18 grs. to grs.
81. Reduce 276,845,300,020,001 grs. to lbs.
82. Reduce 20 T. 19 cwt. 3 qrs. 19 lb. 4 oz. 3 dr. to dr.
83. Reduce 259,030,278,001,005, dr. to lbs.
84. Reduce 274,300,000,000,274,300,000,000 sec. to centuries.

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OBSERVATIONS ON DIVISION, FOR ADVANCED PUPILS.

§ XXXIV. Some arithmeticians employ a different order of arrangement, in preparing numbers for division, from that prescribed by our rule. The latter, however, is the more common and convenient.

The process of division differs materially from those which have been previously explained, in the manner of commencing. In all other cases we begin on the *right*, but here we find it necessary to commence on the left. The reason of this is, that remainders are often left in the higher order, which must, necessarily, be carried down to the lower.

When the divisor is large, we cannot always, at once, determine the quotient figure. When this is the case, it is convenient to divide the first figure of the dividend (or the first two if necessary), by the first figure of the divisor. Then, increase this figure of the divisor by 1, and divide by it again, in like manner. Two quotient figures will thus, usually, be found, and, from these (with the intermediate numbers), the true quotient figures may easily be selected. If one figure of the divisor be not sufficient to determine, two, or three may be employed.

Besides the mode of proof, already given, we have the following.

Since the divisor and the integral part of the quotient, multiplied together, reproduce the dividend, with the exception of the remainder, it is plain that if we take the remainder from the dividend, and divide the dividend, so diminished, by the quotient, the result will be the divisor. In case the remainder is less than the quotient, we need not subtract it from the dividend, before employing this method; for, on dividing by the quotient we shall obtain the divisor, with the same remainder; but when the remainder is greater than the quotient, we *must* subtract it, or we shall obtain a result, greater than the divisor, with a less remainder than before. Hence, it is best always to subtract the remainder first.

Another mode of proof is as follows. We see that in the process of Division, the dividend is exhausted by repeated subtractions, until the remainder left, is too small to contain the divisor. Now these numbers, thus taken from the dividend, together with the final remainder, ought to equal the dividend again. The order in which they stand in the work, shows the order in which they have been taken away, and in which they ought, therefore, to be added. In the following illustration, a star is placed against the numbers to be added.

```
29)7583(261
    58*
    ———
    178
    174*
    ———
      43
      29*
      ——
      14*
    ———
    7583
```

This mode is the same, in principle, with that in §XXXII, for the numbers with the star are the remainder, and the products of the divisor, by the several quotient figures; standing only in a different order of arrangement.

From this, we have a mode of proof by rejecting 9s, on the principle of that in addition. It is as follows:

I. REJECT THE 9s FROM THE REMAINDER, AND FROM EACH NUMBER SUBTRACTED DURING THE DIVISION.

II. REJECT THE 9s FROM THE SUM OF THE EXCESSES, THUS OBTAINED.

III. REJECT THE 9s FROM THE DIVIDEND, AND IF THEIR EXCESS AGREE WITH THE LAST, THE OPERATION MAY BE CONSIDERED AS CORRECTLY PERFORMED.

Or, as the divisor and quotient, multiplied, make up, (with the remainder,) the dividend, we obtain another rule, from those in Multiplication and Addition combined.

I. REJECT THE 9s FROM THE DIVISOR AND QUOTIENT, MULTIPLY THE EXCESSES THUS OBTAINED, AND REJECT THE 9s FROM THEIR PRODUCT.

II. REJECT THE 9s FROM THE REMAINDER, ADD THIS EXCESS TO THE LAST, AND REJECT THE 9s FROM THE SUM.

III. **REJECT THE 9s FROM THE DIVIDEND, AND IF THE EXCESS THUS OBTAINED, AGREE WITH THE LAST, THE OPERATION MAY BE CONSIDERED AS CORRECTLY PERFORMED.**

It was mentioned in multiplication, that processes were much shortened by the use of Logarithms. The same may be said of Division. And as multiplication is performed by the *addition* of Logarithms, so is division, on the other hand, by *subtraction*.

We will now give a few instances of

ABBREVIATED DIVISION.

ART. I. To divide by 10, we cut off one figure from the right of the dividend. But 5 is half of ten, and therefore 5 is contained in any number as often as 10 is contained in a number *twice as great.* Thus 5 is contained in 15 as often as 10 is in 30; and 5 is contained in 756 as often as 10 is contained in *twice* 756. But twice 756 is 1,512: and 10 is contained in 1,512, 151 $\frac{2}{10}$ times. Therefore, 5 is contained in 756, 151 $\frac{2}{10}$ times. Hence, *to divide by* 5,

MULTIPLY THE DIVIDEND BY 2, AND CUT OFF ONE FIGURE FROM THE RIGHT OF THE PRODUCT. THE REMAINING FIGURES WILL BE THE QUOTIENT. THE FIGURE CUT OFF WILL BE SO MANY TENTHS, TO BE ANNEXED TO THE QUOTIENT.

ART. II. To divide by 100, we cut off two cyphers. But 25 is a quarter of 100. 25 will, therefore, be contained in any number as often as 100 is contained in *four times that number.* Thus, 25 is contained in 100, as often as 100 is contained in 400. And 25 is contained in 1,874, as often as 100 is contained in 1,874×4=7,496. But 100 is contained in 7,496, 74 $\frac{96}{100}$ times. Therefore, 25 is contained in 1,874, 74 $\frac{96}{100}$ times. Hence, *to divide by* 25,

MULTIPLY THE DIVIDEND BY 4, AND CUT OFF TWO FIGURES ON THE RIGHT OF THE PRODUCT. THE REMAINING FIGURES WILL BE THE QUOTIENT; AND THE FIGURES CUT OFF WILL BE SO MANY HUNDREDTHS, TO BE ANNEXED TO THE QUOTIENT.

ART. III. 9 is 1 less than 10. Therefore, 9 will be contained in any number just as often as 10, and will leave, besides, *just as many* 1s *over.* If, then, we find how many 9s there are *in the ones over*, and add them to the number of 10s, we shall obtain the whole number of 9s in the given number. For example, in 270 there are 27 tens. Of course, there are 27 nines, and 27 *ones over.* In these 27 *ones* are 3 nines. Therefore, in 270, there are 27+3=30 nines.

Again, in 193 are 19 tens and 3 remainder. Therefore, there are 19 nines, and 19 *ones over*, and 3 remainder. In the 19 ones are 2 nines and 1 over. Therefore, in 193 are 19+2=21 nines, and 1 and 3 remainders=21 nines and 4 remainder. Take now an example.

Divide 8,875 by 9.	First by 10, thus,	887	5
	That quotient by 9, thus,	98	5
	Add these quotients and remainders, thus,	985	10

Therefore, there are 985 nines in 8,875, and 10 remainder. But this remainder contains 1 nine and 1 over. Therefore, in 8,875, are 986 nines and 1 remainder, or 986 $\frac{1}{9}$ nines. But this is not all the contraction we can make in this division. The student will at once perceive, that in dividing the quotient 98 by 9, we might employ the artifice used in dividing 8,875. That is, we might first divide it by 10, and then divide the resulting quotient by 9. In this way, we should obtain three quotients and three remainders, to be added together, thus:

First divide by 10, thus,	887	5
This quotient by 10, thus,	88	7
This quotient by 9, thus,	9	7
	—	—
Add these quotients and remainders, thus,	984	19
	2	18
	—	—
	986	1

But here we obtain a remainder, which contains 2 nines. If these be subtracted from the 19, and 2 added to the quotient 984, we obtain the same result as before, viz. 986 $\frac{1}{9}$.

But the student will see that we might have used the same artifice in dividing the second quotient, and, for the same reason, a third, fourth, fifth, &c., if we should obtain so many, so that *no division by 9 is necessary, at all*; thus,

First divide by 10, thus,	887	5
This quotient by 10, thus,	88	7
This quotient by 10, thus,	8	8
This quotient by 10, thus,	0	8
	—	—
Then add,	983	28
From the remainder 28, take three 9s and add them to the 983 nines. Then, as before, we have,	3	27
	—	—
	986	1

Thus, we have made the division by 9 very simple. It can be made more simple still. If in adding the remainders, we had *carried* from the sum, to the first column in the quotients, exactly as though the numbers were not separated, we should have carried 2, that is, 2 tens, which is 2 units more than 2 nines. We should have, therefore, taken 2 units too much from the remainder. But when we carried, if we had set down the 2, that is, *the same figure which we carried*, under the remainders' column, to be afterwards united with the remainder, this would have counterbalanced the error. This is done in the following.

887	5
88	7
8	8
0	8
—	—
985	8
	2
—	—
	10
1	9
—	—
986	1

Here we add the column of remainders, exactly as if it was a units' column in Addition. We have 2 to carry. We therefore put a 2 under the 8, and carry the 2 to the first column of quotient figures. Adding the 8 and 2, we have 10, which contains 1 nine. This 1 nine we subtract from the remainder, and add to the quotient; and then we have the same result as before.

We have been thus full upon the division by 9, because it is the same in principle with that which follows, by 99,999, &c. 99 is 1 less than 100. Therefore, 99 will be contained in any number, just as often as 100, with just as many *ones over*. If we find how many 99s there are contained *in these ones over*, and add them to the number of 100s, it will give us the whole number of 99s in the given number. This case, then, is just like the last, except that we must divide repeatedly by 100, instead of by 9, thus,

How often is 99 in 37,645.

Divide by 100, thus,	376	45
This quotient by 100, thus,	3	76
This quotient by 100, thus,	0	03
	—	—
We observe the same rule here in carrying as before.	380	24
		1
	—	—
	280	25

The case of 999 is the same, except that we divide repeatedly by 1,000; of 9999, except that we divide by 10,000, &c.

Hence, *to divide by a number consisting of* 9, *or of several 9s*:

I. DIVIDE REPEATEDLY BY A UNIT OF THE NEXT HIGHER ORDER, UNTIL YOU OBTAIN 0 FOR A QUOTIENT FIGURE.

II. ADD THE SEVERAL QUOTIENTS, AND LIKEWISE THE SEVERAL REMAINDERS THUS OBTAINED, CARRYING FROM THE REMAINDERS TO THE QUOTIENTS, AS IN COMMON ADDITION, AND PLACING THE FIGURE SO CARRIED, UNDER THE UNITS' COLUMN OF THE REMAINDERS.

III. TO THE TOTAL REMAINDER OBTAINED BY THIS ADDITION, ADD THIS FIGURE, AND IF THE SUM EQUAL OR EXCEED THE GIVEN DIVISOR, SUBTRACT THAT DIVISOR FROM IT, AND ADD 1 TO THE SUM OF THE QUOTIENTS. THE RESULTING QUOTIENT AND REMAINDER WILL BE THE TRUE ONES SOUGHT.

ART. IV. By a similar method we may abbreviate Division, when the units figure of the divisor is not a 9. In order to understand this, we will premise, that the difference between any number and a unit of the next higher order, is called the *complement* of the number. Thus, 2 is the complement of 98, because 100—98 =2. And 3 is the complement of 97, because 100—97=3, and so on. Now since 98 is 2 less than 100, 98 will be contained in any number just as often as 100 is contained in the same number, with just as many 2s over. Then if we find how often 98 is contained in these *twos over*, and add this number to the number of 100s, we shall obtain the whole number of 98s in the given number. Of course, after dividing by 100, we must multiply the quotient by 2, and divide it by 98. And as we may use the same artifice as before, in this second division, and then in the third, and so on, we find that this case only differs from that in Art. III, in requiring us to multiply by 2 our successive quotients, obtained from repeated divisions by 100.

This will be rendered clear by an illustration.

Divide 75,432 by 98.

Divide by 100, thus,	754	32
754×2=1,508. 1508÷100=	15	08
15×2=30. 30÷100=	0	30
Quo.	769	70 *Rem.*

We will now give a case, in which it will be necessary to carry from the remainders to the quotients.

Divide 49,999 by 98.

Divide by 100, thus,	499	99
499×2=998. 998÷100=	9	98
9×2=18. 18÷100=	0	18
	510	15
		4
Quo.	510	19 *Rem.*

Here, we had 2 to carry to the quotients. As this 2 was 2 hundreds, and as each hundred is 2 more than the divisor, 98, we must increase the remainder by 2×2=4, which is accordingly done above. Instead, then, of adding to the remainder, as before, *the figure which we carry*, we add *that figure, multiplied by the complement* 2.

It is plain, that if we were dividing by 97, we should be obliged to multiply by 3, wherever, in the above instances, we multiplied by 2. If by 96, we should be obliged to multiply by 4; if by 95, by 5; and so on: that is, we should always multiply by the *complement of the divisor*. Hence, *to divide by a number which consists of 9s, except the unit figure,*

PROCEED BY THE LAST RULE, EXCEPT THAT YOU MULTIPLY EACH SUCCESSIVE QUOTIENT BY THE COMPLEMENT OF THE DIVISOR, BEFORE YOU DIVIDE AGAIN; AND THAT YOU ADD TO THE REMAINDER, THE PRODUCT

OF THE COMPLEMENT AND THE NUMBER CARRIED TO THE QUOTIENT, INSTEAD OF THE NUMBER CARRIED, ITSELF.

Art. V. When we employ long division, we multiply the divisor by each quotient figure, and set down the product before subtracting it from the partial dividend. We may somewhat shorten the process, by subtracting each figure of this product, *as we multiply*, and noting down only the remainder, thus:

Here our first quotient figure is 1. We multiply the divisor by it, and subtract each figure of the product, as we multiply, setting down only the remainder, 221. To this we annex the next figure of the dividend, 3. We then multiply and subtract, as before, and so on. Our final remainder is 740; our quotient 128. In this operation, there is so much to be retained in the mind, that it requires considerable acquaintance with numerical calculations, to render it very useful.

```
764)98532(128
     2213
     ----
     6852
      740
```

Art. VI. When the divisor and dividend are large, it will often be found useful to make a table of products of the divisor, by the nine digits. For each quotient figure must be one of these digits or a cypher; and, by means of the table, we can tell, at a glance, how many times the divisor is contained in each partial dividend. We have, then, the quotient figure and its corresponding product, and, of course, have nothing to do but subtract.

These are the most important contractions in Division. We have now been attending to Notation or Numeration, Addition, Multiplication, Subtraction, and Division. These are called the fundamental or ground rules of Arithmetic, because, by their various combinations, all arithmetical operations are performed. Evolution might, perhaps, seem an exception to this remark; but a little reflection will convince any one, that this process is only a case of division, in which the divisor and quotient are both unknown, but are required to be equal. For division consists in *resolving a number into factors*, and evolution, in *resolving a number into equal factors*.

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COMPOUND NUMBERS.

ADDITION.

MENTAL EXERCISES.

§ XXXV. 1. Four boys gathered chesnuts. The first gathered 2 qts.; the second, 3 qts.; the third, 7 qts., and the fourth, 5 qts. How many pecks did they all gather? Then, 2 qts. and 3 qts. and 7 qts. and 5 qts. are how many pecks?

2. A silversmith melted together several pieces of silver. as follows; one weighing 10 oz.; another 11 oz.; another, 5 oz.; and another 6 oz. How many lbs. did he melt. Then, 10 oz. and 11 oz. and 5 oz. and 6 oz. are how many lbs.?

3. In 3s. 4d. and 5s. 6d. and 2s. 1d. how many shillings and pence?

4. In 4s. 1d. and 8s. 3d. and 2s. 11d. how many shillings and pence?

5. In 16s. 9d. and 7s 10d. how many pounds, shillings and pence?

6. In 3 qts. 1 pt. and 5 qts 1 pt. how many gal. and qts.?

7. In 3 qts. 1 pt. and 2 qts. 1 pt. and 1 qt. 1 pt. how many gal. qts. and pts.?

8. In 2 yds. 2 ft. and 7 yds. 2 ft. how many yds. and ft.?

9. In 6 fur. 35 rds. and 7 fur. 15 rds. how many mls. fur. and rds.?

In the above examples, the numbers have consisted of *several denominations.* Such numbers are called COMPOUND NUMBERS. Numbers consisting of *one denomination only,* are called SIMPLE NUMBERS. Write the following examples.

10. A boy paid for a book 5s. 4d. 2 qrs.; for a bunch of quills, 2s. 3d. 3 qrs.; and for a penknife 3s. 5d. 1 qr. What cost the whole?

Place like denominations under each other; then add the qrs. 1+3+2=6qrs. But 6 qrs.=1d. 2 qrs., because 4 qrs.=1d. Set down the 2 qrs. and carry the 1d. to the pence. There are then 13d.=1s. 1d. Set down the 1d. and *carry* the 1s. to the shillings. The shillings, added, are 11s., which, as they are not enough to make a £. set down under shillings. In like manner, perform the following.

s.	d.	qrs.
5	4	2
2	3	3
3	5	1
11	1	2

11. Bought a watch for 5£. 6s. and a chain for 1£. 19s. How much did I give for both? Ans. 7£. 5s.

12. A man has four farms. The first contains 100 acres, 1 rood, 20 rods; the second 75 acres, 2 roods, 10 rods; the third 150 acres, 15 rods; the fourth 125 acres. 1 rood, 5 rods. How many acres in all? Ans. 451 acres, 1 rood, 10 rods.

13. A merchant bought 4 pieces of cloth, the first containing 10 yds. 2 qrs. 1 nl.; the second 25 yds. 2 qrs. 1 nl.; the third 20 yds. 2 qrs. 2 nls. and the fourth 10 yds. 1 qr. 1 nl. How much in all? Ans. 67 yds. 1 nl.

14. Add together 38 gals. 2 qts. 1 pt. 2 gi.; 16 gals. 1 qt. 3 gi.; 20 gals. 2 qts. 1 pt. 1 gi.; 18 gals. 1 qt. 1 pt.; 7 gals. 1 qt. 2 gi.; 30 gals. 2 qts. 1 pt. Ans. 132 gals.

From these examples we derive the rule *to perform Addition of Compound numbers.*

I. PLACE THE SAME DENOMINATIONS UNDER EACH OTHER.

II. ADD, FIRST, THE LOWEST DENOMINATION. FIND HOW MANY OF THE NEXT HIGHER ARE CONTAINED IN THE SUM, WHICH NUMBER CARRY TO THE NEXT HIGHER, AND PLACE THE REMAINDER UNDER THE DENOMINATION ADDED.

III. PROCEED THUS WITH ALL THE DENOMINATIONS.

EXAMPLES FOR PRACTICE.

15. A man bought land to the amount of 69£ 13s. 5d.; farming implements to the amount of 11£ 10s.; a yoke of oxen for 15£ 6s.; a horse for 13£. 0s. 4d.; a cart for 4£ 17s. 8d. and a saddle for 19s. 4d. 2 qrs. What did the whole cost him; *Ans.* £115; 6; 9½.

16. A silversmith purchased seven ingots of silver. The first weighed 11 lb. 6 oz. 18 dwt.; the second 9 lb. 3 oz. 7 dwt.; the third 4 lb. 7 oz. 9 dwt. 5 gr.; the fourth, 8 lb. 4 oz. 6 dwt.; the fifth, 10 lb. 3 oz. 5 dwt. 19 gr.; the sixth, 7 lb. 9 oz. 0 dwt. 18 gr.; and the seventh 8 lb. 11 oz. 10 dwt. What did the whole weigh; *Ans.* 60 lb. 9 oz. 16 dwt. 18 gr.

17. George lived in Hartford until he was 14 yrs. 3 mo. 4 d. old; then he went to New-Haven, where he remained 8 yrs. 5 mo.; then he went to New-York, where he remained 3 yrs.; then to Philadelphia, where he staid 3 yrs. 2 mo. 1 d. His journeys occupied 4 days? How old was he then? *Ans.* 28 yrs. 10 mo. 1 w. 2 d.

18. A man has, in real estate £304; 5, in one place, and £247; 0; 11, in another; and in personal property, the several sums £34; 19; 7, £7; 18; 5, £45; 0; 6, and 19s. 0d. 3 qrs. How much in all? *Ans.* £640; 3; 5; 3.

19. A man brings to market 4 loads of wood, containing, the first 1 cord 60 ft. 860 in.; the second 1 cord 67 ft. 68 in.; the third 1 cord 30 ft. 300 in.; the fourth 1 cord 30 ft. 631 in. How much in all? *Ans.* 5 cords, 60 ft. 131 in

20. Bought a quantity of goods for £125; 10: paid for freight £3; 19; 6: for transportation, £2; 5: for duties £1; 15; 10: my expenses were £2; 13; 9. What was the whole expense of the goods to me?

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MULTIPLICATION.

MENTAL EXERCISES.

§ XXXVI. 1. A man gave 6d. apiece to four of his children. How many shillings did he give them?

2. 5 boys gathered 3 quarts of walnuts apiece. How many pecks did they all have?

3. Three baskets hold 1 pk. 4 qts. each. How many pks. and qts. will all hold?

4. If one bushel of grain cost 2s. 6d. how much will 2 cost? How much will 3? will 4? 5? 6? 7? 8?

5. Multiply 3 qrs. 4 nls. by 2; by 3; by 4; by 5; by 6; by 7; by 8;

6. Multiply 15 min. by 15 sec. by 4; by 8; by 12;

7. Multiply 1 pt. 3 gi. by 2; by 3; 4; 5; 6; 7; 8; 9;

8. Multiply 9 oz. 12 dwt. 13 gr. by 2; by 3; 4; 5; 6; 7; 8; 9;

9. Multiply 2 in. 2 b. c. by 2; by 3; by 4; 5; 6; 7; 8; 9;

10. Multiply 3 yds. 2 ft. 8 in. by 2; 3; 4; 5; 6; 7; 8; 9;

Perform the following on your slate.

11. A man bought 3 sheep at 1£. 10s. apiece. What cost the whole? Set the multiplier under the multiplicand, and multiply as in simple numbers, observing only *to carry from one denomination to another, as in Addition of Compound Numbers.*

£.	s.
1	10
	3
4	10

12. How much wool in 3 packs, each pack weighing 2 cwt. 2 qrs. 13 lb. A. 7 cwt. 3 qrs. 11 lb.

13. What is the value of 5 cwt. of raisins, at 2£. 1s. 8d. pr. cwt. A. 10£. 8s. 4d.

14. What is the weight of 3 doz. silver spoons, each doz. weighing 2 lb. 6 oz. 12 dwt. 3 gr. A. 7 lb. 7 oz. 18 dwt. 9 grs.

15. In 8 bales of cloth, each bale containing 12 pieces, and each piece 27 yds. 1 qr. 2 nls., how many yds.? A. 2,628.

From these examples, we derive the following rule.

PLACE THE MULTIPLIER UNDER THE MULTIPLICAND, MULTIPLY EACH DENOMINATION SEPARATELY, BEGINNING WITH THE LOWEST, AND CARRY AS IN ADDITION.

EXAMPLES FOR PRACTICE.

16. What cost 12 bu. of apples, at 1s. 9d. per bu.? A. £1. 1s.

17. What cost 9 lbs. of cinnamon, at 11s. 4½d. per lb.? A. £5; 2; 4; 2.

18. FORMS OF BILLS.

Sheffield, Jan. 1, 1828.

MR. ROBERT F. BARNARD,

BOUGHT OF WM. B. SAXTON.

4 gals. wine, at 8s. 7d. pr. gal. - - - - - -
7 reams paper, at 17s. 9½d. pr. ream, - - -
8 yds. broadcloth, at 1£. 7s. 9½d. pr. yd. - - -
12 bu. wheat, at 1s. 9d. pr. bu. - - - - - - -

Rec'd. payment, £20; 2; 2½

WM. B. SAXTON.

Hartford, March 7, 1830.

MR. WILSON WHITON,

BOUGHT OF PACKARD & BUTLER.

1 set Waverley Novels,	£5; 8; 10
3 Atlantic Souvenir, for 1830, at 13s. 6d.	2; 0; 6
1 Byron's works,	1; 0; 0
1 Johnson's do.	1; 15; 0

Rec'd. payment, £10; 4; 4

For P. & B.

WILLIAM TRUSLY.

19. If 1 load of hay weigh 1 T. 10 cwt. 2 qrs. 20 lb. 5 oz. 15 dr. what will 33 loads weigh?

20. Multiply 27 gals. 1 qt. 1 pt. 3 gi. by 28.

21. Multiply 67 yds. 3 qrs. 3 nls. by 72.

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SUBTRACTION.

MENTAL EXERCISES.

§ XXXVII. 1. A goldsmith having 1 lb. of silver, melted up 7 oz. of it. How much had he left?

2. A boy having 3 shillings, gave away 1s. 6d. How much had he left?

3. A pitcher contains 3 quarts of cider, and a man fills 3 pint tumblers from it. How much cider is left in the pitcher?

4. Take 2 gi. from 1 pt. From 1 qt. From 1 gal.

5. Take 3 nls. from 1 qr. From 1 yd. From 1 E. E. From 1 E. F.

6. Take 3 oz. from 1 lb. From 1 lb. 1 oz. From 2 lb. 2 oz.

7. Take 19 gr. from 1 ℈. From 1 ℈. 6 gr. From 3 ℈. 12 gr.

8. Take 3d. from 1 shilling. From 1s. 1d. From 3s. 2d.

9. Take 15 minutes from 1 hour. From 1h. 5m. From 7h. 7m.

10. Take 5 drams from 1℔. 1℥. From 6℥. 3ʒ. From 9℔. 1℥. 4ʒ.

Write the following examples.

11. A man's property amounted to 6,872£. 15s., and he lost a ship worth 1,539£. 17s. What was he then worth?

As we cannot take 17s. from 15s. we must *borrow* a £.=20s. This 20s. added to 15s. makes 35s., from which, 17 being taken, 18 are left. As we borrowed a £, we must make the pounds 1 less; that is, 6,871 instead of 6,872. 6871—1539=5332.

£.	s.
6872;	15
1539;	17
5332;	18

12. George had 16s. 8d. and he gave 4s. 9d. for a sled. How much had he left? *Ans.* 11s. 11d.

It will be better, instead of *diminishing the next higher denomination of the minuend,* to *increase that of the subtrahend* by 1; and the result will be the same, for 5 taken from 16, evidently leaves the same remainder, as 4 from 15, viz. 11. This is the best method in compound numbers, because, sometimes, the figure of the minuend may be a cypher. We should not be able to diminish this.

13. A man bought a keg of brandy for 3£. 0s. 6d., but in carrying it home there leaked out as much as would come to 1£. 3s. 8d. What was the value of the remainder ?

Here we borrowed a shilling. To make up for this, we *increased the* 3 *shillings in the lower number* by 1, instead of *diminishing the upper number*, which is a cypher.

£.	s.	d.	
3;	0;	6;	
1;	3;	8;	
1;	16;	10;	*Ans.*

NOTE. The intelligent pupil will perceive, that, as there are no shillings in the minuend, the borrowed shilling comes from the pounds. If we had taken a pound from the 3 pounds, and after borrowing 1s., put down the remaining 19 in place of shillings, there would have been no occasion to increase the lower number. For 19—3=16 and 20—4=16. This however, in compound numbers, would be perplexing.

It will be recollected that this method of allowing, after having borrowed, was noticed in Subtraction of Simple Numbers. (§XXII.)

14. A man commenced business with 1,850£. 0s. 10¼d. and at the end of the year, found that he was worth 2,570£. 9s. 6¼d. How much had he gained ? *Ans.* 720£. 8s. 8d.

15. I borrowed £317; 6, and afterwards paid £178; 18; 5¼. How much was then due ? *Ans.* £138; 7; 6¾.

16. A man purchased cloth to the amount of £27; 11. In turn he gave flour to the amount of £19; 17; 6, and the rest in money. How much money did he give ? *Ans.* £7; 13; 6.

From the above we derive the following rule.

PLACE THE SAME DENOMINATIONS UNDER EACH OTHER. TAKE EACH DENOMINATION IN THE SUBTRAHEND, BEGINNING WITH THE LOWEST, FROM THE SAME IN THE MINUEND, AND TO COMPENSATE FOR BORROWING FROM ONE DENOMINATION TO ANOTHER, INCREASE THE NEXT HIGHER DENOMINATION OF THE SUBTRAHEND BY 1.

EXAMPLES FOR PRACTICE.

18. A merchant had 200 bls. 16 gal. of brandy, of which he sold 82 bls. 15 gal. 1 pt. How much had he left ? *Ans.* 118 bls. 3 qts. 1 pt.

19. A man borrowed £60; 10,: and paid, at one time £17; 11; 6; at another, £9; 8; at another, £7; 9; 6; and at another, 19s. 6½d. How much then remained unpaid ? *Ans.* £25; 1; 5½.

20. I pay a debt of £105; 10, as follows; viz. I give an order on another person for £15; 14; 9, and two notes, one for £30; 0; 6, and another for £39; 11. The rest I pay down. How much do I pay down ?

21. The war between England and America commenced April 19, 1775, and continued until Jan. 20, 1783. How long did it continue ? Set it down thus.

Yrs.	mo.	d.
1783;	0;	20;
1775;	3;	19;

22. From 2 cwt. 2 qrs. 27 lb. 8 oz. 8 dr. take 3 qrs. 0 lb. 9 oz. 7 dr.

23. From 7T. 9 cwt. 3 qrs. 16 lb. 8 oz. 3 dr., take 1T. 11 cwt. 3 qrs. 17 lb. 9 oz. 6 dr.

24. From 15 yrs. 3 mo. 3 w. 2 d. 5 h. 3 m. 3 sec., take 13 yrs. 9 d. 27 sec.

25. From 289 acres, 3 roods 7 rds., take 196 acres 3 roods, 30 rds.

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DIVISION.

MENTAL EXERCISES.

§ XXXVIII. 1. A man divided 4 bushels and 2 pecks of grain, between 2 poor persons. How much had each?

2. A man divided 6 acres, 2 roods, and 8 rods of land into 2 equal fields. How much land was there in each field?

3. Five boys agreed to share 2 qts. and 1 pt. of nuts equally. How much ought each to have?

4. What is the 4th part of 5d.? What is the 6th part of 1d. 2 qrs.?

5. What is the 8th part of 1 lb. Avoidupois?

6. What is the 7th part of 1 lb. 2 oz. Troy?

7. What is the 8th part of 9℥? of 1℔. 4℥?

8. What is the 4th part of 6d.? of 9d.? of 1s. 1d.? of 2s. 2d.?

9. What is the 5th part of 6 gals. 1 qt.? of 12 gals. 2 qts.

10. What is the 9th part of 2 square yds.? of 8 square yds.?

11. What is the 11th part 1£ 2s.? of 3£ 6s.? of 12£ 2s.?

The following are to be written.

12. A man paid 4 labourers an equal sum each. To the whole he gave £10; 8. What was that apiece?

In dividing 10£ by 4, 2£ are left. 4 will not go in 2, but if 2£ be reduced to shillings, 4 will divide the number of shillings. Let this reduction be made, by multiplying by 20, and let the 8s in the given sum be added in, at the same time. The whole is 48s., which divided by 4=12s.

```
 £.  s. £.  s.
4)10 8(2  12
  8
  -
  2
 20
 --
 48
 48
```

Hence, to divide compound numbers,

I. DIVIDE EACH DENOMINATION OF THE DIVIDEND, SEPARATELY BY THE DIVISOR, AND THE SEVERAL QUOTIENTS WILL

BELONG TO THE DENOMINATIONS OF THEIR RESPECTIVE DIVIDENDS.

II. IF A REMAINDER BE LEFT, IN DIVIDING ANY DENOMINATION, REDUCE IT TO THE NEXT LOWER DENOMINATION, ADD IN ALL OF THAT DENOMINATION, IN THE GIVEN DIVIDEND, AND THEN DIVIDE AS BEFORE.

EXAMPLES FOR PRACTICE.

13. If 11 tons of hay cost £23 ; 0 ; 2, what is that per ton ? A. £2 ; 1 ; 10.

14. If 48 lbs. of cheese cost £1 ; 16, what is that per lb. ? A. 9d.

15. If 13 persons pay equally towards a bill of 5£. 8s. 10½d. how much must each pay ? A, 8s. 4½d.

16. If a nobleman's salary be £150,000 a year, what is that a day ? A. 410£ ; 19s. ; 2d.

17. If 1 cwt. of raisins cost £3 ; 10, what is that a lb. A. 7½d.

18. If 12 quarts of wine cost £4 ; 15 ; 6, what is that a qt. A. 7s. 11½d.

19. Divide £115 ; 10, by 90. A. £1 ; 5 ; 8.

20. Divide £136 ; 16 ; 6, by 108. A. £1 ; 5 ; 4.

21. Divide 6 T. 11 cwt. 3 qrs. 19 lb. by 4. A. 1 T. 12 cwt. 3 qrs. 25 lb. 12 oz.

22. Divide 26 lb. 1 oz. 5 dwt. by 24. A. 1 lb. 1 oz. 1 dwt. 1 qr.

23. Divide 666£ 15s. 9d. 1 qr. by 125.

24. Divide 32 yrs. 5 mo. 2 w. 5 d. 17 h. 27 sec. by 306.

25. Divide 441 lb. 3 oz. 16 dwt. 17 gr. by 509.

26. Divide 81 T. 16 cwt. 3 qrs. 25 lb. 15 oz. 13 dr. by 572.

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OBSERVATIONS ON COMPOUND NUMBERS, FOR ADVANCED PUPILS.

§ XXXIX. Man could not have lived long in the earth, without perceiving the necessity of different measures. In estimating dimensions of length, for example, he would find those minute divisions of space, which, for the measurement of small objects were not only convenient but even necessary, very inadequate to his purpose when applied to the height of a tree, or the breadth of a river ; and much more so, when employed to express the elevation of a mountain, or the distance traversed on a long journey. Hence, the origin of different *denominations*. For the purpose of easily comparing the different denominations with one another, and, in some cases, of substituting one for another without altering the value, it seemed best to make each higher denomination such as to contain an *exact number* of the next lower.

Space was, doubtless, first measured by man ; and for this purpose were employed the dimensions of various members of the human body ; as the breadth of the hand, its extent when spread, called the span, the breadth of the nail, and of the thumb, the length of the foot, and of the arm, and also the length of a step or pace. These things seem to have formed the basis of measures of length, in all nations. From the thumb is derived the inch; from the foot, as being, in length, about twelve times the thumb's breadth, the measure still used of the same name; from the arm, as being about three times the length of the foot, the yard; and from the hand, the span and the pace, the measures, respectively bearing those names at the present time. It is said that the yard now in use in England, "was adjusted from the arm of Henry I. in 1,101, and that the old French *pied du roi*, (king's foot) had a similar origin." From these sprung higher denominations; as the mile, being *mille passuum*, that is a thousand paces, &c. From lineal measurements, the transition was easy to those of surfaces and solids.

In the ruder ages, men weighed with the natural balance of the arms and hands; a balance, indeed, quite as rude as the age, in which it was employed. When greater accuracy seemed to become necessary, the *artificial* balance was constructed, on the hint, thus afforded by nature. From what circumstance weights derive their actual, or their comparative sizes, we do not know.

As the process of coining implies weight, money, properly speaking, was not probably employed, until long after the invention of the balance. Its denominations have usually been entirely arbitrary.

The division of time was naturally suggested by the succession of days and nights, by the revolutions of the moon, and the returns of the seasons. The subdivisions into 60s, seem to have had their origin in Ptolemy's sexagesimal Notation. (see § VI.) It has been supposed by some, on the other hand, however, that, since the moon was observed to make 12 complete revolutions in a year, man naturally made the subdivisions of the day, and likewise of the night the same in number. 12 subdivisions of the day, and 12 of the night made 24 *hours*. Then, as the month contained 30 days and 30 nights, making 60 parts in the whole, the subdivisions of the hour were made by 60s. [See American Almanac for 1830.] This, however seems improbable, and the circumstance that the year contains considerably more than 12 revolutions of the moon, and likewise, that one revolution, (which measured the original month,) does not contain 30 days, shows that the supposition has little foundation in fact. The division of time into periods of 7 days, or weeks, has been found to have been very extensively employed, by rude nations.

Any one who will compare the operations on Federal Money, with those on any other set of Compound quantities, which we have exhibited, will be ready to inquire, or will perhaps rather be able to answer the inquiry, why it is that the former are so much more simple and easy, than the latter. It is evidently because the *law of increase*, in Federal Money, is the same as that of simple numbers. The radix of each is 10. The pupil will now perceive how much reason we have to regret, that weights, measures, &c. should have been originally made to increase by ratios so irregular and so inconvenient. But it must be recollected that they had their origin probably, before that of the Decimal Notation; and accident, which, we have seen, determined this Notation, (§ VI.) determined likewise the divisions of Compound Numbers. The ease with which we conduct processes on Federal money, results only from the fact that its ratio is the radix of our scheme of Notation. Then, before this Notation was established, there was nothing to give the decimal division of Compound Numbers any advantage over many others. And, if there had been, it is not to be supposed that rude and barbarous nations would be likely to form systems of division on scientific principles. Present convenience or caprice would be their only guide. Yet notwithstanding all the inconvenience of our present divisions of weights and measures, no question was made of their propriety, until sometime during the last century. Near the commencement of the French revolution, the National convention resolved on a reform. They determined, to destroy, at a blow, the old systems, and to establish others, increasing decimally. This was accordingly done, and, of course, the numerical operations upon Compound Numbers, in France, immediately became as easy, as those upon Simple Numbers. The following is a brief account of

THE FRENCH MONEY, WEIGHTS AND MEASURES.

The National Convention of France, on the first of August, 1793, resolved to introduce a new and uniform system of weights, measures, and generally of all compound quantities. The French Academy of Science, were requested by the Assembly, to draw up such a system; and, accordingly, in the year 1795, they submitted that which we are about to explain. This was immediately adopted, and its use enforced for 16 years, when the alterations took place which we have noticed below.

Commencing with measures of length, the Academy took great pains to obtain some fixed and universal standard. It seemed necessary to determine on some dimension in nature, which should not be liable to change with time, but which, existing always the same, should afford the means of rectifying any error which might, by any accident, or by variations insensibly creeping in, disturb the uniformity of the system. It was, at length, determined that this should be the *ten-millionth* part of the distance from the equator of the earth to the pole, or, in other words, *one ten-millionth part of the quadrant*, (quarter of a circle,) *of the terrestrial meridian.*'

By very accurate measurement, the length of this was discovered to be something more than 3.078 old French feet, or 3.281 English feet, nearly. This was taken as the *unit of length*, and called the METRE. The larger measures were made by multiplying this unit by 10, 100, &c., and the sub-divisions, by dividing it by 10, 100, &c.; in order that the whole might proceed on the decimal scale, and thus, correspond with the decimal Notation of numbers.

FRENCH MEASURES OF LENGTH.

10 milli-metres	make	1 centi-metre.
10 centi-metres	"	1 deci-metre.
10 deci-metres	"	1 METRE, (the unit of length.)
10 METRES	"	1 deca-metre.
10 deca-metres	"	1 hecto-metre.
10 hecto-metres	"	1 kilo-metre.
10 kilo-metres	"	1 myria-metre.

The word *metre* means *measure*. The names of the other denominations are formed from this, by PREFIXES. The prefixes for the *sub-divisions* of the unit, are taken from the Latin; those for the larger measures, from the Greek.

In measuring SURFACE, a SQUARE DECA-METRE, equal to 100 square metres, was made the unit. This unit is called the ARE, from the Latin *area*, meaning surface. The names of the denominations, all of course, graduated by the decimal scale, are as follows, beginning with the least, and proceeding to the greatest.

Centiare, (square metre); Deciare; ARE, (unit of surface); Decare; Hectare; Kilare; Myriare. The square deci-metre, centi-metre, and milli-metre are used for smaller measures.

In measuring CAPACITY, a CUBIC DECI-METRE, equal to .001 of a cubic metre, was made the unit. This unit is called the LITRE, from *litron*, the name of an old French measure, of one-fifth less capacity. The denominations increase decimally, as before, and are Centi-litre; Deci-litre; LITRE, (unit of capacity); Deca-litre; Hecto-litre; Kilo-litre, (cubic metre); Myria-litre. In measuring SOLIDS, a CUBIC METRE was made the unit. This unit is called the STERE, and will be seen to be equal to the Kilo-litre, in the above measures of Capacity. The denominations, increasing decimally, are Deci-stere; STERE, (cubic metre and unit of solidity); Deca-stere. We have given the length of the metre, and from that may be found, if necessary, all the other dimensions. In measuring TIME, the day was divided into 10 hours; each hour, into 100 minutes; and each minute into 100 seconds. Thus a French hour became 2h. 24m. English, or common hours; and the French minute, 1m. 26.4 sec. English. In measuring THE CIRCLE, 100″ made 1′; 100′ made 1°; 100° made 1 quadrant, (quarter of a circle); and 4 quadrants, or 400° made the whole circle. By this division, *one degree* became 54 *minutes* as they are commonly measured. WEIGHTS were connected with measures of extension, *by assuming as the unit of weight, the weight of a* CUBIC CENTI-METRE *of pure* WATER at a medium temperature. This unit was called a GRAMME, and the names of the other denominations were formed, as above, by prefixes. The French MONEY was made very simple. 100 *centiemes*, (hundredths,) make 1 *franc*. The centieme was made to weigh [illegible] grammes, and the silver franc, 5 grammes. Even the thermometer was decimally divided, and the Paris CENTIGRADE THERMOMÈTER has 0 at the freezing, and 100 at the boiling point. Experience proved, however, that these divisions, though they rendered *calculations* extremely easy, and, in fact, reduced all *compound* quantities to *simple*, were not as convenient *in practice*, as many of those, of the systems exploded. On the decimal system, no even quarter, third, or sixth of a foot could be obtained, as on the duodecimal, and as these fractions are of frequent occurrence in practice, much inconvenience was experienced. It was found, too, that "for the operations of weighing, and measuring capacities, the continual division by 2 renders it practicable to make up a given quantity with the smallest number of standard weights and measures, and is far preferable, in this respect, to any decimal scale." [*Report Brit. House of C. June* 24, 1819.] These circumstances rendered the people dissatisfied. The government found it necessary, therefore, to decree, on the 28th March, 1812, that the old names of the *toise*, the *ell* or *aune*, the *foot*, the *inch*, and the *bushel*, should be allowed; but that, in order to preserve the *metrical* system, they should represent new measures, related to that system as follows:

The toise 2 metres.	The inch $\frac{1}{72}$ metre.
The foot $\frac{1}{3}$ metre.	The aune $1\frac{1}{5}$ metres.

The bushel $\frac{1}{8}$ of a hecto-litre.

These measures were likewise subdivided *duodecimally*, instead of *decimally*, on the ground that "the decimal system, though favorable for calculations, is not equally so for the daily operations of the people, and is not easily comprehended." (*Decree, Mar.* 28, 1812.)

Thus we see, that the French weights and measures, at the present day, are, for the most part, only *decimal in theory*, while, *in practice*, they are *duodecimal*. Similar in this respect, in our own country, are the measures of land. Surveyors, by the use of Gunter's chain, are enabled, as has been shown, § XXXIII, to make their calculations *decimally*. But, at the same time, the measures in common use, so far from being on the decimal scale, or from increasing uniformly, are among the most inconvenient we have. This shows the great difficulty of making innovations upon systems in common use, even where convenience seems to demand it.

It should be recollected, that the French decimal system was introduced at the recommendation of philosophers and men of learning, who, having little occasion to engage in those occupations, in which it becomes necessary to make frequent use of weights and measures, over-looked entirely one most important part of their subject, viz. the *practical;* and kept in view only that which most concerned them, viz. the *theoretical.* The result of this experiment has been to show, that it is out of the question ever to think of reducing compound numbers to correspond entirely with the decimal Notation. Much might doubtless be done towards rendering them more uniform and simple than they are *at present,* were it not for the difficulty of the attempt, as well as the inconvenience, and the confusion that must result from any material change in a system so long established, and so interwoven with all the transactions of commerce, as well as the processes of the arts. There has probably never been, in the history of the world, an occasion, on which such a change could have been effected, if we except the French revolution. The people of that country were, at that time, so carried away by their rage for innovation, that they were ready to root up and destroy every thing ancient and venerable; and that, too, for the very reason which induces other nations to cherish and esteem their institutions, viz. their antiquity and long familiiarity. They conceived every thing old, to savor of the *monarchy,* and therefore, they waged against it a most furious warfare. Had this zeal been turned into another channel, it might have been productive of much good. Had the favorable moment been seized for the introduction of the *duodecimal* scale of Notation, (see § VI.) more would have been accomplished towards promoting the cause of science, than it will probably ever again be within the scope of a legislative act to effect.

The subject of weights and measures having been submitted in 1819, by the Prince Regent of Great Britain, to a committee of distinguished men, they reported decidedly against the decimal scale, but proposed some slight alterations in the existing system. Accordingly, in 1825, by act of Parliament, the following changes took place.

The bushel which had been before used in England, was called the *Winchester bushel,* and was a cylinder 8 inches deep, and 18½ inches in diameter, containing 2,150.4 cubic inches. The gallon, by dry measure, therefore, contained one eighth as much, or 268.8 cubic inches. The beer gallon contained 282, and the wine gallon 231 cubic inches.

These measures were abolished, and the bushel ordered to contain 2,217.6 cubic inches, and the gallon, one-eighth as much=277.2 cubic inches. This latter measure is used both for dry and liquid measure.

The capacity of these measures is determined by weighing them, at first empty, and afterwards filled with pure distilled water, at the temperature of 62° of Fahrenheit's thermometer. The bushel ought to hold 80, and the gallon 10 lbs. Avoirdupois.

The weights, however, were first determined from measures of capacity, 19 cubic inches of distilled water, at the temperature of 50°, being declared to weigh 10 oz. or 4,800 grs. Troy, and 7,000 grs. Troy, to be equal to 1 lb. Avoirdupois.

From this, also, we have the relation of the weights Avoirdupois and Troy. 7,000 grs. reduced, make 1 lb. 2 oz. 11 dwt. 16 gr., which expresses the weight of 1 lb. Avoirdupois, in Troy weight. 1 lb. Troy, in Avoirdupois weight, is 13 oz. $2\frac{114}{175}$ dr. Thus the lb. Avoirdupois is greater than the lb. Troy, and the oz. Troy, greater than the oz. Avoirdupois.

It may still be enquired, how is the measure of length determined, on which these measures of capacity and weight depend? It was found that, according to the scale adopted, the pendulum vibrating seconds on a level with the sea, and in a vacuum, at London, was 39.1372 inches, and the French metre, being the ten millionth part of the quadrant of a meridian on the earth, was 39.3694 inches. Hence, there are two natural standards with which it may always be compared, and by which it may be rectified.

In 1,827, the legislature of the state of New-York passed an act, regulating weights and measures. The yard was made the standard of length, and was ordered to be $\frac{1000000}{1086141}$ of the pendulum vibrating seconds at Columbia college, in the city of New-York, this pendulum being 39.10107 inches. The pound was made the unit of weight, and its magnitude determined by the weight of a cubic foot of water, which, at 40° temperature, weighs, on this scale, $62\frac{1}{2}$ lbs., or 1000 oz.

The gallon holds 10lb. and the bushel 80 lb. Hence, in the bushel, are 2,211.84 cubic inches, and in the gallon 276.48 cu. in.

In the state of Connecticut, the bushel contains 2,198 cubic inches, and the gallon, dry measure, $274\frac{3}{4}$. The ale and wine gallons are of the same dimensions with the old English measures of the same kind mentioned above. No scientific standard has been adopted, nor has the mode of testing capacities by weight been established by law. Most of the states of the uinon have been as inattentive to this subject as Connecticut. They have all, indeed, *legislated* on the subject, but their enactments have been made without the advice or assistance of scientific men. It is to be hoped that other states will soon follow the example of New-York.

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PROMISCUOUS EXAMPLES.

MENTAL EXERCISES.

§ XL. 1. $\frac{4}{7}$ of 35 are how many times 4? $\frac{4}{5}$ of 40 are how many times 7?

2. $\frac{9}{10}$ of 60 are how many times 5? $\frac{10}{9}$ of 45 are how many times 8?

3. $\frac{12}{4}$ of 24 are how many times 10? $\frac{11}{7}$ of 42 are how many times 9?

4. $\frac{3}{8}$ of 40 are how many 7ths of 21? $\frac{5}{4}$ of 16 are how many ninths of 72?

5. $\frac{11}{12}$ of 24 are how many fourths of 16? $\frac{12}{5}$ of 40 are how many fifths of 50?

6. $\frac{2}{3}$ of 48 are how many elevenths of 44? $\frac{7}{9}$ of 63 are how many eighths of 72?

7. 24 is $\frac{3}{7}$ of how many times 6? 36 is $\frac{4}{3}$ of how many times 5?

8. 48 is $\frac{3}{5}$ of how many times 4? 72 is $\frac{9}{11}$ of how many times 12?

9. 96 is $\frac{12}{7}$ of how many times 11? 120 is $\frac{10}{6}$ of how many times 5?

10. 77 is $\frac{11}{12}$ of how many fourths of 32? 64 is $\frac{8}{9}$ of how many sevenths of 49?

11. 42 is $\frac{6}{11}$ of how many fifths of 30? 49 is $\frac{7}{8}$ of how many thirds of 36?

12. 39 is $\frac{3}{2}$ of how many twelfths of 48? 60 is $\frac{10}{11}$ of how many eighths of 72?

13. $\frac{3}{4}$ of 24 is $\frac{9}{10}$ of how many times 6? $\frac{4}{5}$ of 15 is $\frac{8}{5}$ of how many times 3?

14. $\frac{9}{8}$ of 32 is $\frac{12}{7}$ of how many times 5? $\frac{12}{11}$ of 44 is $\frac{8}{3}$ of how many times 8?

15. $\frac{12}{15}$ of 45 is $\frac{4}{7}$ of how many times 4? $\frac{16}{3}$ of 15 is $\frac{20}{9}$ of how many times 11?

16. $\frac{7}{8}$ of 64 is $\frac{14}{12}$ of $\frac{8}{9}$ of how many times 7? $\frac{8}{5}$ of 40 is $\frac{4}{3}$ of $\frac{9}{11}$ of how many times 8?

17. $\frac{10}{11}$ of 66 is $\frac{5}{4}$ of $\frac{3}{2}$ of how many times 9? $\frac{9}{10}$ of 80 is $\frac{12}{10}$ of $\frac{5}{7}$ of how many times 9?

18. $\frac{4}{5}$ of 15 is $\frac{6}{20}$ of $\frac{5}{4}$ of how many times 12? $\frac{2}{3}$ of 24 is $\frac{2}{5}$ of $\frac{10}{11}$ of how many times 5?

19. $\frac{3}{4}$ of 40 is $\frac{5}{6}$ of how many 4ths of 28? $\frac{10}{12}$ of 72 is $\frac{5}{4}$ of how many 7ths of 35?

20. $\frac{4}{3}$ of 27 is $\frac{6}{9}$ of how many 3ds of 33? $\frac{7}{4}$ of 16 is $\frac{14}{19}$ of how many 6ths of 18?

21. $\frac{8}{7}$ of 28 is $\frac{2}{3}$ of how many 9ths of 63? $\frac{9}{11}$ of 88 is $\frac{6}{2}$ of how many 3ds of 15?

22. $\frac{8}{7}$ of 35 is $\frac{10}{4}$ of $\frac{8}{15}$ of how many 5ths of 45? $\frac{6}{9}$ of 54 is $\frac{9}{8}$ of $\frac{16}{13}$ of how many 8ths of 56?

23 $\frac{15}{8}$ of 32 is $\frac{5}{6}$ of $\frac{9}{11}$ of how many 6ths of 54? $\frac{13}{8}$ of 32 is $\frac{16}{6}$ of $\frac{4}{7}$ of how many 9ths of 54?

24. $\frac{12}{17}$ of 34 is $\frac{6}{9}$ of $\frac{18}{19}$ of how many 8ths of $\frac{3}{5}$ of 40? $\frac{4}{5}$ of 30 is $\frac{3}{8}$ of $\frac{2}{3}$ of how many 3ds of $\frac{9}{11}$ of 44?

25. $\frac{3}{5}$ of 45 is $\frac{10}{3}$ of $\frac{6}{16}$ of how many 4ths of $\frac{12}{5}$ of 10? $\frac{4}{13}$ of 39 is $\frac{6}{13}$ of $\frac{3}{4}$ of how many 7ths of $\frac{3}{2}$ of 28?

The following are to be written.

26. A man collected debts, of the following amounts; 12£. 9s. 1d. 3 qrs.; 19£. 2s. 11d. 1qr.; 47£. 10s. 9d. 2 qrs.; and 14£. 13s. 5d. How much did he receive in all? *Ans.* 90£. 16s. 3½d.

27. I pay for cloth £14; 19; 6; for flannel, £11; 4; 9; for grain, £25; 10; for sugar, £4; 0; 6; for coffee, £3; 6; 8; and for molasses 19s. 6d. What cost the whole? *Ans.* £60; 0; 11.

28. A farmer brought to market butter, which brought him 54s. cheese, which brought 59s.; a load of wood, 49s. 9d. 2 qrs.; eggs, 39s. 8d.; and apples, 47s. 9d. In part payment he received 4 lb. of tea, at 4s. 9d. pr. lb.; 12 lb. sugar, at 8d pr. lb.; 3 shovels at 6s. apiece; 5 hoes, at 3s. apiece; 8 yds. of cloth, at 1£. 3s. 6d. pr. yd., and the rest in money. How much money did he receive? *Ans.* 2s. 2¼d.

29. What cost 11 T. of hay at 2£. 1s. 10d. pr. T.? *Ans.* £23; 0; 2.

30. 5 men shared equally a prize of £1,444; 1; 6. What was each man's share? *Ans.* £288; 16; 3½.

31. In 9 bales of cloth, each containing 12 pieces, and each piece, 27 yds. 1 qr. 2 nls. how many yds? *Ans.* 2,956 yds. 2 qrs.

32. A man, having £1,000, lost 4½d. What had he left? *Ans.* £999; 19; 7½.

33. A man, failing in trade could only pay £666; 4; 1½. on a debt of £706. How much did his creditor lose? *Ans.* 39; 15; 10½.

34. Bought a hhd. of wine for £67; 4. What was that a gal.? *Ans.* £1; 1; 4.

35. Bought 48 yds. of cloth for £11; 2. What was that a yd.? *Ans.* 4s. 7½d.

36. If a dozen silver spoons weigh 3 lb. 2 oz. 13 dwt. 12 gr. What is the weight of 1 spoon? *Ans.* 3 oz. 4dwt. 11 gr.

37. If 15 yds. of cloth cost $45, what will 47 yds. come to at that rate? *Ans.* $141.

NOTE. If 15 yds. cost $45, 1 yd. will cost *one fifteenth* as much. 45÷15=$3. If 1yd. cost $3 47 yds. will cost 47 times as much. 3×47=$141, *Ans.*

Questions like the last belong to PROPORTION or the RULE OF THREE. (SEE § XCI.) The above mode of solving them is called the SOLUTION BY ANALYSIS, or the ANALYTIC SOLUTION.

38. If 2 gal. of brandy cost $1, what will 50 gal. cost? *Ans.* $25.

39. If 100 men can do a piece of work in 12 days, how many will it take to do it in 3 days? *Ans.* 400.

NOTE. It will take 12 times as many to do it in 1 day; and ⅓ as many to do it in 3 days, as in 1.

How many yds. of cloth, 3 qrs. wide will line 30 yds. that is 5 qrs. wide? *Ans.* 50.

41. How many yds. of matting, 2 ft. 6 in. broad, will cover a floor, 27 ft. long, and 20 ft. broad? *Ans.* 70.

42. If 30 men can perform a piece of work in 11 days, how many men will accomplish a piece, 4 times as large, in ⅕ of the time?

43. If 13 gal. of molasses cost $6.50, what cost 25? *Ans.* $12.50.

44. If 15 lb. of sugar cost $1.875, what cost 29? *Ans.* $3.625.

45. If 21 lbs. of coffee cost $5.25, what cost 57? *Ans.* $14.25.

46. If 5 yards of cloth cost $38.40, what cost 639 yards?

47. If 18 lbs. of sugar cost $1.875, what will 7 barrels come to, each weighing 3 cwt. 3 qrs. 15 lbs. 9 oz.?

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FRACTIONS.

§ XLI. From what has already been said of Fractions, the learner will understand that they are,

I. EXPRESSIONS FOR PARTS OF AN INTEGER OR WHOLE UNIT, OR

II. EXPRESSIONS INDICATING THE DIVISION OF ONE NUMBER BY ANOTHER.

Before proceeding farther, we will lay down a few principles, resulting from the nature of Division, which will assist us in pursuing the subject of Fractions.

If a divisor is contained in a dividend, a certain number of times, the same divisor will be contained in twice that dividend, twice as often; in 3 times that dividend, three times as often; in 4 times that dividend, 4 times as often, and so on.

If, then, the DIVISOR *remains the same,* MULTIPLYING THE DIVIDEND BY ANY NUMBER, MULTIPLIES THE QUOTIENT BY THE SAME NUMBER.

If a divisor is contained in a dividend a certain number of times, the same divisor is contained in half that dividend one half as often; in one third of that dividend, one third as often, and so on.

If, then, the DIVISOR *remains the same,* DIVIDING THE DIVIDEND BY ANY NUMBER, DIVIDES THE QUOTIENT BY THE SAME NUMBER.

If a divisor is contained in a dividend, a certain number of times, twice that divisor is contained in the same dividend, one half as often; 3 times that divisor is contained in the same dividend, one third as often, and so on.

If, then, the DIVIDEND *remains the same,* MULTIPLYING THE DIVISOR BY ANY NUMBER, DIVIDES THE QUOTIENT BY THE SAME NUMBER.

If a divisor is contained in a dividend, a certain number of times, half that divisor is contained in the same dividend, twice as often; one third of that divisor, 3 times as often, and so on.

If, then, the DIVIDEND *remains the same,* DIVIDING THE DIVISOR BY ANY NUMBER, MULTIPLIES THE QUOTIENT BY THE SAME NUMBER.

It seems then, that multiplying the dividend has the same effect on the quotient as dividing the divisor; and that *dividing the dividend has the same effect as multiplying the divisor.*

Of course, *multiplying both divisor and dividend produces opposite effects upon the quotient, and dividing both divisor and dividend produces opposite effects upon the quotient.* Then,

IF THE DIVISOR AND DIVIDEND BE BOTH MULTIPLIED, OR BOTH DIVIDED BY THE SAME NUMBER THE QUOTIENT WILL NOT BE ALTERED.

Now as every Fraction is an instance of division, in which the numerator is the dividend, the denominator the divisor, and *the value of the Fraction, the quotient,* we may apply the above principles to them. Of course,

MULTIPLYING THE NUMERATOR OF A FRACTION, MULTIPLIES THE VALUE, AND DIVIDING THE NUMERATOR DIVIDES THE VALUE. And,

MULTIPLYING THE DENOMINATOR OF A FRACTION DIVIDES THE VALUE, AND DIVIDING THE DENOMINATOR MULTIPLIES THE VALUE. And,

MULTIPLYING THE NUMERATOR PRODUCES THE SAME EFFECT UPON THE VALUE AS DIVIDING THE DENOMINATOR; AND DIVIDING THE NUMERATOR PRODUCES THE SAME EFFECT AS MULTIPLYING THE DENOMINATOR. And,

MULTIPLYING THE NUMERATOR PRODUCES AN OPPOSITE EFFECT UPON THE VALUE, FROM MULTIPLYING THE DENOMINATOR; AND DIVIDING THE NUMERATOR PRODUCES AN OPPOSITE EFFECT FROM DIVIDING THE DENOMINATOR. Therefore,

IF THE NUMERATOR AND DENOMINATOR BE BOTH MULTIPLIED, OR DIVIDED BY THE SAME NUMBER, THE VALUE OF THE FRACTION WILL NOT BE ALTERED.

MENTAL EXERCISES.

§ XLII. 1. Which Fraction has the greatest value $\frac{12}{6}$ or $\frac{6}{3}$? *Ans.* Neither, because 6 is in 12, twice, and 3 is in 6 twice; or $\frac{12}{6}=2$, and $\frac{6}{3}=2$.

2. Which is the greatest $\frac{30}{6}$ or $\frac{15}{3}$? Why?

3. Which is the greatest $\frac{8}{4}$ or $\frac{4}{2}$? Why?

4. Which is the greatest $\frac{6}{8}$ or $\frac{3}{4}$? *Ans.* Neither, for 6 is 3 times 2, or 3 *twos*, and 8 is 4 times 2, or 4 *twos*. It is plain, that 3 *twos* is the same part of 4 *twos* that 3 *ones* is of 4 *ones*, or that 3 is of 4.

5. Which is the greatest $\frac{14}{16}$ or $\frac{7}{8}$? Why? $\frac{9}{6}$ or $\frac{3}{2}$? Why? $\frac{12}{8}$ or $\frac{6}{4}$? Why? $\frac{18}{9}$ or $\frac{6}{3}$? Why? $\frac{10}{12}$ or $\frac{5}{6}$? Why?

We see then, that *Fractions of the same value may be expressed by very different numbers.*

The numerator and denominator of a Fraction are called TERMS.

WHEN A FRACTION IS EXPRESSED BY THE SMALLEST NUMBERS POSSIBLE, IT IS SAID TO BE IN ITS LEAST OR LOWEST TERMS.

When a Fraction is *not* in its lowest terms, it is plain that to make it so, we must *diminish* the terms, in such a manner as not to alter the value of the Fraction. This can be done by dividing both, by some number which will divide them, without remainder. (§ XLI.)

1. Reduce the following Fractions to their lowest terms.
$\frac{9}{15}$; $\frac{18}{24}$; $\frac{30}{40}$; $\frac{15}{18}$; $\frac{35}{45}$; $\frac{12}{24}$; $\frac{25}{45}$; $\frac{12}{36}$; $\frac{25}{75}$; $\frac{18}{27}$; $\frac{27}{36}$.

2. Reduce the following Fractions to their lowest terms. $\frac{45}{50}$; $\frac{21}{9}$; $\frac{10}{12}$; $\frac{16}{18}$; $\frac{14}{20}$; $\frac{9}{27}$; $\frac{30}{35}$; $\frac{7}{28}$; $\frac{8}{64}$; $\frac{9}{63}$; $\frac{7}{63}$; $\frac{15}{60}$; $\frac{14}{42}$; $\frac{13}{39}$; $\frac{14}{36}$; $\frac{18}{48}$.

3. Reduce the following Fractions to their lowest terms. $\frac{15}{45}$; $\frac{11}{44}$; $\frac{11}{66}$; $\frac{22}{99}$; $\frac{33}{77}$; $\frac{12}{72}$; $\frac{24}{84}$; $\frac{63}{18}$; $\frac{56}{48}$; $\frac{81}{90}$; $\frac{70}{77}$; $\frac{35}{55}$; $\frac{13}{52}$; $\frac{12}{96}$; $\frac{100}{150}$.

The following are to be written.

4. Reduce $\frac{725}{850}$ to its lowest terms.

First divide by 5, and then by 5 again. $5)\frac{725}{850}=5)\frac{145}{170}=\frac{29}{34}$, *Ans.*

5. Reduce $\frac{90}{630}$; $\frac{165}{275}$; $\frac{114}{285}$; $\frac{284}{1152}$ and $\frac{99}{297}$; to their lowest terms. *Ans.* $\frac{1}{7}$; $\frac{3}{5}$; $\frac{2}{5}$; $\frac{1}{2}$ and $\frac{1}{3}$.

The rule then, seems to be—

DIVIDE BOTH TERMS OF THE FRACTION BY ANY NUMBER WHICH WILL DIVIDE THEM WITHOUT A REMAINDER, AND THE QUOTIENTS IN THE SAME MANNER, UNTIL NO NUMBER GREATER THAN 1 WILL DIVIDE BOTH, WITHOUT A REMAINDER.

The pupil will soon find, that there are some numbers which cannot be divided without remainder, except by themselves or by 1.

Thus, 7 can be divided without remainder by no numbers except 7 and 1. 13, 17, &c. are such numbers.

A NUMBER WHICH CAN ONLY BE DIVIDED BY ITSELF, OR BY 1, IS CALLED A PRIME NUMBER.

Numbers which admit of division, or of being resolved into factors, we have already seen (§ XII.) are called *composite numbers.*

AN EVEN DIVISOR OF A NUMBER, *that is, one which will divide it without remainder*, IS CALLED A MEASURE OF THAT NUMBER.

Thus, 3 is a measure of 15, because 3 divides 15 without remainder. 7 is a measure of 35, &c.

Every number is a *measure* or *even divisor* of itself.

10 may be divided by 2, 5, or 10. Of course, any number of 10s may be divided by 2, 5, or 10. Then, if the right hand figure of a number be a cypher, that number is divisible, that is, it may be divided by 2, 5, or 10. Of course, if the right hand figure of a number be divisible by 2, the whole number is divisible by 2. And, if the right hand figure of a number be 5, the whole number is divisible by 5.

100 may be divided by 4. Of course, any number of 100s may be divided by 4. Then, if there be two cyphers at the right of a number, the number is divisible by 4. Of course, if 4 will divide two figures on the right of a number, it will divide the whole number.

1,000 may be divided by 8. Of course, any number of 1,000s may be divided by 8. Then, if there be three cyphers at the right of a number, the number is divisible by 8. And, if 8 will divide three figures on the right of a number, it will divide the whole number.

It has been already shown, (§ IX.) that if the sum of the figures, which compose a number, be divisible by 9 or by 3, the whole number is divisible by 9 or by 3.

20 is divisible by 4. Of course, any number of times 20 is divisible by 4. Then, if the *tens be even*, we need only try the right hand figure by 4, to discover whether it will divide the whole number. 200 is divisible by 8. Of course, any number of times 200 is divisible by 8. Then, if the *hundreds be even*, we need only try the two right hand figures by 8, to discover whether it will divide the whole number.

To discover whether a Prime number will divide, we must make actual trial.

1. Find the measures of the following numbers, omitting the number itself, as being of course a measure. 18, 27, 20, 21, 24, 48, 72. *Ans.* Of 18, the measures are 2, 3, 6, 9. Of 27; 3, 9. Of 20; 2, 4, 5, 10. Of 21; 3, 7. Of 24; 2, 3, 4, 6, 8, 12. Of 48; 2, 3, 4, 6, 8 12, 16, 24. Of 72; 2, 3, 4, 6, 8, 9, 12, 18, 24, 36.

2. Find the measures of the following numbers, 108, 120, 432, 936, 846.

3. Reduce the following Fractions to their lowest terms. $\frac{486}{9720}$; $\frac{144}{1728}$; $\frac{324}{648}$; $\frac{1429}{2858}$; $\frac{1644}{2192}$; $\frac{468}{1184}$; $\frac{4746}{38433}$; $\frac{806}{42315}$; $\frac{1158}{2702}$; $\frac{3578164}{32203476}$. *Ans.* $\frac{1}{20}$; $\frac{1}{12}$; $\frac{1}{2}$; $\frac{1}{2}$; $\frac{3}{4}$; $\frac{117}{296}$; $\frac{1582}{12811}$; $\frac{2}{105}$; $\frac{3}{7}$; $\frac{1}{9}$.

4. Reduce the following: $\frac{386}{948}$; $\frac{4932}{8764}$; $\frac{12345}{678910}$; $\frac{24687}{342954}$; $\frac{83808}{95636}$; $\frac{39972}{812322}$; $\frac{998811}{999888611}$; $\frac{103284}{7328472}$.

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§ XLIII. In almost every one of the above instances the pupil has found it necessary to perform several successive divisions. But

if he could have found at once, the *greatest number*, which would divide both terms without remainder, only one division would have been necessary.

A NUMBER WHICH IS A MEASURE OF TWO OR MORE NUMBERS IS CALLED THEIR COMMON MEASURE.

THE GREATEST NUMBER WHICH WILL MEASURE TWO OR MORE NUMBERS IS CALLED THEIR GREATEST COMMON MEASURE.

MENTAL EXERCISES.

1. What is the greatest common measure of 4 and 6? of 8 and 12? of 14 and 16? of 6 and 9?

2. What is the greatest common measure of 4 and 10? of 4 and 14? of 9 and 15? of 10 and 15? of 7 and 21?

3. What is the greatest common measure of 3 and 27? of 30 and 9? of 15 and 20? of 20 and 30? of 30 and 40?

4. What is the greatest common measure of 2 and 20? of 8 and 26? of 6 and 16? of 18 and 24?

5. What is the greatest common measure of 12 and 16? of 16 and 20? of 12 and 18? of 16 and 18?

6. What is the greatest common measure of 35 and 28? of 28 and 21? of 21 and 14? of 14 and 2?

In the case of small numbers like the above, there is no difficulty in finding the greatest common measure. The following are not so easy.

7. Find the greatest common measure of 125 and 375.

We cannot tell what this is at first sight. But we see by the above examples, that the smaller number is *sometimes*, itself the measure sought. Let us try whether this is not the case in the present instance.

We see by this illustration, that 125 is a measure of 375, and as 125 cannot be measured by a number greater than itself, it is, of course, the greatest common measure sought.

125)375(3
375

8. Find the greatest common measure of 144 and 168.

Try first, as before, whether 144 is the measure. 168÷144 leaves 24 remainder. Now if 24 will measure 144, it will also measure 144+24=168. 144÷24=6. 24 then is a common measure of 144 and 168. It is also the greatest common measure, for any number which will measure 144 and 168, will measure their difference. For it is contained an even number of times in 144, and also a greater even number of times in 168. But taking an even number of times any thing from an even number of times the same thing, will leave an even number of times that thing. But 24 cannot be measured by any number greater than itself. Therefore, there can be no greater common measure of 144 and 168, than 24. The explanation would be similar, if several successive divisions took place. Therefore, *to find the greatest common measure of two numbers,*

144)168(1
144
——
24)144(6
144

DIVIDE THE GREATER NUMBER BY THE LESS, THAT DIVISOR BY THE REMAINDER, AND SO ON, ALWAYS DIVIDING THE LAST DIVISOR BY THE

LAST REMAINDER, UNTIL SOME REMAINDER IS LEFT. THE DIVISOR WHICH LEAVES NO REMAINDER IS THE GREATEST COMMON MEASURE.

If it be required to find the greatest common measure of several numbers, find first the greatest common measure of two of them, then the greatest common measure of this common measure and a third, then the greatest common measure of this and a fourth, &c. The last common measure, will be that required. Thus,

8. Find the greatest common measure of 12 and 30 and 9.

6 is the greatest common measure of 12 and 30. 3 is the greatest common measure of 6 and 9, and of course of the three given numbers 12, 30 and 9.

11. Find the greatest com. meas. of 72 and 96. A. 14. Of 330 and 462. A. 66. Of 126 and 342. A. 18. Of 84, 28 and 42. A. 14. Of 34, 746, 69,492 and 38,433. A. 3. Of 1,872, 3,456. 98,712 and 531,711. Of 12,572, 92,4[illegible]6, 2,354 and 34,564.

This principle may be applied with advantage to the reducing of Fractions to lower terms. Take the following examples.

12. Reduce $\frac{39}{52}$ to its lowest terms. 13 is the greatest com. meas. A. $\frac{3}{4}$.

13. Reduce $\frac{84}{132}$. 12 is the greatest com. meas. A. $\frac{7}{11}$.

14. Reduce $\frac{48}{80}$; $\frac{32}{144}$ $\frac{63}{117}$; $\frac{256}{1536}$; and $\frac{2462}{3702}$.

The pupil has, no doubt, by this time observed that there are some numbers which have no common measure greater than one. These numbers are said to be PRIME TO EACH OTHER. This definition must not be confounded with that given of *prime numbers;* for numbers which are themselves composite. may be prime to each other. Thus 25 and 27, both of which are composite, are prime to each other. A number is *prime to itself,* which has no *measure* greater than 1. Numbers are *prime to each other,* which have no *common measure* greater than 1.

We may here make one more suggestion, which will be useful in finding the measures of numbers by trial. 2 and 5, which are prime to each other, are measures of 20. If 20 be resolved into the factors, 5 and 4, we know that 2, which is a measure of 20, must be a measure of 4, since it is not a measure of 5, and has no factor, which is a measure of 5. 20 may therefore be divided successively by 5 and 2, and, consequently, (§ XXIX.) by their product 10. The same may be shown of any two or more numbers, prime to each other, which are measures of a third number. Hence, *If a number be divisible by two or more numbers, which are prime to each other, it is likewise divisible by their product.* If the numbers are not prime to each other, we cannot make this inference.

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MENTAL EXERCISES.

§ XLIV. 1. Change the Fractions $\frac{1}{2}$ and $\frac{1}{4}$ to others having the same denominator. This can easily be done, because we know that $\frac{1}{2}$ is the same as $\frac{2}{4}$.

2. Reduce the Fractions $\frac{1}{3}$ and $\frac{1}{6}$ to the same denominator.

3. Reduce $\frac{3}{4}$ and $\frac{3}{5}$ to the same denominator.

By a COMMON DENOMINATOR is meant the *same* denominator.

4. Reduce $\frac{1}{7}$ and $\frac{2}{21}$ to a common denominator. $\frac{1}{4}$ and $\frac{1}{3}$: $\frac{1}{4}$ and $\frac{1}{2}$: $\frac{2}{9}$ and $\frac{1}{3}$: $\frac{1}{15}$ and $\frac{1}{10}$: $\frac{3}{7}$ and $\frac{5}{14}$.

When the numbers are larger the process cannot be performed in the mind. For example.

5. Reduce $\frac{1}{23}$ and $\frac{3}{24}$ to a common denominator.

When we multiply both terms of a Fraction by the same number, the value is not altered. (§ XLI.) Then we may multiply the 3, and the 24, in the last Fraction by 23, without altering the value. And, we may multiply the 1 and the 23 of the first Fraction by 24, without altering the value.

Thus, 23×3=69 ; and 23×24=552.

Therefore, $\frac{3}{24}=\frac{69}{552}$. And 24×1=24 ; and 24×23=552. Therefore, $\frac{1}{23}=\frac{24}{552}$. But, $\frac{69}{552}$ and $\frac{24}{552}$ have the same denominator.

In this process, we multiplied both terms of each Fraction, by the denominator of the other Fraction.

6. Reduce $\frac{53}{65}$ and $\frac{71}{87}$ to a com. denom. $\frac{77}{98}$ and $\frac{41}{44}$; $\frac{18}{35}$ and $\frac{17}{32}$; $\frac{62}{74}$ and $\frac{31}{35}$; $\frac{52}{65}$ and $\frac{56}{72}$.

It will be found most convenient, to reduce the Fractions first, to their lowest terms.

7. Reduce $\frac{48}{64}$ and $\frac{63}{81}$ to a common denominator.

$\frac{48}{64}=\frac{3}{4}$, and $\frac{63}{81}=\frac{7}{9}$. A. $\frac{27}{36}$ and $\frac{28}{36}$.

8. Reduce $\frac{102}{204}$ and $\frac{27}{108}$ to a com. den. $\frac{180}{360}$ and $\frac{48}{96}$; $\frac{98}{126}$ and $\frac{72}{96}$.

9. Reduce $\frac{3}{4}$ and $\frac{5}{6}$ and $\frac{1}{3}$ to a com. den.

A. $\frac{54}{72}$, $\frac{60}{72}$ and $\frac{24}{72}$.

Here we have three Fractions. If there were only the first two, viz. $\frac{3}{4}$ and $\frac{5}{6}$, we should multiply the terms of $\frac{3}{4}$ by 6 and those of $\frac{5}{6}$ by 4. But if we should multiply these terms, thus increased, by the other denominator, 2, it would not alter the value of the Fractions. And, if, then, we should multiply the terms of $\frac{1}{3}$ by 4 and 6, it would not alter its value. Therefore to reduce Fractions to a common denominater,

MULTIPLY BOTH TERMS OF EACH FRACTION BY THE DENOMINATORS OF ALL THE OTHER FRACTIONS.

NOTE. As all the denominators of the reduced Fractions are alike, when the denominator of one has been found. it may be written down for all the others, without the trouble of Multiplication.

EXAMPLES FOR PRACTICE.

10. Reduce $\frac{13}{14}$, $\frac{15}{16}$, and $\frac{17}{18}$ to a com. den. $\frac{9}{11}$, $\frac{19}{38}$ and $\frac{21}{54}$; $\frac{7}{8}$, $\frac{5}{9}$, $\frac{6}{13}$ and $\frac{5}{16}$; $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ and $\frac{1}{7}$; $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$ and $\frac{7}{8}$; $\frac{1}{12}$, $\frac{2}{14}$, $\frac{3}{15}$, $\frac{6}{28}$, $\frac{7}{21}$, $\frac{9}{18}$ and $\frac{11}{44}$.

§ XLV. A much smaller common denominator, may often be found, than the preceding rule would give. To determine what the smallest possible denominator would be, we must attend to the subject of *multiples.* 15 contains 3, an even number of times exactly. Therefore, 15 is called a *multiple* of 3.

A NUMBER IS CALLED A MULTIPLE OF ANY NUMBER WHICH WILL DIVIDE IT WITHOUT A REMAINDER.

It will be seen that *multiple* and *measure* are merely *relative terms.* Any number is the *multiple* of *its measure*, and the *measure* of *its multiple.* 15 is a multiple of both 3 and 5. For 3 and 5 are *measures* of 15. Hence 15 is called a *common multiple* of 3 and 5.

ANY MULTIPLE OF TWO OR MORE NUMBERS, IS CALLED THEIR COMMON MULTIPLE.

If any two or more numbers be multiplied togther, the product is evidently a common multiple of all; for it may be divided by either of the factors which compose it. If two numbers be *prime* to each other, their product is likewise their *least* common multiple. Thus 3 and 5 are prime to each other, and their product, 15, is their least common multiple. For if you suppose that 3, repeated fewer times, will contain 5, you must suppose that 3 contains some factor, which, multiplied by 4 will produce 5. But, as 3 and 5 are prime to each other, they have *no common factor.* Therefore 5 times 3=15, is the least common multiple of 3 and 5. If there be more numbers than two, the same is true for similar reasons.

Take a case, in which the numbers are not prime to each other. 15 and 10 have 5 for their greatest common measure. Divide one of them, as 15, by this common measure, and the quotient is 3. Multiply this into the other, and the product, 30, is the least common multiple of 15 and 10. For if you suppose that 10, repeated fewer times, as, for example 2 times, will contain 15, you must suppose that 10 contains some factor, which multiplied by 2, will produce 15. But this cannot be, for the quotient 3 was found by dividing 15 by the *greatest* common measure of the two numbers. Therefore, 3 times 10=30, is the least common multiple of 15 and 10.

Hence, to find the least common multiple of two numbers,

DIVIDE ONE OF THE NUMBERS BY THEIR GREATEST COMMON MEASURE, AND MULTIPLY THE QUOTIENT BY THE OTHER.

EXAMPLES FOR PRACTICE.

1. Find the least common multiple of 12 and 16.

Their greatest com. meas. is 4. 16÷4=4. 4×12=48 Ans.

2. Find the least common multiple of 14 and 18. A. 126. Of 9 and 12. A. 36. Of 24 and 30 A. 120. Of 25 and 45. Of 24 and 84. Of 36 and 48. Of 26 and 36. Of 32 and 56. Of 81 and 108.

When there are more than two numbers, after finding the least common multiple of two of them, proceed with this multiple and the third number, to find another common multiple by the rule. After finding for three, proceed in the same manner with the fourth number, and so on.

3. Find the least common multiple of 15, 18 and 24.

The least com. mult. of 15 and 18 is 90; of 90 and 24, 540 *Ans.*

4. Find the least com. mult. of 12, 16 and 30. A. 240. Of 9, 12, 16, 18 and 24. A. 144. Of 6, 10, 25 and 36. Of 224, 648, 936 and 872. Of 828, 333, 756 and 963.

When there are many numbers and particularly if they are large, it is tedious to proceed as above. There is a shorter process.

We have already seen, that when several numbers are *prime to each other*, their product is their least common multiple. If, then, we can contrive to find all the *factors prime to each other*, in the several numbers given, the product of these factors will be the least common multiple, not only of the factors, themselves, but also of the numbers from which we obtained them. These factors must be found by actual trial, thus. *Place the numbers in a row and divide as many of them as possible by any number, which will divide them without a remainder, placing the quotients, with the numbers not divided, likewise in a row under the first. Divide as many numbers in this row as possible, in the same manner as before, and continue this process until no two numbers can be divided by any number greater than 1. The divisors used, and the last row of numbers, will be the factors sought.* Of course,

THE PRODUCT OF THESE FACTORS WILL BE THE LEAST COMMON MULTIPLE REQUIRED.

5. For example: find the least common multiple of 72, 64, 21, 18, and 98.

2	72	64	21	18	98
3	36	32	21	9	49
3	12	32	7	3	49
7	4	32	7	1	49
4	4	32	1	1	7
	1	8	1	1	7

I observe that all, but 21, may be divided by 2. Therefore, I divide by 2 and bring down 21. Then, that all the next row, except 32 and 49, may be divided by 3. Therefore I divide by 3, and bring down 32 and 49. In this way, I proceed, dividing, each time as many as possible. I obtain then the factors 2, 3, 3, 7, 4, 1, 8, 1, 1, 7. Therefore 2×3×3×7×4×8×7=28,224 is the least common multiple of the given numbers. In multiplying, I neglect the factors, 1, 1, 1, since they will not alter the product. The intelligent pupil will at once reply that these factors are not all prime to each other. But, it should be recollected, that, by this process, the factors of some of the numbers become resolved into their *component parts.* If these *component parts* be *re-multiplied*, the compound factors, thus obtained, will be all prime to each other. Thus, the two factors, 3 and 3, were derived, successively from 18. Therefore the compound factor of 18, is 3×3=9. In like manner 2, 4 and 8 were derived successively from 64, and the compound factor is 2×4×8=64. So 7 and 7 were derived from 98, and the compound factor is 49. It will now be seen, that these compound factors, 64, 9 and 49, are all prime to each other; and that they contain all the factors, which are component parts of the given numbers, and no more.

By the process, it will be seen, that when any number is a factor of several numbers, it is excluded from them all, by Division, and is then made *once* a factor in the multiple obtained. If all the given numbers of which it is a factor be not divided by it, then we shall employ it more than once as a factor in the result, and thus make the multiple obtained too large. From this consideration, the pupil will see the necessity of the following *caution* in employing the above mode of operation.

In making each division, choose divisors which will divide AS MANY NUMBERS AS POSSIBLE.

This will be best understood by illustration.

6. Find the least com. mult. of 6, 9, and 24.

6 and 24 can be divided by 9. But all the numbers can be divided by 3. If, then we first divide by 6, we shall leave the 9 undivided by 3, and so obtain too great a multiplier.

First divide by 6.

$$\begin{array}{r|lll} 6 & 6, & 9, & 24, \\ \hline & 1, & 9, & 4, \end{array}$$

We can divide no further. Therefore we should expect 6×9×4=216 to be the least com. mult., which the other operation shows to be incorrect.

First divide by 3.

$$\begin{array}{r|lll} 3 & 6, & 9, & 24, \\ \hline 2 & 2, & 3, & 8, \\ \hline & 1; & 3, & 4, \end{array}$$

Hence, 3×2×3×4=72 is the least com. mult.

None of our common Arithmetics provide against this source of error, and hence the learner is often perplexed by finding his result wrong, when he has strictly followed the directions of his rule. By this mode, perform the following.

7. Find the least com. mult. of 32, 72 and 120. A. 1,440. Of 30,48 and 56. A. 1,680. Of 250, 180 and 540. A. 13,500. Of 375, 125, 320 and 45. Of 872, 16, 98 and 75. Of 196, 762, 1,131, 340 and 460.

In finding a common denominator for several Fractions, we multiply all the denominators together. The common denominator, therefore, is a common multiple of the given denominators. Of course, the *least* com. den. must be the *least* com. mult.

8. Reduce $\frac{5}{6}$ and $\frac{3}{8}$ to the least com. den.

24 is the least com. mult. of 5 and 8. We must therefore bring the Fractions to 24ths. 24 24ths make a whole one. Then $\frac{5}{6}$ of 1 is $\frac{5}{6}$ of 24 24ths$=\frac{20}{24}$; and $\frac{3}{8}$ of 1 is $\frac{3}{8}$ of 24 24ths$=\frac{9}{24}$. A. $\frac{20}{24}$ and $\frac{9}{24}$.

Hence, for the *numerators* of the reduced Fractions, *multiply the com. denom. by each of the given Fractions.* (§ XXXIII.)

9. Reduce $\frac{3}{32}$ and $\frac{5}{28}$ to the least com. den. A. $\frac{21}{224}$ and $\frac{40}{224}$. $\frac{4}{27}$ and $\frac{5}{24}$. A. $\frac{8}{54}$ and $\frac{5}{54}$. $\frac{5}{18}$, $\frac{8}{27}$ and $\frac{17}{96}$. A. $\frac{75}{270}$, $\frac{80}{270}$ and $\frac{57}{270}$. $\frac{8}{9}$, $\frac{7}{58}$, $\frac{8}{19}$ and $\frac{4}{27}$. $\frac{7}{38}$, $\frac{3}{40}$, $\frac{9}{13}$, $\frac{5}{18}$, and $\frac{11}{64}$. $\frac{3}{27}$, $\frac{5}{63}$, $\frac{9}{49}$, $\frac{6}{35}$, $\frac{8}{15}$ and $\frac{3}{320}$.

A shorter mode of finding numerators, may often be employed. Thus,

10. Reduce $\frac{5}{12}$ and $\frac{4}{27}$ to the least com. den. A. $\frac{45}{108}$ and $\frac{16}{108}$.

$$\begin{array}{r|ll} 3 & 12, & 27, \\ \hline & 4, & 9, \end{array}$$

Process for the least com. mult. Then $3\times4\times9=108$ is the least com. mult. 3 and 4 (the first two of these factors) multiplied, make 12, the first given denominator. The remaining factor, being 9, shows that the com. den. is 9 times the given den. 12. Of course the corresponding numerator, in the answer, ought likewise, to be 9 times the given numerator 5. Hence, $9\times5=45$ the first numerator. It will be seen that this is obtained by excluding from the factors of the com. den. just enough to produce 12, the given den., and multiplying the numerator 5, by the remaining factor 9. The process is similar for the other numerator. Thus, $3\times9=27$ the second den. Therefore, exclude 3 and 9, and multiply 4 the second numerator by 4, the remaining factor. Then $4\times4=16$ the second required numerator.

11. Reduce $\frac{5}{48}$, $\frac{17}{56}$ and $\frac{9}{64}$ to the least com. den. A. $\frac{140}{1344}$, $\frac{408}{1344}$, $\frac{189}{1344}$.

12. Reduce $\frac{9}{22}$, $\frac{3}{28}$, $\frac{9}{36}$ and $\frac{13}{52}$ to the least com. den.

When seeking the least com. den. we must of course take care first to bring the given Fractions to their lowest terms.

13. Reduce $\frac{12}{27}$, $\frac{30}{48}$, $\frac{77}{84}$, and $\frac{40}{135}$ to the least com. den. A. $\frac{96}{216}$, $\frac{135}{216}$, $\frac{198}{216}$, and $\frac{64}{216}$.

14. Reduce $\frac{3}{9}$, $\frac{8}{24}$, $\frac{7}{49}$, $\frac{9}{81}$, $\frac{24}{84}$ and $\frac{32}{96}$ to the least com. den.

Sometimes the denominator of one of the given Fractions may be a measure of that of another. In this case, by multiplying both terms of the former, it may be brought to the same denominator with the latter: or, if the numerator of the latter admits of being divided, we may bring this one to the same denominator with the former. Common denominators, and sometimes a least common denominator may often be readily found in this way. Sometimes both Division and Multiplication may be used, but we leave the pupil, in this respect, to exercise his own ingenuity.

15. Take, for example the Fractions $\frac{1}{9}$ and $\frac{4}{18}$. Multiplying the former by 2 gives us $\frac{2}{18}$ and $\frac{4}{18}$, having a com. den. Dividing the latter by 2 gives $\frac{1}{9}$ and $\frac{2}{9}$, having a least com. den.

16. Reduce $\frac{1}{2}$ and $\frac{15}{18}$ to a com. den. A. $\frac{9}{18}$ and $\frac{15}{18}$.

17. Reduce $\frac{3}{4}$ and $\frac{6}{16}$ to a com. den. A. $\frac{12}{16}$ and $\frac{6}{16}$. To a least con. den. A. $\frac{6}{8}$ and $\frac{3}{8}$.

18. Reduce $\frac{2}{3}$ and $\frac{3}{12}$ to a least com. den. A. $\frac{8}{12}$ and $\frac{3}{12}$. $\frac{3}{5}$ and $\frac{4}{15}$. A. $\frac{3}{5}$ and $\frac{2}{5}$. $\frac{5}{15}$ and $\frac{2}{3}$. $\frac{3}{7}$ and $\frac{2}{28}$. $\frac{4}{5}$ and $\frac{6}{25}$. $\frac{6}{7}$ and $\frac{4}{56}$. $\frac{5}{6}$ and $\frac{6}{24}$.

19. Reduce $\frac{2}{3}$ and $\frac{4}{9}$ and $\frac{14}{27}$ to a least com. den.

20. Reduce $\frac{1}{4}$ and $\frac{2}{3}$ and $\frac{5}{12}$ and $\frac{2}{24}$ to a least com. den.

ADDITION.

MENTAL EXERCISES.

§ XLVI. 1. George gave $\frac{1}{4}$ of an orange to his sister, $\frac{3}{4}$ to his brother, and then had $\frac{3}{4}$ himself. How many fourths had he at first?

2. William bought $\frac{2}{6}$ of a quart of chesnuts at one place, $\frac{3}{6}$ at another, and $\frac{5}{6}$ at another. How many sixths did he buy in all?

3. A man has $\frac{3}{4}$ of an acre of ground in his yard, $\frac{5}{4}$ in his garden, and $\frac{9}{4}$ in his corn lot. How many fourths has he in all?

4. A man gave $\frac{2}{5}$ of a bushel of rye to one person, $\frac{3}{5}$ to another, and $\frac{4}{5}$ to another. How many fifths did he give away?

5. How many eighths in $\frac{1}{8}+\frac{2}{8}+\frac{3}{8}$? In $\frac{7}{8}+\frac{6}{8}+\frac{3}{8}+\frac{5}{8}$?

6. How many twelfths in $\frac{4}{12}+\frac{2}{12}+\frac{3}{12}$? In $\frac{1}{12}+\frac{9}{12}+\frac{7}{12}$?

In the above examples, the denominators are alike, and the process of Addition is very easy, since we only have to add the numerators. But when the denominators are different, we cannot add in this way. But, by § XLIV, we can *make the denominators alike*; that is, we can reduce the Fractions to a *common* denominator.

7. Add $\frac{1}{2}$ and $\frac{2}{3}$. $\frac{1}{2}$ and $\frac{1}{3}$. $\frac{1}{7}$ and $\frac{1}{2}$. $\frac{4}{5}$ and $\frac{2}{3}$.

8. Add $\frac{9}{10}$ and $\frac{1}{2}$. $\frac{3}{5}$ and $\frac{1}{3}$. $\frac{5}{7}$ and $\frac{1}{10}$. $\frac{2}{3}$ and $\frac{7}{8}$.

The following are to be written. It is always best to reduce Fractions to their lowest terms before adding. The sum may often be reduced to a whole or mixed number. When this cannot be done, it may often be reduced to lower terms.

9. Add $\frac{7}{21}$, $\frac{9}{36}$ and $\frac{8}{48}$. A. $\frac{9}{12}=\frac{3}{4}$.

10. Add $\frac{35}{45}$, $\frac{63}{81}$ and $\frac{16}{36}$. A. 2.

11. Add $\frac{96}{120}$, $\frac{60}{108}$, $\frac{72}{84}$ and $\frac{36}{48}$. A. $2\frac{1213}{1260}$.

12. Add $\frac{21}{63}$, $\frac{24}{192}$, $\frac{49}{56}$, $\frac{33}{55}$ and $\frac{39}{52}$. A. $2\frac{41}{60}$.

When mixed numbers occur, it is plain that we must *add the whole, and the Fractional parts separately*. The two sums must then be united into one.

13. Add $13\frac{2}{3}$ and $5\frac{2}{5}$ $13+5=18$. $\frac{2}{3}+\frac{2}{5}=\frac{16}{15}=1\frac{1}{15}$. $18+1\frac{1}{15}=19\frac{1}{15}$ A.

14. Add $17\frac{1}{2}$ and $16\frac{4}{7}$. A. $33\frac{1}{14}$.

15. Add $15\frac{1}{9}$, $14\frac{3}{8}$, $\frac{3}{4}$ and $\frac{2}{3}$. A. $30\frac{65}{72}$.

16. Add $\frac{7}{12}$, $19\frac{8}{27}$, $\frac{14}{12}$, $9\frac{5}{24}$, $\frac{11}{132}$ 252 and $1\frac{5}{48}$.

A. $283\frac{191}{432}$.

17. Add $\frac{39}{80}$, $\frac{76}{80}$, 259, $547\frac{369}{720}$, $329\frac{38}{40}$, 54 and $3\frac{1}{20}$. A. $1{,}194\frac{19}{20}$.

18. Add $13\frac{4}{15}$, $17\frac{3}{20}$, $30\frac{5}{6}$, 99, $101\frac{7}{42}$, $20\frac{12}{76}$, $19\frac{36}{162}$, and $15\frac{144}{1728}$. A. 216.

19. Add $137\frac{2}{5}$, $26\frac{9}{15}$ and $243\frac{2}{7}$. A. $407\frac{2}{7}$.

In this example, reducing $\frac{9}{15}$ to its lowest terms, it becomes $\frac{3}{5}$ which added to $\frac{2}{5}$ makes $\frac{5}{5}$, or 1. It is therefore unnecessary to reduce $\frac{2}{5}$ and $\frac{9}{15}$ to the same denominator with $\frac{2}{7}$. Such artifices may often be employed.

20. Add $3\frac{35}{42}$, $4\frac{14}{16}$, $3\frac{24}{28}$, $9\frac{12}{72}$, $16\frac{19}{152}$ and $\frac{5}{35}$. A. 38.

21. Add $18\frac{24}{32}$, $\frac{13}{52}$, $17\frac{25}{45}$, 11, $6\frac{36}{108}$ and $5\frac{7}{63}$. A. 59.

22. Add $13\frac{12}{36}$, 17, $2\frac{26}{30}$, $1\frac{36}{45}$, 14 and $53\frac{11}{33}$. A. $102\frac{1}{3}$.

23. Add $4\frac{28}{98}$, $3\frac{36}{132}$, $10\frac{50}{130}$, $7\frac{27}{39}$, $6\frac{27}{117}$, $8\frac{48}{104}$, $1\frac{34}{221}$, $2\frac{19}{247}$, $4\frac{22}{121}$, $5\frac{84}{154}$ and $8\frac{52}{91}$. A. $60\frac{6}{7}$.

◆

MULTIPLICATION.

MENTAL EXERCISES.

§ XLVII. 1. If 1 dollar will buy $\frac{1}{3}$ of a bushel of wheat how many thirds will 2 dollars buy?

2. If 1 cent will buy $\frac{1}{6}$ of an orange, how much will 5 cents buy?

3. If 1 bushel of oats cost $\frac{1}{3}$ of a dollar, what will 2 bushels cost?

4. If 1 apple cost $\frac{1}{4}$ of a cent, what will 3 apples cost?

5. A boy gave $\frac{1}{7}$ of a pine-apple to one of his companions, and 3 times as much to another, how much did he give the last?

6. 4 times $\frac{1}{6}$ are how many 6ths? 7 times $\frac{1}{8}$?

7. 3 times $\frac{1}{5}$ are how many 5ths? 6 times $\frac{1}{7}$?

8. 10 times $\frac{3}{4}$ are how many 4ths? 5 times?

9. 5 times $\frac{2}{3}$ are how many 3ds? 6 times?

In the above examples, a Fraction is the multiplicand, and a whole number the multiplier; in other words, it is required to multiply a Fraction by a whole number. As Multiplication is only a repeated Addition of the multiplicand, (§ XI.) it is plain, that, in this case, it may be performed like Addition of Fractions, in the last section. Thus,

10. A man had $\frac{3}{4}$ of a dollar in each pocket, and he had 4 pockets. How many fourths of a dollar had he?

Repeat $\frac{3}{4}$, 4 times by Addition, thus, $\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}=\frac{12}{4}$ *Ans.*

Hence it appears, that *Multiplication of a Fraction by a whole number, may be performed by a repeated Addition of the Numerator of the multiplicand.* But this repeated Addition is Multiplication of the Numerator. (§ xl.)

Hence, to multiply a Fraction by a whole number,

I. MULTIPLY THE NUMERATOR OF THE FRACTION BY THE WHOLE NUMBER.

This was also shown in § XLI. In the same §, likewise, it was shown, that *dividing the denominator multiplies the value.*

Hence, another rule.

II. DIVIDE THE DENOMINATOR OF THE FRACTION BY THE WHOLE NUMBER.

NOTE. This is altogether *the best rule,* and should *always be used* when the denominator admits of Division.

In performing the following examples for practice, let the pupil divide the denominator, in all cases, where it is practicable. The results should be reduced, if possible, to whole or mixed numbers as in Addition.

11. Multiply $\frac{25}{96}$ by 4. A. $\frac{25}{24}=1\frac{1}{24}$.

12. Multiply $\frac{328}{7853}$ by 7,853. A. $\frac{328}{1}=328$. Hence,

A Fraction is multiplied into a number equal to its denominator, by removing the denominator.

14. How much is 40 times $\frac{71}{120}$. A. $\frac{71}{3}=23\frac{2}{3}$.

13. How much is 27 times $\frac{31}{45}$.

27 is a composite number, having the factors 9 and 3. We may multiply by these factors, successively, instead of multiplying by 27 at once. (§ XI.) Now 9 will divide the denominator 45, though 27 will not. Therefore, $\frac{31}{45}\times 9=\frac{31}{5}$, and $\frac{31}{5}\times 3=\frac{93}{5}=18\frac{3}{5}$ A. This mode is often convenient.

15. A man bought 12 yards of cloth at $5\frac{3}{15}$ dollars a yard. How much did the whole come to. A. $\$62\frac{2}{5}$.

To multiply a mixed by a whole number, *multiply the whole, and fractional parts separately, and unite the products.*

16. Multiply $3\frac{5}{60}$ by 42. A. $129\frac{1}{2}$.

17. Multiply $7\frac{3}{80}$ by 40. A. $281\frac{1}{2}$.

18. Multiply $\frac{15}{237}$ by 471. A. $29\frac{64}{79}$.

19. Multiply $\frac{967}{1000}$ by 138. A. $133\frac{223}{500}$.

20. What will $1\frac{233}{477}$ hogsheads of wine come to at \$1.23 per gallon? A. $\$115.34\frac{8}{53}$.

21. What will $4\frac{3}{8}$ tons of iron come to at \$4.00 pr. cwt.? A. \$350.

22. What will $8\frac{3}{5}$ hhds. of molasses come to at \$0.48 pr. gal.? A. $\$360.06\frac{2}{5}$.

23. What cost $7\frac{4}{38}$ cwt. of sugar at $0.08 pr. lb.?
A. 63.66\frac{4}{19}$.

24. What cost $23\frac{17}{35}$ pipes of brandy at $1.43 pr. gal.?
A. $4.231.65$\frac{3}{5}$

◆

§ XLVIII. In the last section you were taught to multiply a Fraction by a whole number. Multiplying a whole number by a Fraction, is exactly the same thing, for, (§ xi.) it matters not which factor is made the multiplicand nor which the multiplier. This case was noticed, § xxxiii.

To this case belong such questions as these: what is ¾ of 24; what is ⅗ of 40, &c.: that is *all questions where a whole is given to find a certain part.*

EXAMPLES FOR PRACTICE.

1. Three men, A. B. and C. drew a prize in a lottery, of $25,000.00, A. owned ½ the ticket; B. ⅓; and C. ⅙. What amount ought each to have? *Ans.* A. ½ of $25,000=$12,500. B. ⅓ of $25,000=$8,333.33⅓, and C. ⅙ of $25,000=$4,166.66⅔.

2. For the same ticket was paid $16.00. What did each pay?
Ans. A. $8.00. B. $5.33⅓. C. $2.66⅔.

3. A man, worth $32,750.25, wished to invest $\frac{9}{16}$ of it in bank capital. How much would he have left? A. $14,328.23$\frac{7}{16}$.

4. At ¾ of a dollar a yard, what cost 164 yards of linen?
A. $123.00.

5. At 75 cts. a yard, what cost 164 yards of linen?
75 cts. is ¾ of a dollar; therefore the question is the same as the last.

6. At 50 cts a yard, what cost 230 yards of cambric?
50 cts. is ½ a dollar, and ½ of 230=$115.00 A.

7. At 4d. a pr. pt. what cost 135 pts. of molasses?
4d. is ⅓ of a shilling, and ⅓ of 135=45s.=£2; 5, A.

8. At 33⅓ cts. pr. lb. what cost 15,618 lbs. of coffee? A. $5,206.
33⅓ cts. is ⅓ of a dollar, and ⅓ of 15,618=$5,206, A.

9. At 16⅔ cts. pr. lb. what cost 5,766 lbs. of sugar? A. $991.
16⅔ cts. is $\frac{1}{6}$ of a dollar, and $\frac{1}{6}$ of 5,766=$991.00 A.

10. At 25 cts. pr. lb. what cost 9,496 lbs. of raisins? A. $2,374.
25 cts. is ¼ of a dollar, and ¼ of 9,495=$2,374 A.

11. At 5s. a yard what cost 3,656 yds. of cloth? A. £914.

12. At 6d. pr. lb. what cost 2,864 lbs. of sugar? A. £71; 12.

In the above examples, the price is an *even part* of a higher denomination, as a dollar, a shilling, &c. An even part is called an ALIQUOT part. Hence, *when the price of a single thing, (as a gallon, pound, &c.) is an aliquot part of a higher denomination of money,*

MULTIPLY THE QUANTITY GIVEN, BY THE FRACTION EXPRESSING THAT PART.

NOTE. This rule may be found in other books, under the head of PRACTICE; which name is given it on account of its *practical utility.*

ALIQUOT PARTS OF A DOLLAR.

Cts.				Cts.			
50	is	$\frac{1}{2}$	of a dollar.	12½	is	$\frac{1}{8}$	of a dollar.
33⅓	is	$\frac{1}{3}$	of a dollar.	10	is	$\frac{1}{10}$	of a dollar.
25	is	$\frac{1}{4}$	of a dollar.	8⅓	is	$\frac{1}{12}$	of a dollar.
20	is	$\frac{1}{5}$	of a dollar.	6¼	is	$\frac{1}{16}$	of a dollar.
16⅔	is	$\frac{1}{6}$	of a dollar.	5	is	$\frac{1}{20}$	of a dollar.

NOTE. When the price is not an *aliquot* part, but some other simple fractional part, (as in ex. 5 above,) that Fraction may be used as a multiplier in the same manner. The pupil's ingenuity will enable him to discover many abbreviations, which we have not room to mention.

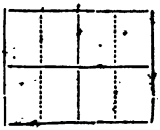
13. At 5 cts. pr. lb., what cost 360 lbs. of pork? A. $18.00.
14. At 8⅓ cts. pr. lb., what cost 6 bls. of beef? A. $100.00.
15. At 25 cts. pr. qt. what cost 478 gals. of brandy? A. $478.00.
16. At 6¼ cts. pr. pt. what cost a barrel of rum? A. $15.75.

◆

MENTAL EXERCISES.

§ XLIX. 1. A man having divided a lot of ground into 4 equal parts, divided each part into 2 equal portions. What part of the whole lot was one of these portions? What is $\frac{1}{2}$ of $\frac{1}{4}$?

The figure represents the lot, cut by the black lines into 4 equal parts, or fourths. These fourths are each cut into 2 equal parts, or halves, by the dotted lines. It will be seen that there are 8 of these halves. But when a thing is divided into 8 equal parts, each part is called an eighth. $\frac{1}{2}$ of $\frac{1}{4}$, then is $\frac{1}{8}$. This is *multiplying a Fraction by a Fraction.*



2. A boy had $\frac{3}{4}$ of an orange, and told his brother he would give him $\frac{1}{2}$ of what he had, if he could tell what part of the whole orange that would be. How much did he give him? What is $\frac{1}{2}$ of $\frac{3}{4}$?

3. A boy had $\frac{1}{2}$ of an apple, and gave away [illegible] of what he had. How much did he give away? What is $\frac{1}{3}$ of $\frac{1}{2}$?

4. A man having $\frac{1}{2}$ of a bushel of oats, fed his horse $\frac{2}{3}$ of them. What part of a whole bushel did he give the horse? What is $\frac{2}{3}$ of $\frac{1}{2}$?

5. A man owning $\frac{2}{3}$ of a bed of ore, sold $\frac{1}{4}$ of his share. What part of the whole did he sell? What is $\frac{1}{4}$ of $\frac{2}{3}$?

6. A man owning $\frac{2}{3}$ of the capital of a trading establishment, sold $\frac{3}{4}$ of his share. What part of the whole capital did he sell? What is $\frac{3}{4}$ of [illegible]?

7. What is $\frac{1}{6}$ of $\frac{1}{3}$? $\frac{1}{6}$ of $\frac{2}{3}$? $\frac{5}{6}$ of $\frac{2}{3}$? $\frac{3}{6}$ of $\frac{2}{3}$?

8. What is $\frac{1}{7}$ of $\frac{1}{5}$? $\frac{1}{7}$ of $\frac{3}{5}$? $\frac{2}{7}$ of $\frac{3}{5}$? $\frac{4}{7}$ of $\frac{4}{5}$?

The following are to be written.

9. What is $\frac{1}{9}$ of $\frac{1}{15}$? To obtain a ninth part of a number, we must divide by 9. But (§ XLI.) multiplying the denominator divides the value. Hence, $\frac{1}{9}$ of $\frac{1}{15}=\frac{1}{9\times15}=\frac{1}{135}$ A. $\frac{2}{9}$ of $\frac{1}{15}$? A. $\frac{2}{135}$. $\frac{2}{9}$ of $\frac{4}{15}$? A. $\frac{8}{135}$.

10. What is $\frac{5}{17}$ of $\frac{7}{18}$? First find $\frac{1}{17}$ of $\frac{1}{18}=\frac{1}{306}$: then $\frac{1}{17}$ of $\frac{7}{18}=7$ times as much$=\frac{7}{306}$; then $\frac{5}{17}$ of $\frac{7}{18}=\frac{35}{306}$ A. This is the ANALYTIC MODE of solution.

11. What is $\frac{21}{22}$ of $\frac{27}{29}$? A. $\frac{567}{638}$. $\frac{15}{56}$ of $\frac{3}{7}$? A. $\frac{45}{392}$.

In every case, the pupil will observe, that the answer is found by multiplying the two numerators together, and likewise the two denominators together. Hence, the rule,

MULTIPLY THE NUMERATORS TOGETHER, FOR THE NUMERATOR OF THE PRODUCT, AND THE DENOMINATORS TOGETHER, FOR THE DENOMINATOR OF THE PRODUCT.

If mixed numbers occur in the multiplier or multiplicand, it is best, when the integral parts are small, to reduce them to improper Fractions; when otherwise, to multiply separately the whole and Fractional parts of the multiplier, into the whole and Fractional parts of the multiplicand.

A case of the unperformed multiplication of two or more Fractions, that is, a Fraction of a Fraction (like the expressions, $\frac{3}{4}$ of $\frac{1}{2}$, $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{4}{5}$, &c.) is called a COMPOUND FRACTION. A Fraction, not connected with others in this way, is called a SIMPLE FRACTION. Some of the members of a compound Fraction, may sometimes be whole or mixed numbers. Thus $\frac{2}{3}$ of $3\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{3}{4}$ of 2, are compound Fractions. Compound Fractions may be reduced to simple ones by multiplying numerators together, and denominators together. Whole numbers are of course made factors in the numerator. In this reduction, if the numerator of one Fraction be equal to the denominator of another, both may be neglected. (§ XLI.) Thus, in reducing $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, I may neglect the two 3s, and likewise the two 2s, and the resulting Fraction will be $\frac{1}{4}$. So, likewise, if any numerator has a common *factor*, or *measure*, with any denominator, they may be divided by it before reducing.

Thus, $\frac{36}{37}$ of $\frac{21}{48}$ of $\frac{74}{75}$. 36 and 48 may both be divided by 12; 21 and 75 by 3; and, as 37 measures 74, it may be neglected and 74 divided by it. The Fraction will then be $\frac{3}{1}$ of $\frac{7}{4}$ of $\frac{2}{25}$. 2 and 4 may still be divided, and it becomes $\frac{3}{1}$ of $\frac{7}{2}$ of $\frac{1}{25}=\frac{21}{50}$.

In the Addition and Subtraction of Fractions, it is evident that compound Fractions should first be reduced to simple ones. It will be observed that the word OF between two Fractions signifies the same thing as the sign ×.

13. A man owned $\frac{4}{25}$ of a factory, and sold $\frac{4}{11}$ of his

share. What part of the whole capital did he sell? A. $\frac{24}{275}$. What part did he then own? A. $\frac{20}{275}=\frac{4}{55}$.

14. During a storm, the master of a vessel was obliged to throw overboard $\frac{7}{15}$ of the cargo. A. owned $\frac{1}{3}$ of the cargo; what part of the whole cargo was his share of the loss. A. $\frac{7}{45}$. What had he left? A. $\frac{1}{5}$ of the whole cargo.

15. At $\$\frac{4}{19}$ pr. yd. what cost $13\frac{2}{7}$ yds. of cotton cloth? A. $\$2\frac{106}{133}$.

16. If 1 lb. of cotton cost $\$\frac{1}{8}$, what cost $15\frac{43}{85}$ lbs.? A. $\$1\frac{319}{340}$.

17. At $\$\frac{37}{285}$ pr. qt., what cost 4 gals. $1\frac{2}{15}$ qts. of rum? A. $\$2\frac{959}{4275}$.

18. A and B bought a cord of wood together; A paid 3 dollars, and B, 4. What part of the whole did each pay? How many solid feet ought each to have? *Ans.* A. $54\frac{6}{7}$ and B. $73\frac{1}{7}$.

19. Two men, E and F, traded in company. E furnished $2,000 and F, $1,000 of the capital. What part of the whole did each furnish? They gained $864. What was each one's share? A. E's $576, F's $288.

20, A, B and C traded in company and gained $300. A's capital was $600, B's $400, C's $200. What part of the gain ought each to have? What was each man's share? A's $150, B's $100, C's 50.

Questions like the last three, belong to what is called FELLOWSHIP. BY FELLOWSHIP IS MEANT A CO-PARTNERSHIP, OR JOINT INTEREST; as when several men are trading together.

Fellowship is not, however, confined to trading companies, but embraces all cases, whatever, in which a distribution of money or property, or an adjustment of gain or loss is to be made among several individuals in a certain proportion.

The money or the value of the articles put into the common fund, is called STOCK or CAPITAL

The profit or loss to be shared or divided is called a DIVIDEND.

A company is said to *declare a dividend*, when public notice is given of the amount due on each share of stock. After a dividend has been *declared*, the company is bound in law to make payment of the amount to the owners of stock, on or after the day when it is declared to be due. From the above we have the rule.

FIND WHAT FRACTIONAL PART OF THE WHOLE CAPITAL, EACH PARTNER'S CAPITAL IS, AND MULTIPLY THE WHOLE DIVIDEND BY IT.

It will be seen that many questions in Fellowship have nothing to do with Capital or Stock. Such are the division of prize money among a crew, of estates among heirs, the assessment of taxes, the distribution of a bankrupt's estate among creditors, &c. &c.—for all which things, see § XCII.

21. A, B, and C freight a vessel with 300 tons. In a storm, the crew were obliged to throw 100 tons overboard. What was each man's loss, allowing that A owned 90 tons, B, 120, and C, the rest?

ANALYTIC SOLUTION. First find the loss on 1 ton. The loss on 1 ton is $\frac{1}{300}$ of the loss on 300 tons, and is therefore $\frac{1}{300}$ of 100 tons$=\frac{100}{300}=\frac{1}{3}$ of a ton. The loss, then, is $\frac{1}{3}$ of a ton on every ton of freight, and, as A owned 90 tons, he ought to lose 90 times $\frac{1}{3}=\frac{90}{3}=30$ tons. B ought to lose 120 times $\frac{1}{3}=\frac{120}{3}=40$ tons; and C ought to lose 100—30—40=30 tons.

22. A, B, C and D own, respectively, $300, $500, $900 and $300 stock in trade. They gain $200. What is each man's share? A. $30, $50, $90, $30.

NOTE. To solve it analytically, find the gain first on $1, &c. The pupil will soon find this mode pleasing, simple, and satisfactory.

23. Three men, A, B and C, purchased a hogshead of wine; for which A paid $45, B, $44, and C, $100. How many gallons ought each to receive?

Ans. A 15 gals. B 14$\frac{2}{3}$ gals. C 33$\frac{1}{3}$.

24. Four men purchased a field for $6,000. The first paid $1,200, the second, $2,000, the third, $1,000, and the fourth, the rest. The field contained 600 acres. What was each man's share? A. The first, 120, the second, 200, the third, 100, and the fourth, 180 acres.

25. A and B trade together. A furnishes $100 of capital, and B, $200. At the end of 2 months B withdraws his capital, but A goes on to the end of a year from the commencement of the partnership. The gain is then found to be $32. How much ought each man to have?

This case differs from the preceding, because the stocks are in trade *different lengths of time.*

ANALYTIC SOLUTION. A's capital, $100, is in trade 12 months, and gains a certain sum. 12 times as much capital would have gained the same sum in 1 month: that is $100×12=$1,200. Hence, *A's gain is the sum which* $1,200 *would acquire* (gaining at the same rate) *in* 1 *month.* B's capital $200 is in trade 2 months. 2 times as much capital would have gained the same sum in 1 month: that is, $200×2=$400. Hence, *B's gain is the sum which* $400 *would acquire* (at the same rate) *in* 1 *month.* Hence, *A's and B's gains together are* *what* $\$1,200+\$400=\$1,600$ *would acquire in* 1 *month.* Now, if \$1,600 gain \$32, \$1 will gain $\frac{1}{1600}$ as much $=\$\frac{32}{1600}=\$\frac{1}{50}$. Then \$1,200 will gain 1,200 times as much $=\$\frac{1200}{50}=\$24$, A's gain; and \$400 will gain 400 times as much as $\$1=\$\frac{400}{50}=\$8$, B's gain.

Solution by FRACTIONAL PARTS, OR RATIOS. We find above, that A's gain is what \$1,200 would acquire in 1 month, and B's, what \$400 would acquire in 1 month. Therefore, the whole gain, \$32, is what $\$1,200+\$400=\$1,600$ would acquire in 1 month. A's part of this is $\frac{1200}{1600}=\frac{3}{4}$ of $\$32=\$24$; B's is $\frac{400}{1600}=\frac{1}{4}$ of $\$32=\$8$. Hence, the rule,

MULTIPLY EACH PARTNER'S CAPITAL BY ITS TIME, AND ADD THE PRODUCTS. FIND WHAT FRACTIONAL PART EACH ONE'S PRODUCT IS OF THIS SUM, AND MULTIPLY THE WHOLE DIVIDEND BY IT.

26. A and B hire a pasture for \$36. A put in 23 oxen for 27 days, and B 21 oxen for 35 days. How much ought each to pay? A. A. $\$16.48\frac{912}{1356}$, B. $\$19.51\frac{444}{1356}$.

27. Two merchants traded in partnership. A put in at first \$200, and at the end of 8 months put in \$100 more. B put in at first \$550, but at the end of 4 months, he took out \$140. At the end of 18 months they have gained \$526. What is each man's share?

A. A's $\$192.95\frac{70}{1254}$, and B's $\$333.04\frac{1184}{1254}$.

28. Three graziers hire a pasture for \$60.50. A puts in 5 sheep for $4\frac{1}{2}$ months, B, 8 for 5 months, and C, 9 for $6\frac{1}{2}$ months. What ought each to pay?

A. A \$11.25, B \$20 and C \$29.25.

29. Three persons make a joint stock. A puts in \$185.66, and B \$98.50, and C \$76.85. They gain \$222. What is each one's share? A. A's $\$114.17\frac{83}{36101}$, B's $\$60.57\frac{6243}{36101}$, C's $\$47.25\frac{29775}{36101}$.

30. A bankrupt is indebted to A \$277.33, to B \$305.17, to C \$152, and to D \$105. His estate is worth only \$677.50. How must it be divided? A. A $\$223.81\frac{2580}{8395}$, B $\$246.28\frac{615}{8395}$, C $\$122.66\frac{6930}{8395}$ and D $\$84.73\frac{6665}{8395}$.

31. A, with \$1,000 capital, began trade Jan. 1, 1829. He took in B as a partner, with \$1,500 capital on the 1st. of March following. Three months after, they admit C, with \$2,800 capital, and Jan. 1, 1830 they find the whole gain \$1,776.50. What sum belonged to each?

A. A $\$457.46\frac{364}{466}$, B $\$571.83\frac{222}{466}$, C $\$747.19\frac{346}{466}$.

SUBTRACTION.

MENTAL EXERCISES.

§ L. 1. A boy had $\frac{5}{8}$ of a quart of chesnuts, and gave away $\frac{3}{8}$. How many eighths had he left?

2. A man bought $\frac{9}{10}$ of a barrel of flour, and gave $\frac{2}{10}$ to some labourers. How much had he left?

3. Two boys together gathered $\frac{7}{8}$ of a peck of strawberries, and one of them took $\frac{3}{8}$ of a peck, as his share. How many had the other?

4. $\frac{7}{9}$ from $\frac{8}{9}$ leaves how many ninths?

5. $\frac{3}{7}$ from $\frac{6}{7}$ leaves how much? $\frac{3}{5}$ from $\frac{4}{5}$? $\frac{8}{12}$ from $\frac{11}{12}$? $\frac{5}{6}$ from $1\frac{1}{6}$? $\frac{5}{6}$ from $2\frac{2}{6}$? $\frac{9}{11}$ from $8\frac{1}{11}$? $\frac{6}{7}$ from $5\frac{3}{7}$? $\frac{9}{12}$ from $6\frac{2}{12}$? $\frac{2}{9}$ from $4\frac{7}{9}$?

But we shall often be required to make subtractions between Fractions whose denominators are different. In this case we may reduce them to a *common* denominator, and then we can subtract as above.

6. Take $\frac{1}{7}$ from $\frac{1}{3}$. $\frac{1}{7}$ is $\frac{3}{21}$, and $\frac{1}{3}$ is $\frac{7}{21}$.

7. Take $\frac{2}{5}$ from $\frac{5}{6}$. $\frac{3}{4}$ from $\frac{7}{9}$. $\frac{1}{4}$ from $\frac{3}{5}$. $\frac{1}{3}$ from $\frac{9}{16}$.

8. Take $1\frac{1}{4}$ from $2\frac{4}{5}$. $2\frac{1}{7}$ from $5\frac{8}{14}$. $3\frac{1}{2}$ from $5\frac{1}{3}$.

Let the following be written.

9. From $\frac{14}{45}$ take $\frac{1}{2}$. A. $\frac{13}{90}$.

10. From $\frac{22}{27}$ take $\frac{2}{3}$. A. $\frac{12}{81}$.

11. From $\frac{51}{52}$ take $\frac{37}{39}$. A. $\frac{5}{156}$.

12. From $2\frac{61}{63}$ take $1\frac{13}{27}$. A. $1\frac{92}{189}$.

NOTE. When mixed numbers occur, perform the subtraction of the whole, and fractional parts *separately*, if it can be done.

13. From $20\frac{13}{15}$ take $6\frac{2}{5}$. A. $14\frac{7}{15}$.

14. From $37\frac{11}{14}$ take $20\frac{2}{7}$. A. $17\frac{1}{2}$.

15. From $30\frac{65}{72}$ take $15\frac{1}{8}$. A. $15\frac{7}{9}$.

16. From $21\frac{1}{4}$ take $6\frac{2}{3}$. A. $14\frac{7}{12}$.

NOTE. When the Fraction in the subtrahend is greater than that in the minuend, it will be necessary to borrow a unit. Sometimes, likewise, there will be no Fraction in the minuend. In this case, it will also be necessary to borrow.

17. From $225\frac{9}{11}$ take $71\frac{11}{12}$, A. $153\frac{119}{132}$.

18. From $621\frac{3}{4}$ take $497\frac{5}{6}$. A. $123\frac{11}{12}$.

19. From $525\frac{6}{13}$ take $326\frac{9}{14}$. A. $198\frac{149}{182}$.

20. From $2,983\frac{287}{456}$ take $1,843\frac{369}{370}$. A. $1,139\frac{53323}{84360}$.

21. From $21\frac{45951}{46952}$ take $13\frac{227840}{616907}$. A. $8\frac{18650377717}{28965017464}$.

Hence, *to perform Subtraction of Fractions,*

REDUCE THE FRACTIONS TO A COMMON DENOMINATOR, FIND THE DIFFERENCE OF THE NUMERATORS, AND WRITE IT OVER THE COMMON DENOMINATOR.

22. A man bought $27\frac{2}{3}$ yds. of cloth, and had $19\frac{2}{21}$ yds. of it made into clothes. How much was left?

A. $8\frac{4}{7}$ yds.

23. A merchant had $56\frac{97}{100}$ gals. of brandy in a cask, and $7\frac{3}{7}$ gals. leaked out. How much had he left?

A. $49\frac{379}{700}$ gals.

24. A man had a lot containing $97\frac{1}{3}$ acres of ground, from which he fenced off $14\frac{3}{7}$ acres. How many acres were left in the lot? A. $81\frac{19}{21}$ Acres.

DIVISION.

MENTAL EXERCISES.

§ LI. 1. At $\frac{1}{5}$ of a dollar a yard, how many yards of calico can be bought with $\frac{2}{5}$ of a dollar? Hom many with $\frac{3}{5}$? With $\frac{4}{5}$? With $\frac{9}{5}$? With $\frac{17}{5}$? With $\frac{24}{5}$?

2. At $\frac{1}{7}$ of a dollar a bushel, how many bushels of oats can be bought for $\frac{4}{7}$ of a dollar? How many for $\frac{6}{7}$? For $\frac{8}{7}$? For $\frac{10}{7}$?

3. $\frac{1}{4}$ is contained in $\frac{3}{4}$ how many times?

4. $\frac{2}{3}$ are in $\frac{4}{3}$ how many times? In $\frac{6}{3}$?

5. $\frac{3}{5}$ are in $\frac{6}{5}$ how many times? In $\frac{9}{5}$?

6. $\frac{2}{7}$ are in $\frac{4}{7}$ how many times? In $\frac{8}{7}$? In $\frac{6}{7}$?

7. $\frac{3}{11}$ are in $\frac{6}{11}$ how many times? In $\frac{12}{11}$? In $\frac{9}{11}$.

The pupil will see, that he has been finding how often one Fraction is contained in another; that is, he has been *dividing a Fraction by a Fraction*. In the above examples, the denominators are the same. If they were not the same, they might be made so by § XLIV. Hence, to divide a Fraction by a Fraction,

I. REDUCE THE FRACTIONS TO A COMMON DENOMINATOR, AND DIVIDE THE NUMERATOR OF THE DIVIDEND BY THAT OF THE DIVISOR.

8. Divide $\frac{3}{4}$ by $\frac{3}{8}$. The least com. den. (§XLV.) is 8. The Fractions, then, are $\frac{6}{8}$ and $\frac{3}{8}$. Then, $6 \div 3 = 2$ *Ans.*

9. Divide $\frac{5}{7}$ by $\frac{7}{8}$. Frac. reduced $\frac{40}{56}$ and $\frac{49}{56}$. $40 \div 49 = \frac{40}{49}$. *Ans.*

10. Divide $\frac{3}{4}$ by $\frac{1}{2}$. $\frac{7}{8}$ by $\frac{9}{10}$. $\frac{5}{6}$ by $\frac{4}{9}$. $\frac{3}{8}$ by $\frac{5}{7}$.

It will be observed that no use, whatever, is made of the *Common Denominator*, and that this denominator, *is, in fact, lost*, in the answer. It is very manifest, then, that, *it is of no use to find this denominator* ITSELF, *if we only find the numerators*, AS THOUGH *we were reducing the fractions to a common denominator.* For these numerators are the only numbers concerned in the process. Thus,

11. Divide $\frac{2}{27}$ by $\frac{1}{35}$. Find the numerators *as though* for a common denominator; which is done (§ XLIV.) by multiplying 35 into 2=70, and 27 into 1=27. Then $70 \div 27=\frac{70}{27}=2\frac{16}{27}$ *Ans.*

The answer obtained, is evidently the same we should have found, if we had *turned over*, or *inverted* the Divisor, and then, *multiplied the two upper numbers together for a numerator, and the two lower for a denominator.* $\frac{2}{27}\times\frac{35}{1}=\frac{70}{27}=2\frac{16}{27}$ as before. But this process *after inverting the Divisor*, is exactly like the rule for Multiplication of Fractions. (§ XLIX.) Hence, to divide a Fraction by a Fraction,

II. INVERT THE DIVISOR AND PROCEED AS IN MULTIPLICATION.

12. Divide $\frac{13}{14}$ by $\frac{15}{16}$. *Ans.* $\frac{208}{210}=\frac{104}{105}$.
13. Divide $\frac{1824}{7368}$ by $\frac{912}{2456}$. *Ans.* $\frac{2}{3}$.
14. Divide $\frac{9876}{10859}$ by $\frac{3292}{9555}$. *Ans.* $\frac{28665}{10859}=2\frac{6937}{10859}$.
15. Divide $\frac{258611}{1002001}$ by $\frac{3030301}{6060604}$. A. $\frac{1567338861044}{3036364632301}$.
16. Divide $2\frac{1}{5}$ by $\frac{1}{3}$. *Ans* $\frac{33}{5}=6\frac{3}{5}$.

NOTE. When a mixed number occurs, it is to be reduced to an *improper Fraction*. (§ XXVII.)

17. Divide $5\frac{1}{3}$ by $8\frac{1}{7}$. *Ans.* $\frac{112}{171}$.
18. How often is $2\frac{9}{10}$ contained in $3\frac{1}{2}$.
Ans. $\frac{70}{58}=\frac{35}{29}=1\frac{6}{29}$.
19. How often is 6 contained in $13\frac{1}{3}$. 6 may be made an improper Fraction by placing 1 under it, thus $\frac{6}{1}$.
Ans $\frac{40}{18}=\frac{20}{9}=2\frac{2}{9}$.

NOTE. It will be best usually to reduce the divisor, and, sometimes the dividend to its lowest terms before dividing.

20. Divide $\frac{4}{18}$ by $\frac{24}{144}$. $\frac{24}{144}=\frac{1}{6}$. *Ans.* $\frac{4}{3}=1\frac{1}{3}$.
21. Divide $\frac{15}{18}$ by $\frac{1}{3}$. $\frac{15}{18}=\frac{5}{6}$. *Ans.* $\frac{5}{2}=2\frac{1}{2}$.

◆

MENTAL EXERCISES.

§ LII. 1. A man bought cotton cloth, to the amount of 5 dollars, at $\frac{1}{5}$ of a dollar a yard. How many yards did he buy ?

2. A boy bought marbles at $\frac{1}{2}$ a cent apiece, to the amount of 12 cents. How many marbles did he buy ?

3. How many fifths are there in 4 ? In 5 ?

4. How many sevenths are there in 3 ? In 4 ?

5. A man put 6 barrels of beer into kegs that held $\frac{2}{3}$ of a barrel each. How many kegs did it take ?

6. How many times $\frac{2}{5}$ in 10 ? $\frac{3}{5}$ in 9 ? $\frac{4}{5}$ in 8 ? $\frac{2}{3}$ in 4 ? $\frac{3}{4}$ in 6 ?

7. How often is $\frac{4}{5}$ in 6? $\frac{3}{4}$ in 5? $\frac{7}{8}$ in 3? $\frac{9}{10}$ in 6?

8. How often is $\frac{3}{8}$ in 11? $\frac{4}{7}$ in 9? $\frac{6}{7}$ in 7? $\frac{4}{7}$ in 4?

In the above examples, it is required to find how often a Fraction is contained in a whole number, that is, *to divide a whole number by a Fraction.* The quotient will be seen to be larger than the dividend, and it ought so to be whenever a proper Fraction is the divisor; for (§ XLI.) the smaller the divisor, the greater the quotient. There are twice as many half-pints in a pail of water, as there are pints. If I divide a whole number by 1, the quotient will be the same whole number. Thus 1 is in 8, 8 times; 1 is in 12, 12 times, &c. If I divide by a number less than 1, then, I ought to obtain a greater quotient. Thus $\frac{1}{2}$ is in 8, 16 times; $\frac{1}{2}$ is in 12, 24 times, &c.; in which cases, I have a quotient, twice as great as my dividend.

9. At $\frac{2}{9}$ of a dollar a piece, how many spools of thread can be bought with 2 dollars?

First find the ninths in $2=\frac{18}{9}$. Then say $\frac{2}{9}$ are in $\frac{18}{9}$, 9 times. *Ans.*

Hence, to divide a whole number by a Fraction,

MULTIPLY THE WHOLE NUMBER BY THE DENOMINATOR OF THE FRACTION, AND DIVIDE THE PRODUCT BY THE NUMERATOR.

11. Divide 24 by $\frac{18}{21}$. *It is best to reduce the Fraction to its lowest terms,* $\frac{18}{21},=\frac{6}{7}$. Ans. $\frac{28}{1}=28$.

12. Divide 84 by $\frac{96}{103}$. Ans. $90\frac{1}{8}$.

13. Divide 22 by $2\frac{3}{4}=\frac{11}{4}$. Ans. 8.

NOTE.—*The divisor, if a mixed number, must always be reduced to an improper Fraction.*

21. Divide 37 by $4\frac{2}{5}$. A. $8\frac{9}{22}$. 53 by $8\frac{5}{9}$. A. $6\frac{15}{77}$. 127 by $9\frac{4}{5}$. 948 by $8\frac{5}{7}$. 1,847 by $15\frac{12}{25}$.

To this case belong such questions as these; 4 is $\frac{1}{2}$ of what number; 6 is $\frac{3}{8}$ of what number, &c.; that is, *all questions where a certain part is given, to find a whole.*

14. 125 is $\frac{5}{9}$ of what number? Ans. 225.

Perhaps it may be well for beginners, to accustom themselves to solve these examples by analysis, thus. If 125 is 5 ninths, what is one ninth? Ans. 25. If 25 is $\frac{1}{9}$, what is $\frac{9}{9}$? Ans. 225.

15. 178 is $\frac{2}{7}$ of what number? Ans. 623.

16. 256 is $\frac{8}{11}$ of what number? Ans. 352.

17. 1,874 is $\frac{2}{15}$ of what number? Ans. 14,055.

18. 29,864 is $\frac{8}{47}$ of what number? Ans. 175,451.

19. 425,763,891 is $\frac{63}{3469}$ of what number?
Ans. 23,444,046,633.

20. 7,587,648 is $\frac{72}{327}$ of what number?

21. 1,343,826 is $\frac{1832}{5834}$ of what number?

MENTAL EXERCISES.

§ LIII. 1. A lady, having $\frac{5}{8}$ of a yard of ribbon, cut it into 5 equal parts. How much was there in each part?

2. Thomas had $\frac{4}{5}$ of a pint of chesnuts, and distributed them equally among 4 of his companions. How many had each?

3. A man divided $\frac{7}{8}$ of an acre of ground into 7 equal parts. How much was there in each part?

4. A boy divided $\frac{5}{7}$ of an orange among 5 of his companions. How much did he give each?

5. A grocer put $\frac{6}{7}$ of a hogshead of brandy into 6 casks, putting an equal quantity into each. How much did he put in each?

6. Divide $\frac{6}{8}$ by 6. $\frac{6}{8}$ by 3. $\frac{5}{9}$ by 5. $\frac{4}{7}$ by 4. $\frac{4}{7}$ by 2. $\frac{8}{9}$ by 8. $\frac{8}{9}$ by 4. $\frac{8}{9}$ by 2. $\frac{11}{12}$ by 11. $\frac{4}{5}$ by 2. $\frac{4}{5}$ by 4. $\frac{3}{5}$ by 3. $\frac{4}{6}$ by 4. $\frac{12}{13}$ by 6. $\frac{12}{13}$ by 4. $\frac{12}{13}$ by 3. $\frac{12}{13}$ by 2.

In the above examples, a whole number is the divisor, and a fraction, the dividend; that is, it is required to divide a fraction by a whole number. Let the following be written.

7. Divide $\frac{762}{943}$ by 3. Dividing the numerator divides the value. (§ XLI.) Ans. $\frac{254}{943}$.

8. Divide $\frac{216}{574}$ by 27. A. $\frac{8}{574}$. $\frac{345}{765}$ by 9. A. $\frac{38}{765}$.

9. Divide $\frac{297}{645}$ by 2. 2 will not divide the numerator, but (§ XLI.) multiplying the denominator divides the value. A. $\frac{297}{1290}$.

10. Divide $\frac{272}{587}$ by 4. A. $\frac{68}{587}$. $\frac{395}{763}$ by 5. A. $\frac{79}{763}$.

Hence the rules,

I. DIVIDE THE NUMERATOR OF THE FRACTION BY THE WHOLE NUMBER. OR,

II. MULTIPLY THE DENOMINATOR OF THE FRACTION BY THE WHOLE NUMBER.

11. Divide $\frac{7896437}{576787777}$ by 7,896,437. A. $\frac{1}{576787777}$.

12. Divide $\frac{18}{25}$ by 8. $8=4\times2$. We may divide by these factors successively. (§ XXIX.) $\frac{18}{25}\div2=\frac{9}{25}$. $\frac{9}{25}\div4=\frac{9}{100}$ Ans. This method is often convenient.

13. Divide $\frac{273}{560}$ by 9. A. $\frac{91}{1680}$. $\frac{976}{1143}$ by 32. A. $\frac{61}{2286}$. $\frac{432}{1275}$ by 81. A. $\frac{16}{3825}$.

14. Divide 14 $\frac{21}{23}$ by 7. *Divide the whole number and Fraction separately.* *Ans.* 2$\frac{3}{23}$.

15. Divide 575 $\frac{125}{238}$ by 25. Ans. 23 $\frac{5}{238}$.

16. Divide 65 $\frac{1}{3}$ by 7.

If you divide the whole number by 7, you obtain a remainder of 2; for 7 is contained in 65, 9 times, and 2 over. Now this remainder 2 is $\frac{6}{3}$, (§ XXVII.) and $\frac{6}{3}$ added to $\frac{1}{3}$ makes $\frac{7}{3}$. $\frac{7}{3} \div 7 = \frac{1}{3}$. The answer then is $9\frac{1}{3}$.

Hence, when a remainder is left from dividing the whole number, it must, with the given Fraction, be reduced to an improper Fraction, and then divided.

17. Divide $5\frac{3}{4}$ by 23. $5\frac{3}{4} = \frac{23}{4}$. $\frac{23}{4} \div 23 = \frac{1}{4}$.

18. Divide $25\frac{4}{21}$ by 20. A. $1\frac{109}{420}$. $114\frac{1}{5}$ by 280. A. $\frac{571}{1400}$. $184\frac{1}{2}$ by 7. A. $26\frac{5}{14}$. $7{,}646\frac{11}{24}$ by 24. A. $318\frac{347}{576}$.

19. Divide $88\frac{1}{11}$ by 17. A. $5\frac{2}{11}$. $476\frac{4}{5}$ by 8. A. $59\frac{3}{5}$. $175\frac{3}{8}$ by 23. A. $7\frac{5}{8}$. $253\frac{19}{21}$ by 17. A. $14\frac{334}{357}$. $5{,}650\frac{4}{231}$ by 215. A. $26\frac{13864}{49665}$.

◆

MENTAL EXERCISES.

§ LIV. 1. What part of a hogshead of wine is 1 gallon? is 57 gallons?

2. What part of a gallon is 1 quart? 3 quarts?
3. What part of an hour is 1 minute? 5 minutes?
4. What part of a week is 1 day? 2 days? 3 days?
5. What part of a yard of cloth is 1 quarter?
6. What part of a minute is 1 second? 10 seconds?

The following are to be written.

7. What part of a pound is 3d. 1 qr.? Compound numbers cannot be used in a Fraction. We must therefore reduce 3d. 1 qr. to qrs.=13 qrs. In 1£. are 960 qrs. Therefore, (§ XXVIII.) 3d. 1 qr. is $\frac{13}{960}$ of a pound.

8. What part of 1 cwt. is 3 qrs. 15 lb.? A. $\frac{99}{112}$.

9. What part of 1 hogshead of wine, is 11 gals. 1 qt. Ans. $\frac{45}{252} = \frac{5}{28}$.

This process is called *bringing whole numbers of lower denominations, to Fractions of a higher*. The rule seems to be,

REDUCE THE GIVEN NUMBERS TO THE LOWEST DENOMINATION MENTIONED FOR A NUMERATOR, AND AN INTEGER OF THE NEXT HIGHER DENOMINATION, TO THE SAME DENOMINATION, FOR A DENOMINATOR.

NOTE. Reduce the Fraction, thus obtained to its lowest terms.

10. What part of a pound is 13s. 4d.? Ans. $\frac{160}{240} = \frac{2}{3}$.

11. What part of 1 cwt. is 3 qrs. 15 lb. 14 oz.?
Ans. $\frac{799}{896}$.

12. What part of 1 bu. is 3 pks. 7 qts. 1 pt. *Ans.* $\frac{63}{64}$.

13. What part of 15 pipes is 25 gals.? *Ans.* $\frac{5}{378}$.

Note. In this example 15 pipes must be reduced to gals.; or, which is better, divide both 15 and 25 by 5, and then reduce the quotient 3 pipes to gals. Dividing both by 5 will not affect the answer, (§ XLI.) since, both numerator and denominator are equally divided.

14. What part of 2 miles is 7 fur. 11 in. 2 b. c,?
Ans. $\frac{33271}{76032}$.

15. What part of 1 mo. is 22 d. 15 h. 1 m.? A. $\frac{32581}{43200}$.

16. What part of the whole duration of the world have you lived?

17. The earth's diameter, is 7,911$\frac{3}{4}$ miles in diameter nearly, and the highest peak of the Himmaleh mountains, is estimated to be 27,677 ft. above the level of the ocean. What part of the whole diameter of the earth is the height of this mountain?

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MENTAL EXERCISES.

§ LV. 1. What part of a penny is $\frac{1}{3}$ of a farthing? $\frac{2}{3}$ of a farthing?

2. What part of a shilling is $\frac{1}{4}$ of a penny? $\frac{3}{4}$ of a penny?

3. What part of a pound is $\frac{3}{8}$ of a shilling? $\frac{2}{5}$ of a shilling?

4. What part of a penny is $\frac{2}{5}$ of a farthing?

5. What part of a gallon is $\frac{2}{3}$ of a quart?

It is evident, that as fractions are not in their *nature* different from other numbers, but only in their *form*, we need no other rules for reducing them from one denomination to another, than those, given for whole numbers. Hence, *to reduce a Fraction from a lower to a higher denomination,*

Divide as in reduction ascending of whole numbers.

6. Reduce $\frac{2}{9}$ of a shilling to the Fraction of a £.
Ans. $\frac{2}{180}=\frac{1}{90}$.

Note. The results should always be reduced to their lowest terms.

7. Reduce $\frac{4}{7}$ of a dwt. to the Fraction of a lb. Troy.
Ans. $\frac{4}{1680}=\frac{1}{420}$.

8. Reduce $\frac{7}{8}$ of a lb. Avoirdupois to the Fraction of a cwt. *Ans.* $\frac{1}{128}$.

9. What part of a week is $\frac{3}{4}$ of an hour? *Ans.* $\frac{1}{224}$.

10. Reduce $5\frac{1}{2}$ furlongs to the Fraction of a mile.
Ans. $\frac{11}{16}$.

11. What part of a mile is $\frac{7}{8}$ of a b. c.?

12. Reduce $27\frac{3}{4}$ gals. to the Fraction of a butt.

13. Reduce $65\frac{3}{5}$ qts. to the Fraction of a bu.

◆

MENTAL EXERCISES.

§ LVI. 1. $\frac{1}{2}$ of a penny is how many farthings? $\frac{1}{3}$ of a penny?

2. $\frac{1}{6}$ of a shilling is how many pence? $\frac{1}{3}$ of a shilling?

3. $\frac{1}{9}$ of a shilling is how many pence? how many pence and farthings?

4. $\frac{1}{4}$ of a lb. Troy is how many oz.? $\frac{1}{6}$ of a lb.?

The rule, of course, for the reason assigned in the last section, is the same as in whole numbers. Hence, *to reduce a Fraction from a higher denomination to a lower,*

MULTIPLY AS IN REDUCTION DESCENDING OF WHOLE NUMBERS.

5. Reduce $\frac{1}{288}$ of a £ to the Fraction of a penny.
A. $\frac{240}{288}=\frac{5}{6}$.

6. Reduce $\frac{1}{1008}$ of a hhd. to the Fraction of a qt.

7. Reduce $\frac{5}{1008}$ of a cwt. to the Fraction of an oz.
A. $\frac{80}{9}=8\frac{8}{9}$.

8. What part of a pt. is $\frac{5}{171}$ of a bar.?
A. $\frac{1280}{171}=7\frac{83}{171}$.

9. What part of a minute is $\frac{1}{1441}$ of a day? A. $\frac{1440}{1441}$.

10. What part of an E. E. is $\frac{7}{8}$ of a yd? A. $\frac{7}{10}$,

First reduce it to the Fraction of a quarter by (§ LV.) and then to the Fraction of an E. E.

11. What part of a hhd. is $\frac{3}{4}$ of a puncheon?
A. $\frac{252}{252}=1$.

12. What part of a guinea is $\frac{25}{26}$ of a £? A. $\frac{250}{273}$.

13. Reduce $1624\frac{6}{11}$ pun. to tier. A. $3249\frac{1}{11}$.

14. Reduce $19421\frac{1}{13}$ tier. to pun. A. $9710\frac{2}{13}$.

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MENTAL EXERCISES.

§ LVII. 1. In $\frac{1}{12}$ of a shilling how many pence? In $\frac{3}{12}$?

2. In $\frac{1}{3}$ of a shilling how many pence? In $\frac{2}{3}$?
3. In $\frac{1}{7}$ of a shilling how many pence? In $\frac{6}{7}$?
4. In $\frac{1}{3}$ of a peck, how many qts.? In $\frac{2}{3}$?

In the last section, you were required to reduce *Fractions of one denomination*, to *Fractions of another*; the present case only differs, in requiring you find *whole numbers*. Hence, *to reduce a Fraction to whole numbers of inferior denominations*,

MULTIPLY AS IN REDUCTION DESCENDING OF WHOLE NUMBERS, REDUCING THE FRACTION, WHENEVER IT BECOMES IMPROPER, TO A WHOLE OR MIXED NUMBER, AND CONTINUING THE REDUCTION ONLY OF THE FRACTIONAL PART.

5. How much is $\frac{5}{7}$ of a hhd. of wine? A. 45 gals.
6. Find $\frac{23}{87}$ of a £. A. 5s. 3d. $1\frac{69}{87}$ qrs.
7. Find $\frac{248}{823}$ of 1 cwt. A. 1 qr. 5lb. 11 oz. $15\frac{759}{823}$ dr.
8. Find $\frac{486}{1273}$ of a week. A. 2 d. 16 h. 8 m. $17\frac{919}{1273}$ sec.
9. Find $\frac{107}{640}$ of a ℔ Apothecaries' weight. A. 2℥. 3 gr.

* * *

DECIMALS.

§ LVIII. According to our system of Numeration, numbers increase from right to left in a tenfold proportion; that is, a figure becomes *ten times greater than before*, on being moved *a single place* toward the left; *one hundred times*, on being moved *two places; one thousand times*, on being moved *three places*, and so on. But as numbers *increase* from *right to left* in tenfold proportion, so they *decrease* from *left to right* in the same manner; that is, a figure becomes only *the tenth part of its former value*, on being moved *a single place* towards the right; *the hundredth part*, on being moved *two places*, and so on.

Take for example the number 5, 5.

If it be moved one place to the left, it becomes *ten times as great*, that is, fifty, 50.

If it be moved another place, it becomes ten times as great as fifty, or *a hundred times as great* as at first, that is, five hundred, 500.

Thus, we may go on, as far as we please, and show that each removal to the right, multiplies the number by 10, or makes it *ten times as great*. If this be true, then, it will, likewise, be true, that removing *the other way*, will divide the number by 10, or make it *a tenth part as great*.

Let us take five hundred, the number we ended with. 500.

Remove the significant figure 5, one place to the right, and it becomes fifty, *the tenth part* of five hundred. 50.

Remove it another place, and it becomes five, the tenth part of fifty, or *the hundredth part* of five hundred. 5.

Each of these removals, we see, divides the number by 10, or makes it *the tenth part as great* as before. We have now brought the number back to the units' place. But there is plainly no reason why we should stop here. If a point be made, to distinguish this place, it is evident that we may *continue* to remove the number in this manner to the right, calling it at every removal, *a tenth part of what it was before.* Then, the first place below units would be *tenths*, and the *five* instead of being five *units* would be *five tenths.* .5

The next place would be *tenths of tenths*, or *hundredths;* and the five would become five hundredths. .05

We use a cypher here, to show that the 5 is in the second place below units. If there were no cypher, it would stand in the first place, and be .5=*five tenths*, as above.

The next place would be *tenths of hundredths* or *thousandths.* .005

The next place, *tenths of thousandths*, or *ten-thousandths.* .0005

There is no reason to prevent our continuing this mode of writing to any distance.

Thus, we see, we have *a new series of orders*, less than units, or whole numbers, which increase towards the left, and decrease towards the right in ten-fold proportion exactly like whole numbers. These are called DECIMALS, from the Latin word, *decimus*, which means *tenth.*

We have given some examples above, which contain but one significant figure. The following contain more. The pupil should endeavour to read the figures without looking on the names, which should be covered over. If he succeeds without assistance, he will be more likely to possess a clear knowledge of the Notation.

Five tenths and two hundredths, or fifty-two hundredths. .52

NOTE. We read this *fifty-two hundredths*, because the 5 is ten times greater than if it stood in the hundredths' place; that is, than 5 hundredths. It is therefore, *ten-times* 5 hundredths or *fifty* hundredths; and two hundredths more make fifty-two hundredths. The same explanation will apply in the following.

Seven tenths and four hundredths, or seventy-four hundredths	.74
Eight hundredths and six thousandths, or eighty-six thousandths	.086
Nine thousandths and two tenths of thousandths, or ninety-two tenths of thousandths.	.0092
Seventy-one hundredths	.71
Eighty-nine thousandths	.089
Ninety thousandths	.090
Two hundred and thirty thousandths	.230
Nine hundredths	.09
Twenty-three hundredths	.23
One tenth	.1
Ten hundredths	.10
Two tenths	.2
Two hundred, thousandths	.200

From these latter examples it will be observed, that *the value of a decimal is not altered by annexing cyphers;* for 1 tenth is the same as 10 hundredths and 2 tenths is the same as 200 thousandths, the significant figure, being just as far from the units' place in one case as in the other. But, as has been observed above, cyphers *prefixed*, (between the unit's place and the significant figures,) materially affect the value, since every cypher, thus prefixed, removes each significant figure, one place farther from the units' place, and, of course diminishes its value ten fold. As these principles are important, we state them concisely as follows,

I. CYPHERS ON THE RIGHT OF A DECIMAL DO NOT ALTER THE VALUE. But

II. EACH CYPHER ON THE LEFT DECREASES THE VALUE TEN FOLD.

Thus. .5, .50, .500, .5000, and .50000, are all equal in value, being each equal, as is evident to ½. But .5, .05, .005, .0005, .00005, and .000005 decrease in ten fold proportion.

The pupil will now see the use of the point or period, which we have employed to separate decimals from whole numbers. If he were required to read this decimal, .15, he would call it 15 hundredths, but if there were no point, as here, 15, he would call it 15 (units or whole numbers,) simply. The point, then, enables him to determine what *name* and what *value* belongs to a decimal, by showing him its distance from the units' place. As it *separates* decimals from whole numbers, it is usually called a SEPARATRIX. The following examples contain whole numbers and decimals mixed, and are therefore called MIXED NUMBERS.

Three, and five tenths 3.5
Four, and seventy four hundredths 4.74
8.93 7.625 9.842 7.6324 5.55555.
82.594 96.006 825.0304 9004.010203 827.34359.

NOTE. When speaking of Federal Money, we remarked that, dollars and dimes might all be read as dimes; dollars, dimes, and cents might all be read as cents; and dollars, dimes, cents, and mills might all be read as mills. The reason of this is that Federal Money increases in *ten fold proportion*, in the same manner as decimals. For the same reason *whole numbers* and *tenths* may all be read together as tenths; *whole numbers*, *tenths* and *hundredths* may all be read together as hundredths; and so on to any distance. Thus 2.7 may be read *two, and seven tenths*, or *twenty seven tenths;* 3.25 may be read *three and twenty five hundredths*, or *three hundred and twenty five hundredths*, &c.

The pupil will perhaps be ready to demand, what are decimals but Fractions. Fractions (§ XXVI.) are expressions for parts of numbers, and so are decimals. We reply that *decimals* ARE *Fractions*, and are often called DECIMAL FRACTIONS. The only differerence between them and any other Fractions is, that they increase and decrease just like whole numbers, by *tens*, and therefore we are not obliged to write down their denominators. If we please, however, we may write the denominators, and this we shall easily do, from observing the name of the decimal. Thus,

$2.5=2\frac{5}{10}$. $.75=\frac{75}{100}$. $.8639=\frac{8639}{10000}$. $27.93485=27\frac{93485}{100000}$, and so on.

It will be seen by these examples that,

THE DENOMINATOR OF A DECIMAL ALWAYS CONSISTS OF A UNIT, OR 1, WITH CYPHERS ANNEXED.

Fractions having other denominators, are called VULGAR FRACTIONS. It will be further seen by the above, that,

THE NUMBER OF CYPHERS IN THE DENOMINATOR OF A DECIMAL IS THE SAME AS THE NUMBER OF PLACES BELONGING TO THE DECIMAL.

Or, *the distance of the lowest figure of the decimal from the separatrix, determines the number of cyphers in the denominator.* When the denominator is actually written, cyphers on the left of the decimal may be neglected; but when it is written, and afterwards *removed*, the cyphers must of course be carefully restored. In the following, let the denominators be removed, and the numerators writen decimally.

1. $\frac{5}{10}$
2. $\frac{5}{100}$
3. $\frac{55}{100}$
4. $\frac{5}{1000}$
5. $\frac{555}{1000}$
6. $\frac{5}{10000}$
7. $\frac{5555}{10000}$
8. $2\frac{7}{10}$
9. $2\frac{7}{10000}$
10. $2\frac{707}{1000}$
11. $47\frac{81}{1000}$
12. $63\frac{954}{10000}$
13. $194\frac{123}{10000000}$
14. $271\frac{1}{1000000}$
15. $\frac{1}{10000}$
16. $\frac{2}{100000}$
17. $\frac{13}{100000}$
18. $\frac{276}{1000000}$
19. $\frac{349}{10000000}$
20. $\frac{8073}{100000000}$
21. $\frac{891056}{1000000000}$

When decimals are written with their denominators, like Vulgar Fractions, it often happens that they can be reduced to lower terms. Let the following be thus written and reduced as low as possible.

1.	.2	*Ans.* $\frac{1}{5}$	7.	13.18	13.	143.95
2.	.5	" $\frac{1}{2}$	8.	17.265	14.	164.175
3.	1.25	" $1\frac{1}{4}$	9.	37.408	15.	279.234
4.	3.75	" $3\frac{3}{4}$	10.	19.005	16.	348.112
5.	29.085	" $29\frac{17}{200}$	11.	20.0004	17.	793.452
6.	16.0015	" $16\frac{3}{2000}$	12.	39.00002	18.	345.858

Note. We have mentioned Federal Money as being similar to decimals. In fact, the denominations of Federal Money *are* decimals. The dollar is considered the unit or whole number; dimes stand in the tenths' place, cents in the hundredths', and mills in the thousandths'. The French weights, measures, &c., we have likewise seen are on the principles of decimals. (§ XXXIX.) The advantage in the use of decimals consists in the uniformity of their denominators, and in their regular increase by 10 like whole numbers. Hence, we have always to *carry for* 10; and the operations of the four ground rules are as simple as in whole numbers.

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§ LIX. 1. Change $\frac{1}{2}$ a dollar to a decimal.

Here we wish to find how many *tenths*, *hundredths*, &c. of a dollar, there are in $\frac{1}{2}$ a dollar. Now a tenth of a dollar is a dime; hence, we have to inquire how many *dimes* there are in $\frac{1}{2}$ a dollar. $\frac{1}{2}$ a dollar is 10 times $\frac{1}{2}$ a dime, because 10 dimes make a dollar. Then $\frac{1}{2}$ a dollar is $\frac{10}{2}$ of a dime=5 dimes=\$0.5 *Ans.*

2. Change $\frac{1}{2}$ to a decimal.

This question is the same as the last, except that it is not Federal Money. We wish here to find how many *tenths*, *hundredths*, &c. there are in $\frac{1}{2}$ a unit. $\frac{1}{2}$ a unit is 10 times $\frac{1}{2}$ a tenth, $=\frac{10}{2}$ of a tenth, $=.5$ *Ans.*

3. Change $\frac{1}{8}$ of a dollar to a decimal. A. \$0.125.

4. Change $\frac{1}{8}$ to a decimal. A. .125

5. Change $\frac{1}{16}$ to a decimal. A. .0625

Note. If we multiply by 10, to reduce the Fraction to tenths, it becomes $\frac{10}{16}$ of a tenth, which not being a whole tenth, we are obliged to place a cypher in the tenths' place, and proceed as before.

6. Change $\frac{5}{25}$ to a decimal. A. .2

Note. It is best to reduce the vulgar Fraction, first, to its lowest terms.

7. Change $\frac{125}{625}$ to a decimal. A. .2

8. Change $\frac{15}{30}$ to a decimal. A. .5

9. Change $\frac{7}{28}$, $\frac{5}{8}$, $\frac{9}{45}$, and $\frac{3}{16}$ to decimals.
A. .25, .625, .2 and .1875

10. Change $\frac{12}{480}$, $\frac{2}{40}$, and $\frac{16}{1600}$ to decimals.
A. .025, .05 and .01

11. Change $\frac{9}{720}$, $\frac{27}{540}$, $\frac{15}{300}$, and $\frac{18}{144}$ to decimals.

12. Change $\frac{3}{600}$, $\frac{8}{6450}$, $\frac{5}{8000}$, and $\frac{7}{128}$ to decimals.

Another mode of explaining the above operation may be given.

13. Change $\frac{7}{16}$ to a decimal.

If both numerator and denominator of a Fraction be multiplied by the same number, the value will not be altered, (§ XLI.) I may therefore multiply both the 7 and the 16 in the above Fraction, by 10, 100, 1,000 or any other number, without altering the value. Multiply both, then, by 10,000 and the Fraction becomes $\frac{70000}{160000}$. Now (§ XLI.) I may *divide* both numerator and denominator by the same number without altering the value. Divide them both then, by 16, and the Fraction becomes $\frac{4375}{10000}$=(§ LVIII.) .4375 *Ans.*

Either mode of explanation gives us the following rule.

ANNEX CYPHERS TO THE NUMERATOR AND DIVIDE BY THE DENOMINATOR. THE DECIMAL WILL CONSIST OF AS MANY PLACES AS THERE ARE CYPHERS ANNEXED.

NOTE. *If there are not as many quotient figures as the rule requires, prefix cyphers enough to make out the number.* An improper Fraction must, of course, be first reduced to a mixed number.

14. Reduce $\frac{1}{625}$ to a decimal. A. .0016
15. Reduce $\frac{7}{250}$ to a decimal. A. .028
16. Reduce $\frac{9}{160}$ to a decimal. A. .05625
17. Reduce $\frac{1}{3}$ to a decimal. A. .3333333+

NOTE. We see here, that we may go on forever, and the decimal will continue to repeat 33, &c.

18. Reduce $\frac{2}{11}$ to a decimal. A. .18181818181818+

NOTE. This decimal goes on, repeating like the other; but it repeats two figures, 18, instead of one, as before.

Decimals, which continually repeat the same figures, are called REPEATING DECIMALS, or REPETENDS.

When one figure is repeated, the decimal is called a SINGLE REPETEND; when two or more, a COMPOUND REPETEND.

Repeating decimals are also called CIRCULATES, or CIRCULATING DECIMALS. Properly, the term *circulate*, belongs to compound repetends.

When other decimal figures precede the repetend, it is called a MIXED REPETEND; when otherwise, a PURE, or SIMPLE REPETEND.

Repetends are also called INFINITE DECIMALS; decimals which terminate, or come to an end, are called FINITE.

In single repetends, the repeating figure is commonly written only once, with a point over it, thus, $.\dot{3}$. In compound repetends, all the repeating figures are once written, and a point placed over both the first and last, thus, $.\dot{1}\dot{8}$. This notation shows us all the figures that repeat, and if it is necessary to extend the decimal lower, for the purposes of Addition, Multiplication, &c. we can write it down immediately, without the trouble of calculation. For further particulars on this subject see § LXIX.

19. Change $\frac{1}{7}$ to a decimal. A. $.\dot{1}4285\dot{7}$
20. Change $\frac{2}{3}$ to a decimal. A. $.\dot{6}$
21. Reduce $\frac{9}{1125}$ to a decimal. A. .008
22. Reduce $\frac{11}{16}$ to a decimal. A. .6875
23. Reduce $\frac{17}{20}$ to a decimal. A. .85
24. Reduce $\frac{1}{32}$ to a decimal. A. .03125
25. Reduce $\frac{1}{27}$ to a decimal. A. $.\dot{0}3\dot{7}$

26. Reduce $\frac{1}{26}$ to a decimal. A. $.0\dot{3}8461\dot{5}$

27. Reduce $\frac{3}{29}$, $\frac{6}{23}$, $\frac{16}{47}$, $\frac{1}{1728}$, $\frac{3}{256}$, $\frac{7}{512}$, $\frac{8}{960}$, $\frac{5}{5.6}$, and $\frac{3}{87}$ to decimals.

◆

§ LX. 1. Reduce 1 pt. to the decimal of a gallon.
Ans. 1 pt.$=\frac{1}{8}$ gal.$=.125$

2. Reduce 9 hours to the decimal of a day.
Ans. 9h.$=\frac{9}{24}=\frac{3}{8}$ day$=.375$

3. Reduce 2 ft. 6 in. to the decimal of a yard.
Ans. 2ft. 6in.$=$30in.$=\frac{30}{36}=\frac{5}{6}$yd.$=.8\dot{3}$

4. Reduce 5 fur. 16 rds. to the decimal of a mile.
Ans. .675

5. Reduce 12s. 6d. 3 qrs. to the decimal of a pound.
Ans. .628125

NOTE. When decimals, whether finite or infinite run on to a great number of places, it will usually be sufficient to take them to three or four, and neglect the rest. For the lower decimals become so small that they are of little importance. If on carrying the decimal as many places as you wish, the remainder left is less than half the divisor, the decimal, as it stands will be sufficiently accurate; if it be more, increase the last figure of the decimal by 1. It will be seen that neither of these decimals will be perfectly accurate, the former, being too small, and the latter too large. When a decimal is thus taken too small, the sign + is usually annexed to it, signifying that the true decimal is *more*. When it is taken too large, the sign — is annexed, signifying that the true decimal is less. Decimals written with these signs are called APPROXIMATES, because they are only an *approximation* to the truth.

6. Reduce 47£. 16s. $7\frac{3}{4}$ d. to a decimal expression.
A. £$47.832291\dot{6}$

NOTE. It would be sufficient to say £47.832+. We have carried the decimals several places, throughout these examples, for the purpose of testing the accuracy of the pupil.

7. Reduce 83£; 19; $5\frac{1}{2}$ to a decimal.
Ans. £$83.97291\dot{6}$ or £83.973—.

8. Reduce 2 gals. to the decimal of a hhd.
Ans. $\dot{0}3174\dot{6}$ or .032—.

9. Reduce 15s. 9d. 3 qrs. to the decimal of a pound.
Ans. .790625 or .79+ or .791—.

For practical purposes, there is a shorter mode of reducing shillings, pence and farthings to the decimal of a pound. It is not perfectly accurate, but sufficiently so, in most cases. Take the last example.

As 1 shilling is $\frac{1}{20}$ of a pound, so 2 shillings are $\frac{2}{20}=\frac{1}{10}$ of a pound. For every 2 shillings, then, we have 1 tenth in the decimal, and, as 15s. contains 2s. 7 times, we shall have .7. There is a shilling over. This is *a half of* $\frac{1}{10}$, or *a half of* $\frac{10}{100}=\frac{5}{100}$=.05. For 15s. then, we have the decimal .75. Thus far, the method is perfectly accurate.

1 farthing is $\frac{1}{960}$ of a £., because 1£. contains 20×12 ×4=960 farthings. Now 960+$\frac{1}{24}$ of itself=1000.—Therefore, any number of farthings, or 960ths. of £1., increased by $\frac{1}{24}$ of itself, will express the same value in 1,000ths of £1. If the farthings be 12, a 24th. part of them will be ½: if more than 12, more than ½. In this case, then, if we add 1 to the farthings, they will be 1,000ths., with an error, less than ½ a 1,000th. If they are 36, a 24th. part will be 1½; if more, more than 1½. In this case, then, if we add 2 to the farthings, they will be 1,000ths. with an error, less than ½ a thousandth. If they are 24 or 48, a 24th. is 1, or 2, which added, gives 1,000ths exactly.

Now in the example above, we have 9¾d.=39 qrs.=$\frac{39}{960}$ of a pound, =$\frac{41}{1000}$ of a pound =.041, the decimal for the pence and farthings. Hence the whole decimal, for shillings, pence and farthings, is .75+.041=.791, corresponding to one of the answers above.

Hence, to express shillings, pence and farthings in the decimal of a pound,

CALL EVERY 2 SHILLINGS, 1 TENTH, EVERY ODD SHILLING 5 HUNDREDTHS, AND THE FARTHINGS IN THE PENCE AND FARTHINGS, SO MANY THOUSANDTHS, ADDING 1 IF THE NUMBER IS BETWEEN 12 AND 36, AND 2, IF ABOVE 36.

NOTE. As 48 farthings make a shilling, we never have occasion to go above this number. The above mode of finding the decimal of a pound, is called finding the decimal BY INSPECTION; that is, *by sight*, or without the trouble of a written calculation. Take the following examples.

10. Reduce 17s. 6d. 3qrs. to the decimal of a pound? *Ans.* £0.878.

11. Reduce £19; 8; 7; 2, to a decimal expression. *Ans.* £19.431.

12. Reduce £18; 16; 9½, to a decimal expression. *Ans.* £18.840.

13. Reduce 13s. 3d. 1qr. to the decimal of a pound. *Ans.* £0.664.

14. Reduce 14s. 8d. 3qrs.; 19s. 10½d.; 16s. 7¾d. and 18s. 9¼d. to decimals.

15. Reduce £16; 17; 3½; £11; 19; 5¾; £13; 9; 1, and £15; 8; 2, to decimals.

ADDITION.

§LXI. 1. A man paid debts as follows; to A, $37.3, to B, $94.05, to C, $127.063, and to D, $1,843.375. How much did he pay in all?

We have given this first example, in *Federal Money*, because the pupil will see at once that dimes should be added to dimes, cents to cents, &c. Of course he will place the numbers so that these denominations shall stand under each other.

$ 37.3
$ 94.05
$ 127.003
$1,843.375
Ans. $2,101.728

2. Add 37.3; 94.05; 127.003, and 1,843.375.

These numbers are the same as those in the last example, except that they are not Federal Money. They must therefore evidently be written down and added as before; for as dimes must be added to dimes, cents to cents, &c., so tenths must be added to tenths, hundredths to hundredths, &c. This, it will be observed, brings all the decimal points in the given numbers under each other: *and the point in the result*, of course, *immediately under them.* Hence the rule,

WRITE THE NUMBERS SO THAT THE DECIMAL POINTS MAY BE UNDER EACH OTHER; ADD AS IN WHOLE NUMBERS, AND PLACE THE POINT IN THE ANSWER DIRECTLY UNDER THE OTHER POINTS.

NOTE. Repetends should be extended, at least as far, and usually one place farther, than the longest finite decimal, before adding. When numbers are given in vulgar Fractions, they must be reduced to decimals, before addition takes place; and when the numbers are compound, the lower denominations should be made decimals of the higher.

EXAMPLES FOR PRACTICE.

3. A man purchased at one time, $7\frac{1}{3}$ cwt. of sugar; at another, $17\frac{1}{2}$ cwt.; at another, $15\frac{5}{8}$ cwt.; at another, $20\frac{2}{7}$ cwt. How many cwt. did he purchase in all? A. 60.744+

4. In 3 pieces of cloth, are contained as follows: in the first, 25 yds. 3 qrs. 2 na.; in the second, 19 yds. 1 qr. 3 na.; and in the third, 17 yds. 1 qr. 1 na. What is the whole amount in yards and decimals. A. 62.625.

5. In one cask are contained 28 gals. 2 qts. 1 pt.; in another, 36 gals. 3 qts.; in another, 27 gals. 3 qts.; and in another, 16 gals. 1 pt. How much in all? A. 109.25 gals.

6. One cask of sugar weighs 1 cwt. 2 qrs. 14 lb.; another, 3 cwt. 14 lb.; another, 2 cwt. 1 qr.; and another, 1 cwt. 14 lb. How much do all weigh? A. 8.125 cwt.

7. If I travel 35 mls. 3 fur. 20 rds., one day; 41 mls. 2 fur. 10 rds., another; 17 mls. 4 fur., another; and 26 mls. 3 fur. 20 rds., another; how far do I travel in all? A. 109.65625 mls.

8. If I pay $$21\frac{3}{4}$ for one hhd. of molasses; $$19\frac{5}{8}$ for another; $$17\frac{1}{2}$ for another; $23\frac{7}{8}$ for another; and $18\frac{1}{4}$ for another; how much do I give for all? A. $101.00

9. Add $12.34565; $7.891; $2.34; $14 and $0.0011. A. $36.57775.

10. Add .014; .9816; 1.32; 2.15914; .72913 and 3.0047.
A. 8.20857.

11. Add 27.148; 918.73; 14,016; 294,304.001; 7,138 and 221.701.
A. 309,488.2938.

12. Add 312.984; 21.3918; 2,700.42; 3.173; 27.2 and 581.07.
A. 3,646.2388.

◆

MULTIPLICATION.

§ LXII. 1. At $1.25 per yd. what cost 3 yds. of cloth?

This may be performed, either by Addition or by Multiplication.

BY ADDITION.	BY MULTIPLICATION.
$1.25	$1.25
1.25	3
1.25	
$3.75 *Ans.*	$3.75 *Ans.*

We have exhibited both modes, to show where the point ought to be placed, as this is the only difficulty in decimal calculations. When the *multiplier* is a *whole number*, then, *there must be as many decimals in the product, as in the multiplicand.*

2. At $125 a pipe, what cost .3 of a pipe of wine?

$125	$125
3	.3
$375	$37.5 *Ans.*

On the left we have found what 3 *whole pipes* cost, by multiplying by 3. Now .3 is the tenth part as much as 3 whole ones. Of course, we must take the tenth part of the price of 3 whole pipes, for the price of .3 of a pipe. But (§ XXX.) cutting off one figure divides by 10. Of course, we have one decimal place in the product by .3 This is shown on the right. When the *multiplicand* is a *whole number*, then, *there must be as many decimals in the product, as in the multiplier.*

3. At $1.25 per lb. what cost .3 pound of tea?

If it were 3 *whole pounds*, we should obtain $3.75, as in Ex. 1. But it is the *tenth part* of 3 lb., being .3 lb., and therefore, the price will be a *tenth part as much*=$0.375 *Ans.*

When *neither factor* is a *whole number*, then, *there must be as many decimals in the product as in both factors.* Observation of the following, may, perhaps, render the principle more clear. We take the same numbers, making them at first *whole*, and moving the decimal point *one place* at each step.

125	12.5	1.25	.125	.125	.125	.0125
3	3	3	3	.3	.03	.03
375	37.5	3.75	.375	.0375	.00375	.000375

Hence the rule,

MULTIPLY AS IN WHOLE NUMBERS, AND POINT OFF AS MANY DECIMALS IN THE PRODUCT, AS THERE ARE IN BOTH FACTORS.

NOTE. From observing the three latter illustrations, the pupil will see that *when there are not as many figures in the product as the rule requires, cyphers must be prefixed to make out the number.*

EXAMPLES FOR PRACTICE.

4. At \$$3\frac{9}{20}$ pr. lb., what cost 16 lb. of tea? A. \$55.20.
5. At \$2 pr. gal. what cost $\frac{3}{5}$ gal. of wine? A. \$1.20.
6. At \$5.27 pr. yd., what cost $7\frac{1}{50}$ yds. of cloth? A. \$36.9954.
7. At $3\frac{1}{2}$ cts. pr. lb., what cost 7,853 lbs. of rice? A. \$274.855.
8. At \$$13\frac{3}{5}$ a bl., what cost $46\frac{1}{5}$ bls. of sugar? A. \$627.12.
9. Multiply .004 by .04. A. .00016.
10. Multiply 24.61 by .0529. A. 1.301869.
11. Multiply .0007853 by .035. A. .0000274855.
12. Multiply .051 into .091. A. .004641.
13. Multiply .0217 into .00431. A. .000093527.

NOTE. The truth of the rule will further appear, by writing the decimals in the form of vulgar Fractions, and multiplying by the rule § XLIV. Thus, multiply .02 by .03 $.02=\frac{2}{100}$ and $.03=\frac{3}{100}$ $\frac{2}{100}\times\frac{3}{100}=\frac{6}{10000}=.0006$, where the decimal places of the product are equal to those in both factors.

14. Multiply .02 by 10. A. .2
15. Multiply .003 by 100. A. .3

It will be seen that multiplying a decimal by 10 does not alter the significant figures, but only moves the separatrix one place towards the right. This makes the multiplicand 10 times greater, because it causes every figure to stand *one place higher*. So, multiplying by 100 moves the separatrix *two* places towards the right; by 1,000, three places, &c. Hence, to multiply by a unit with cyphers annexed,

REMOVE THE SEPARATRIX AS MANY PLACES TO THE RIGHT AS THERE ARE CYPHERS IN THE MULTIPLIER, ANNEXING CYPHERS IF NECESSARY.

NOTE. If the multiplier be *any number* with cyphers annexed, it may be considered as a composite number, one of whose factors is 10, 100, 1000, &c., and we may first employ this factor, according to the above rule, and afterwards, the other, as usual.

16. Multiply .38179 by 10; by 100; by 1,000; by 10,000.
17. Multiply 1.876 by 1,000; by 10,000; by 1,000,000.
18. Multiply 17.9 by 1,00; by 1,000; by 100,000,000.

◆

§ LXIII. It often happens that, in practice, we have no occasion for as many decimal places as the rule will give us. The lower decimals are so small that they may be rejected as of no importance. We will now propose a mode, by which we may obtain decimals, accurately as far as we please without the trouble of finding the others.

1. Multiply 13.1346 by 12.2876.

COMMON METHOD.

13.1346	
12.2875	
7	88076
91	9422
1050	768
2626	92
26269	2
131346	
161.392	71096

CONTRACTION.

13.1346
6782.21
131.346
26.269
2.627
1.050
.092
.008
161.392

I wish to perform this multiplication, and retain only three places of decimals. Now I know (§ LXII.) that when the multiplier is whole numbers I have just as many decimals in the product as in the multiplicand. If then I multiply only three decimal figures of the multiplicand by the 2 in the units' place of the multiplier, I shall have only *three* decimal places in the product, thus obtained. If I multiply *four* decimal figures of the multiplicand by the 1 in the *tens'* place of the multiplier, =10, I shall have four decimal places in the product, but the last one, (being a cypher) may be neglected, leaving *three* places, as before. If I multiply two figures of the multiplicand by the 2 in the *tenths'* place in the multiplier, I shall have *three* decimal places in the product. (§ LXII.) If I multiply, in like manner, *one* decimal place by the 8 in the *hundredths'* place, I shall have again *three* decimal places in the product. So, if I go on, multiplying *one place less in the multiplicand, for every place lower in the multiplier,* I shall still have *three decimal places in the product.* I may therefore arrange all these products, thus obtained, *with their right hand figures under each other,* and put the separatrix, in each, *three places from the right.* They may then, it is obvious, be added as they stand; and the sum will be the total product, with only three decimal places.

It is a convenient way of determining where to *commence* multiplying with each figure, to put the units' place of the multiplier under that decimal of the multiplicand, which you wish should be the last in the product; and then write all the other figures before and after it, *in an inverted order,* as in the contraction above. Then in employing each figure of the multiplier, commence with the figure of the multiplicand *immediately above it,* neglecting all the figures to the right, and place the right hand figures of the several products exactly under each other. They will then stand in proper order to be added. Thus, in the last example, units stand under thousandths; and, of course, the first figure of the product is thousandths; tenths stand under hundredths, and the last figure of the product is in like manner thousandths; hundredths under tenths, which produce thousandths, as before, &c.

One thing more. In order to ensure sufficient accuracy, when we begin to multiply by any figure, we must first multiply the *preceding* figure of the multiplicand by the figure we are using as a multiplier, and *carry from that product,* to the first figure which we set down; always carrying for the *nearest number of tens,* whether above or below. Thus when multiplying by the 2 (units) in the above example, the first figure in the product is 2×3=6. But to this we carry 1, making 7, because 2×4 (the preceding figure) =8, which is nearer to 10 than it is to 0. In multiplying by 8, (tens,) the first figure obtained is 8×1=8, to which we carrry 2, making 10, because 8×3=24, which is nearer to 2 tens than to 3 tens, &c. Hence, the rule.

I. WRITE THE UNIT FIGURE OF THE MULTIPLIER UNDER THE DECIMAL PLACE WHICH IS TO BE LAST IN THE PRODUCT, AND THE OTHER FIGURES IN AN INVERTED ORDER.

II. IN MULTIPLYING, REJECT ALL THE FIGURES IN THE MULTIPLICAND, TO THE RIGHT OF THE FIGURE YOU ARE MULTIPLYING BY; CARRY TO THE FIRST FIGURE OF THE PRODUCT FOR THE NEAREST NUMBER OF TENS CONTAINED IN THE PRODUCT OF THE PRECEDING FIGURE IN THE MULTIPLICAND, AND SET DOWN THE RIGHT HAND FIGURES OF THE SEVERAL PRODUCTS UNDER EACH OTHER.

III. THESE PRODUCTS, ADDED AS THEY STAND, WILL GIVE THE TOTAL PRODUCT, WITH THE REQUIRED NUMBER OF DECIMAL PLACES.

NOTE. If the multiplier, when thus written, extends below the multiplicand, fill out the multiplicand, or suppose it to be filled out with cyphers annexed.

3. Multiply 27.14986 by 92.41035, retaining four decimal places in the product. A. 2,508.9280

4. Multiply 480.14936 by 2.72416, retaining four decimal places. A. 1,308.0037

5. Multiply 73.8429753 by 4.628754, retaining five decimal places. A. 341.80097

6. Multiply 8,634.875 by 843.7527, retaining only the integers in the product. A. 7,285,699

◆

§ XLIV. 1. What is the value of .75 of a pound sterling in shillings, pence, &c.?

1£ contains 20s. Therefore $\frac{1}{100}$ of a pound contains $\frac{20}{100}$ of a shilling, and $\frac{75}{100}$ of a pound contains 20 times as many hundredths of a shilling=$\frac{1500}{100}$ of a shilling=15s. This is performing the operation as in vulgar Fractions. It is obviously the same, whether the denominator 100 be expressed or not.

$$\begin{array}{r} £0.75 \\ 20 \\ \hline \text{A. } 15.00\text{s.} \end{array}$$

2. Reduce .17525£ to lower denominations.
A. 3s. 6d. 0.2688 qrs.

Hence, to reduce decimals of higher, to whole numbers, and decimals of lower denominations,

MULTIPLY AS IN REDUCTION DESCENDING, RESERVING THE WHOLE NUMBERS OF EACH DENOMINATION, AND CONTINUING THE REDUCTION OF THE DECIMAL ONLY.

3. Reduce £0.040625 A. 9¾ d.
4. Reduce £0.5724 A. 11 s. 5 d. 1.504 qrs.
5. Reduce £0.85251 A. 17 s. 0 d. 2.4 qrs.
6. Reduce .8125 of a shilling. A. 9¾ d.
7. Reduce .875 hhd. of wine. A. 55 gal. 0 qt. 1 pt.
8. Reduce .875 of a yd. A. 3 qrs. 2 na.
9. Reduce .76442 lb. Troy. A. 9 oz. 3 dwt. 11 gr.
10. Reduce .617 cwt. A. 2 qrs. 13 lb. 1 oz. 10.6 gr.
11. Reduce .089 mile. A. 28 rds. 2 yds. 1ft. 11.04 in.
12. Reduce .9075 acre. A. 3 roods, 25.2 rds.
13. Reduce .712 furlong. A. 28 rds. 2 yds. 1 ft. 11.04 in.
14. Reduce .002084 lb. Troy. A. 12.00384 gr.
15. Reduce .046875 lb. Avoirdupois. A. 12 dr.
16. Reduce .142465 year of 365 days. A. 51.999725 days.
17. Reduce .569 year of 365 days. A. 207 d. 16 h. 26 m. 24 sec.
18. Reduce .6725 cwt. A. 2 qrs. 19 lb. 5 oz.
19. Reduce .61 tun of wine. A. 2 hhd. 27 gal. 2 qts. 1 pt.
20. Reduce .8322916 £. A. 16 s. 7 d. 2.999936 qrs.

In reducing decimals of a £. we may evidently reverse the rule § LX, thus,

DOUBLE THE TENTHS' FIGURE FOR SHILLINGS, AND IF THE HUNDREDTHS BE 5, OR MORE, DEDUCT THE 5, AND ADD ANOTHER SHILLING. CALL THE REMAINING FIGURES, IN THE HUNDREDTHS' AND THOUSANDTHS' PLACES, FARTHINGS, DECREASING THEM BY 1, IF BETWEEN 12 AND 36, AND BY 2, IF ABOVE 36.

NOTE. The decimal below thousandths is neglected. But if it amount to more than half a thousandth, it is best to increase the 1,000ths. by 1.

21. Reduce .8971 £. .13763 cwt. .19843 ml. .15634 yd. .71 lb. Troy. 71 lb. Avoirdupois. .71 ℔. Apothecaries'. .8934 week. .9193 month of 30 d. .7346 rd. .9874 hhd. of wine. .9874 hhd. ale. .7759 hhd. beer. .8557 oz. Troy. .9365 £. .59347 £.

SUBTRACTION.

§ LXV. 1. I have a debt of \$95.763, and I make a payment of \$87.665. How much is remaining due?

Of course, dimes must be taken from dimes, cts. from cts. &c. Hence, the numbers must be written as in Addition, and the point placed in the same manner.

```
$95.763
 87.665
-------
  8.098   Ans.
```

Hence, to subtract decimal numbers,

WRITE THE NUMBERS AS IN ADDITION, SUBTRACT AS IN WHOLE NUMBERS, AND PLACE THE SEPARATRIX AS IN ADDITION.

2. From a piece of cloth containing $47\frac{3}{8}$ yds. a merchant sold $23\frac{9}{25}$ yds. How much was left? A. 24.015

3. On a debt of \$383.00 there was paid \$47.25. How much remained unpaid? A. \$335.75

4. On a debt of $\$193\frac{2}{5}$, there was paid $\$87\frac{5}{8}$. How much remained unpaid? A. \$105.775

5. From 1,153$\frac{2}{7}$ tons of iron, there were sold $684\frac{5}{11}$ tons. How much remained unsold? A. 468.8312 tons.

6. From $37\frac{2}{3}$ gals. of oil, were sold $28\frac{9}{17}$ gals. How much remained? A. 9.1372 gals.

7. A man having $759\frac{621}{800}$ bu. of wheat, sold to one person $47\frac{6}{20}$ bu.; to another, $87\frac{15}{32}$; to another, $94\frac{8}{25}$; to another, $387\frac{1}{16}$. How much had he left? A. 143 bu.

8. A man had $\$16\frac{73}{200}$ of which he spent as follows: for a load of hay, $\$6\frac{4}{25}$; for a load of grain, $\$7\frac{7}{40}$; for 3 bu. of corn, $\$\frac{17}{60}$ pr. bu.; and for a load of wood, \$2. What had he left? A. \$0.18

9. A merchant sold a barrel of flour for $\$2\frac{5}{12}$; 5 gals. of molasses for $\$\frac{11}{93}$ pr. gal.; and 6 gals. of wine for

$\$\frac{198}{276}$ pr. gal. In payment, he received a load of wood, worth $\$2\frac{2}{15}$, and 2 bu. of wheat worth $\$\frac{13}{36}$ pr. bu.; and the rest in money. How much money did he receive? A. $2.428, nearly.

10. From $87\frac{24}{37}$ take $36\frac{31}{44}$. From $927\frac{83}{130}$ take $179\frac{54}{91}$. From $169\frac{423}{463}$ take $47\frac{36}{95}$. From $829\frac{37}{81}$ take $33\frac{94}{830}$. From $334\frac{627}{865}$ take $234\frac{33}{44}$. From $973\frac{58}{71}$ take $864\frac{999}{1008}$.

11. From 72.345 take 63.1345. From 39.38463 take .27953. From 125.125125 take 25.025025. From 380.613401 take 1.7834. From 830.595003 take .000004. From .00001 take .00000001.

◆

DIVISION.

§ LXVI. 1. If four gallons of wine cost $8.24, what cost 1 gallon?

We must divide by 4. The fourth part of 8 dollars is 2 dollars, and the fourth part of 24 cts., that is, 24 hundredths of a dollar is 6 cts.=6 hundredths=.06

```
4)8.24
  ----
  2.06
```

2. At $2.06 pr. gal., how many gals. of wine may be bought with $8.24?

Here we have decimals both in the divisor and dividend. But the 8.24 is 824 hundredths, and the 2.06 is 206 hundredths.

```
2.06)8.24(4
     8.24
     ----
```

Dividing *hundredths by hundredths* will evidently give whole numbers, just as dividing pints by pints, quarts by quarts, or units by units, gives whole numbers. 824÷206=4, as seen above.

From observation of these two examples, we see that *when the divisor is a whole number, the quotient has just as many decimals as the dividend.* And *when the divisor has as many decimals as the dividend, the quotient is a whole number.*

3. At $0.206 a yard, how many yds. of calico will $8.24 buy?

Here the divisor has three decimal places. If the dividend had as many, we should have a whole number quotient. But we may put a cypher on the right of the dividend, without altering its value. (§ LVIII.) The decimal places will then be equal, and the quotient 40 will be whole numbers.

```
.206)8.240(40
     8 24
     -----
        00
```

4. At $30.5 pr. hhd., how many hhds. of molasses can be bought for $76.25?

As the divisor has, here, one decimal place, I know that, until one decimal place in the dividend has been employed, the quotient will be whole numbers, and afterwards, decimals. A comma is placed here, after the 2, to show the limit of whole numbers. The pupil will find it convenient to employ it.

```
30.5)76.2,5(2.5
     610
     -------
     1525
     1525
```

5. Divide .15 by 7.5.

```
7.5).1,50(0.02
     150
     ———
```

I find here, that, taking one decimal place in the dividend will give no quotient figure. Hence, the quotient contains *no whole numbers*. In like manner, by taking two places, I find that the quotient contains *no tenths*. I then annex a cypher, and obtain 2 in the quotient, which, of course, is 2 *hundredths*. Hence, the general principle,

THE DECIMAL PLACES IN THE DIVISOR AND QUOTIENT TOGETHER, MUST BE JUST EQUAL TO THE DECIMAL PLACES IN THE DIVIDEND.

This results, likewise from the principles of multiplication; for the divisor × the quotient = the dividend (§ xxv.), and hence their decimal places together ought to equal those of the dividend (§ LXII.)

Hence, the rule,

DIVIDE AS IN WHOLE NUMBERS, AND POINT OFF DECIMALS ENOUGH TO MAKE THE PLACES IN THE DIVISOR AND QUOTIENT TOGETHER, EQUAL TO THOSE IN THE DIVIDEND.

NOTE. If there be not enough figures in the quotient, prefix cyphers to make out the number. If the division is performed as in the above illustrations, the cyphers will be placed in the quotient during the division, which is best.

6. At \$$1\frac{1}{2}$ pr. lb., how many lbs. of indigo will \$$27\frac{171}{200}$ buy? A. 18.57

7. If 1 gal. of wine cost \$$1\frac{3}{8}$, how many gals. will \$$14\frac{3}{4}$ buy? A. 18.

8. If I pay \$$316\frac{15}{16}$ for $92\frac{1}{5}$ cords of wood, what is that a cord? A. \$$3\frac{7}{16}$.

9. If I pay \$$35\frac{37}{64}$ for 12 lb. 15 oz. of tea, what is that a pound? A. \$$2\frac{3}{4}$.

10. At \$$3\frac{1}{4}$ a yd., how many yds. of broadcloth can I buy for \$$64\frac{3}{16}$. A. $19\frac{3}{4}$.

11. If $71\frac{9}{16}$ yds. of cloth cost \$$168\frac{71}{80}$, what is that a yard? A. \$$2\frac{9}{25}$.

12. If 16 lb. 9 oz. of sugar cost \$$2\frac{9}{128}$, what is that pr. pound? A. \$$\frac{1}{8}$.

13. Divide 226.827 by 8.13. A. 27.9.
14. Divide 1 by 1.25. A. .8.
15. Divide 1 by 562.5. A. $.001\dot{7}$.
16. Divide 3.464 by 2,706.25. A. .00128.
17. Divide 313.5 by 861.3.
18. " 913.41 by 3,864.
19. " 8,193.2 by 41,621.1
20. Divide 27,941.2 by 18.2.
21. " 101,001 by .00001.
22. " 21 by 269,843.21.

Divide 298.7461 by 10. A. 29.87461. By 100. A. 2.987461.

NOTE. The above example shows, that dividing by 10 does not alter the significant figures, but only removes the decimal point one place towards the left. In like manner, dividing by 100, removes the separatrix two places to the left; by 1,000, three places, and so on. Hence, when the divisor is a unit, with cyphers annexed, REMOVE THE SEPARATRIX AS MANY PLACES TO THE LEFT AS THE DIVISOR CONTAINS CYPHERS, PREFIXING CYPHERS, IF NECESSARY.

NOTE. If the divisor be *any number* with cyphers annexed, it may be treated as a composite number, one of whose factors is 10, 100, 1,000, or a similar number; and we may first divide by this factor, according to the above rule, and afterwards by the other. (§ XXIX.)

23. Divide 7.52 by 10.
24. " 25.9 " 100.
25. " 1 " 1,000.
26. " 300 " 10.
27. " 45.962 " 1,000.
28. Divide 27.3 by 20.
29. " 15.9 " 2,000.
30. " 350.4 " 600,000.
31. " 27.854 " 360,000.
32. " 941.31 " 87,000,000.

◆

§ LXVII. As we know, (LXVI.) that the quotient will be whole numbers until as many decimals are employed in the dividend as there are in the divisor, and that all figures, afterwards obtained in the quotient, are, of course, decimals, we can always tell what will be the place of the first quotient figure; that is, whether it will be hundreds, tens, units, tenths, hundredths, thousandths, &c.

Now, for ordinary purposes, it has been remarked, we frequently do not need many decimal places. A shorter mode of division, in such cases, may be employed.

1. Divide 82.931453 by 78.124.

COMMON METHOD.

```
78.124)82.931,453(1.061
      78.12|4
      -----|-
        480|745
        468|744
        ---|----
         12|0013
          7|8124
          -|----
          4|1889
```

CONTRACTION.

```
78.12,4)82.93,1453(1.061
          78.12
          -----
           481
           469
           ---
            12
             7
             -
             5
```

My divisor, I observe, has three decimal places. Taking three, likewise, in the dividend, I find the divisor will go, and therefore, conclude that there will be one place of whole numbers in the quotient. I wish to retain *three decimal places* in the quotient. The quotient, then, will consist of four figures. Now, though not perfectly accurate, it is sufficiently so, to take four figures on the left of the divisor, and divide by them, instead of using the whole. Thus, I obtain one quotient figure. By the common method, I should now bring down another figure of the dividend. But it will be sufficiently accurate to *shorten the divisor* one figure; that is, cut off a figure from the right of it. For this makes the divisor *ten times less*, (§ XXX.) as annexing a figure to the number divided, would make it ten times greater, nearly. (§ XV.) In like manner, for the third figure of the quotient, I take off another figure from the divisor, instead of bringing down another from the dividend, and so on.

In multiplying the divisor by the quotient figure, *we must carry*, as in multiplication, (§ LXIII.) to the first figure of the product, *for the nearest number of tens*, from the product of the *last neglected figure of the divisor*. Thus, in the above example, in multiplying by 6, we should have 6×8=48, but to this we carry 1, because 6×1 (the last figure cut off,)=6, which is nearer 10, than 0. Hence, the first figure of the product is 9, instead of 8. Hence, the rule,

I. TAKE FROM THE LEFT OF THE DIVISOR AS MANY FIGURES AS YOU WISH IN THE QUOTIENT, AND, FOR THE FIRST QUOTIENT FIGURE, DIVIDE BY THESE AS USUAL.

II. FOR EACH FOLLOWING QUOTIENT FIGURE, DIVIDE THE LAST REMAINDER BY THE PRECEDING DIVISOR, DIMINISHED ONE FIGURE ON THE RIGHT.

III. IN MULTIPLYING THE DIVISOR BY THE QUOTIENT FIGURE, CARRY FROM THE PRODUCT OF THE QUOTIENT FIGURE, INTO THE LAST NEGLECTED FIGURE OF THE DIVISOR, FOR THE NEAREST NUMBER OF TENS.

2. Divide 721.17562 by 2.2574321, so that the quotient may contain three decimals. A. 319.467.

3. Divide 2,508.92806505178 by 92.4103576, so that there may be six figures in the quotient. A. 27.1498.

NOTE. The pupil will see that there must be two places of whole numbers; consequently, there will be but four decimals.

4. Divide 12.169825 by 3.14159, so that the quotient may have six figures. A. 3.87377.

5. Divide 87.076326 by 9.365407, so that the quotient may have seven decimals. A. 9.2976559.

NOTE. When the divisor has not as many figures as are required, begin dividing by the whole divisor, as usual, and so continue, until the figures yet to be found, equal those in the divisor; after which, employ the contraction.

6. Divide 934.8179832 by 93.217, so as to have seven figures in the quotient. A 10.02840.

7. Divide 1,133.7649 by 1.8888434, retaining only the integers.

◆

§ LXVIII. We have given one method of reducing whole numbers of lower denominations to decimals of higher. (§ LX.) The following mode, however, will often be most convenient.

1. Reduce 3 qts. 1 pt. to the decimal of a qt.

This operation is most conveniently performed by setting down the numbers as here; the least denomination highest, and the others in order. Then divide the numbers, as in Reduction Ascending, annexing cyphers, as if reducing a vulgar Fraction to a decimal, and join each quotient to the next denomination with the decimal point between them. Hence, the rule,

2	1.0 pt.
4	3.500 qts.
	.875 A.

DIVIDE THE DENOMINATIONS, BEGINNING WITH THE LEAST, AS IN REDUCTION ASCENDING, ANNEXING CYPHERS, AND JOIN EACH SUCCESSIVE QUOTIENT AS A DECIMAL TO THE NEXT HIGHER DENOMINATION.

EXAMPLES FOR PRACTICE.

2. Reduce 3 bu. 1 pk. to the decimal of a quarter. A. .40625.
3. Reduce 10 oz. 18 dwt. 6 gr. to the decimal of a lb. A. $.9\dot{1}$.
4. Reduce 1 gal. of wine to the decimal of a hhd. A. $.\dot{0}1587\dot{3}$.
5. Reduce 17 yds. 1 ft. 6 in. to the decimal of a ml. A. $.009943\dot{1}\dot{8}$.
6. Reduce 8 rds. 4 yds. 2 ft. 3 in. 1 b. c. to the decimal of a fur. A. $.221\dot{6}3299\dot{6}$.
7. Reduce 3 qrs. 1 na. to the decimal of a yd.
8. Reduce 11 lb. 3 oz. 2 dr. to the decimal of a qr.
9. Reduce 9 oz. 6 dwt. 2 gr. to the decimal of a lb.
10. Reduce 2 h. 13 min. 17 sec. to the decimal of a day.
11. Reduce 4 fur. 5 rds. 2 yds. 2 ft. 1 b. c. to the decimal of a mile.
12. Reduce 3 wks. 2 d. 5 h. 3 m. 1 sec. to the decimal of a month; (30 days.)

For the more advanced pupils who study this book, we insert the following observations on

CIRCULATING DECIMALS.

§ LXIX. Repetends which begin at the same place, are called Similar Repetends. Thus $.\dot{3}$ and $.\dot{7}$, or $.8\dot{1}3\dot{4}$ and $.5\dot{3}86\dot{1}$ are similar repetends.

Those which begin at different places, are called Dissimilar Repetends. Thus, $.\dot{6}$ and $.4\dot{1}6\dot{9}$, $.82\dot{7}$ and $.963\dot{4}5\dot{6}$ are all dissimilar.

Those which end at the same place, are called Conterminous Repetends. Thus, $.8\dot{3}$ and $.\dot{7}\dot{9}$ and $.\dot{0}\dot{5}$ are conterminous.

Those which begin and end at the same places, are called Similar and Conterminous Repetends. Thus, $.8\dot{0}5\dot{4}$ and $.6\dot{1}7\dot{5}$ are similar and conterminous.

Repeating decimals arise from division. They sometimes occur in reducing vulgar Fractions to their equivalent decimals, and sometimes in decimal Division.

In reducing vulgar Fractions to decimals, we annex cyphers to the numerator, and divide by the denominator, (§ LIX.) annexing cyphers to each remainder, in order to continue the division. Now, as every remainder must, of course, be less than the divisor, it is plain that *there can be only as many different remainders in any division, as there are numbers smaller than the divisor.* There are, it is likewise manifest, as many numbers smaller than any given divisor, as there are units in that divisor, *less one.* Thus, there are two numbers smaller than 3; four numbers smaller than 5, &c. Now, a decimal will begin to repeat, whenever we obtain a remainder like any remainder previously found since we began to annex cyphers. For, then, the same dividends, and, of course, the same quotient figures, begin to come round. Hence, *when a decimal repeats, the repetend can never have more places than there are units in the divisor, less one.* If the remainder becomes 0, the decimal terminates, or is *finite.* Hence, also,

Every infinite decimal, obtained from the division of finite numbers, must be a repeating decimal.

The intelligent pupil may here be ready to inquire, "in what manner can I determine whether a given vulgar Fraction will produce a finite or a repeating decimal?" This question will now be answered.

Case. I. Finite Decimals. In our inquiries on this subject, we must take Fractions in their lowest terms; if they be not so, they must, first, be reduced. For the sake of simplicity, we will take the case, when the numerator is 1. This, in the division, by annexing cyphers, is made 10, 100, 1,000, &c. 10 is composed of the prime factors 2×5. Hence, the Fractions $\frac{1}{2}$ and $\frac{1}{5}$ will, of course, be finite. $\frac{1}{10}$ will, likewise, obviously, be finite. Moreover, each of these decimals will terminate in *one place;* for it will only be necessary to annex a single cypher, in order to reduce them. If the denominator be $2\times2=4$, the decimal will terminate, but not in a single place, because 2 is a factor in 10, only once, and in 4, twice. In order, then, to make 2 twice a factor in the dividend, (or numerator,) we must annex another cypher to the 10; making it $100=10\times10=2\times5\times2\times5=4\times5\times5=4\times25$, where, we see, 4 is a factor. Hence, $\frac{1}{4}$ will terminate in two places. From looking over the above, we see that, for similar reasons, if the denominator be 5×5

$=25$, in order to have 5 a factor twice in the dividend, two cyphers must likewise be annexed, and the decimal will terminate in two places, thus, .04. $\frac{1}{100}$ will manifestly require two cyphers, and, therefore, likewise terminate in two places, thus, .01. In like manner, if the denominator be $2\times2\times2$ or $5\times5\times5$ or $10\times10\times10$, three cyphers will be necessary, and the decimal will terminate in three places, and so on.

The whole of the preceding may be summed up as follows:

IF THE DENOMINATOR OF A FRACTION CONSIST ENTIRELY OF 2s, OR OF 5s, OR OF 10s AS FACTORS, THE CORRESPONDING DECIMAL WILL BE FINITE; AND WILL CONSIST OF AS MANY PLACES AS THERE ARE 2s, 5s OR 10s AS FACTORS IN THE DENOMINATOR.

If the denominator be $2\times10=20$ it is manifest from the preceding, that one cypher must be annexed for the 10, and another for the 2. The decimal, then, will terminate in two places. For similar reasons, if the denominator be $5\times10=50$, the decimal will terminate in two places. $2\times5=10$; hence this case is embraced in what has gone before. Hence,

WHEN THE DENOMINATOR OF A FRACTION CONSISTS ENTIRELY OF 2s, 5s AND 10s MULTIPLIED TOGETHER, THE CORRESPONDING DECIMAL WILL BE FINITE; AND WILL HAVE ONE PLACE FOR EVERY 10, WHICH IS A FACTOR, AND ONE FOR EVERY REMAINING 2 OR 5, WHICH IS A FACTOR IN THE DENOMINATOR.

NOTE. If the numerator be greater than 1, the same will evidently hold true; only the decimal will be as many times greater, as the numerator contains units.

CASE II. SIMPLE REPETENDS. It will be evident without much reasoning, that any denominator which is *prime to* 10, will produce an *infinite* decimal. For, if it be prime to 10, it will also be prime to 10 times 10, to $10\times10\times10$, &c., since each of these products contains only the factors of 10 repeated, (viz. 2 and 5,) and (§ XLIII.) neither of these is a factor of the denominator. OR, it is manifest that the smallest number of 10s, which 3 will measure, is 3 *tens*, or 30. The smallest number that 13 will measure is 13 tens, or 130; that 27, 270; that 91, 910; that 21, 210, &c. For, take the last, viz. 21. This number will measure no other number, which does not contain all its *prime factors*, (viz. 3 and 7.) But 10 contains neither, and 21 is the smallest number which contains both. Therefore, no smaller number of 10s, than 21 *tens* $=210$ can be measured by 21. Hence the principle,

NO NUMBER, PRIME TO 10, WILL MEASURE A LESS NUMBER OF 10s, THAN IT CONTAINS UNITS.

Now in reducing a Vulgar Fraction to a decimal, each remainder is less than the denominator, or divisor, and, of course, each dividend contains fewer 10s than the divisor contains units. Hence, it follows, that the divisor, if it be prime to 10, will never measure any dividend, and, of course, the decimal will be infinite, and being infinite, must repeat. Furthermore, DIVIDENDS OF DIFFERENT MAGNITUDES LEAVE DIFFERENT REMAINDERS.

For every dividend is a number of 10s. If, then, it be supposed that two dividends of different magnitudes leave the same remainder, let the less be taken from the greater, and there will remain a number of 10s, which the divisor will exactly measure. But each dividend contains fewer 10s than the divisor contains units; and the same must of course be true of their difference. Therefore, according to the principle just laid down, it is absurd to suppose that the divisor will measure this difference.

Now, in reducing a Fraction whose denominator is prime to 10, if our successive remainders are different, our dividends must be so too, (since the dividends are formed by annexing cyphers to the remainders,) and these, in turn will produce different remainders again; so that we never can obtain the same remainder, nor of course the same dividend, until the number in the numerator, with which we began, recurs as a remainder. That is, *the decimal will not repeat, until the remainder left, is equal to the original numerator.* But every infinite decimal, obtained by division, must repeat. Therefore, we must necessarily obtain, after a time, the original numerator, as a remainder. Hence the principle,

WHEN THE DENOMINATOR OF A FRACTION IS PRIME TO 10, THE CORRESPONDING DECIMAL WILL BE INFINITE, AND WILL BEGIN TO REPEAT WHEN THE NUMERATOR RECURS AS A REMAINDER.

NOTE. In this case the repetend begins in the first place of decimals, and is, of course, a *pure* or *simple Repetend.*

If the numerator be 1, the decimal repeats when the remainder becomes 1. Of course, if we were, then, to take the original numerator, with all the cyphers which have been annexed to it since the divisor commenced, and from this to subtract the remainder 1, the denominator would exactly measure the number left. This number would be a series of 9s. For example, in reducing $\frac{1}{7}$, the remainder 1 occurs after annexing six cyphers. Take, then, 1,000,000 and subtract this remainder, 1, from it, and there is left 999,999, which 7 will exactly measure. It will be seen, that there are just as many 9s as there were cyphers annexed, and of course, as there are decimal places in the repetend. Hence, when the denominator of a Fraction is prime to 10, to determine of how many places the corresponding *pure circulate* will consist,

DIVIDE A SERIES OF 9s, BY THE DENOMINATOR UNTIL NO REMAINDER IS LEFT. THE NUMBER OF 9s USED WILL SHOW THE NUMBER OF PLACES IN THE CIRCULATE.

CASE III. MIXED REPETENDS. When the denominator is not itself prime to 10, but contains *a factor*, which *is* prime to 10, we shall best understand the nature of the corresponding decimal, by separating the denominator into two factors; one of which shall be the entire part that is prime to 10. The other part, of course, must be composed of 2s separately, or 5s separately, as factors; or of 2s and

5s multiplied together. Now, in the reduction, instead of dividing by the whole denominator at once, we may divide successively by these two factors; employing that which is prime to 10, last. Dividing by the factor composed of 2s, 5s, &c., will produce a finite decimal as a quotient. In dividing this quotient by the other factor, it is evident, that, (whatever figures we first obtain,) when we arrive at the end of the significant figures in the number divided, we shall be in the same situation as above, when reducing a Fraction whose denominator was simply prime to 10. Hence, the circulate will begin where the previously found finite decimal ended. Hence, also, the resulting decimal will be a *mixed repetend.* Hence, the general principle,

WHEN THE DENOMINATOR OF A FRACTION IS ENTIRELY COMPOSED OF FACTORS OF 10, THE CORRESPONDING DECIMAL WILL BE FINITE; WHEN IT IS PRIME TO 10, A PURE REPETEND; WHEN IT IS NOT PRIME, BUT CONTAINS A FACTOR WHICH IS PRIME TO 10, A MIXED REPETEND.

From what has been said, the pupil will easily be able to determine where the repetend will begin in the latter case, and also, of how many places it will consist. For, it will begin after as many places as there are 10s, 5s, or 2s, factors in the denominator, and will extend as many places as it requires 9s for the factor prime to 10, to measure.

Hence, the general rule for determining the nature of the decimal, corresponding to any Vulgar Fraction.

I. DIVIDE THE GIVEN DENOMINATOR BY 10 AS OFTEN AS POSSIBLE, AND AFTERWARDS BY 2 OR 5 AS OFTEN AS POSSIBLE WITHOUT REMAINDER. IF THE LAST QUOTIENT BE 1, THE DECIMAL WILL BE FINITE, AND WILL CONTAIN AS MANY PLACES AS THERE WERE DIVISIONS.

II. IF THE LAST QUOTIENT BE NOT 1, THE DECIMAL WILL BE INFINITE. THEREFORE, DIVIDE A SERIES OF 9S BY THAT QUOTIENT, UNTIL NO REMAINDER IS LEFT. THE NUMBER OF 9S USED WILL SHOW THE NUMBER OF PLACES IN THE REPETEND; WHICH WILL BEGIN AFTER AS MANY PLACES AS YOU AT FIRST PERFORMED DIVISIONS BY 2, 5 OR 10.

NOTE. The Fraction, it must be recollected, must be in its lowest terms.

EXAMPLES FOR PRACTICE.

1. Let the Fraction be $\frac{1}{32}$, $\frac{3}{128}$, $\frac{9}{256}$, $\frac{17}{51}$, $\frac{18}{36}$, $\frac{13}{1728}$, $\frac{144}{1728}$.
2. Let the Fraction be $\frac{6}{720}$, $\frac{9}{453}$, $\frac{18}{940}$, $\frac{235}{8450}$, $\frac{2345}{8912}$, $\frac{1783}{1987}$.
3. Let the Fraction be $\frac{3}{1825}$, $\frac{139}{172}$, $\frac{12598}{688450}$, $\frac{781200}{9876000}$.
4. Let the Fraction be $\frac{187}{389}$, $\frac{543267}{1888884}$, $\frac{17943814532}{98333344445}$.

◆

§ LXX. It is evident from what has been said, that any number prime to 10, will divide exactly a series of 9s. Now, if the quotient, obtained by such a division, be multiplied into the dividing number, the same series of 9s will be produced again. This quotient is evidently the circulate, which would be obtained by reducing a Fraction, whose numerator is 1, and whose denominator, the dividing number. Now, if both terms of this Fraction were to be multiplied by this quotient, or circulate, the Fraction, without being altered in value, (§ XLI.) would be changed to one whose numerator would be the circulate itself, and whose denominator, just as many nines as the circulate has places. To illustrate by an example. $999999 \div 7 = 142857$, and $\frac{1}{7}$ reduced to a decimal $= .\dot{1}4285\dot{7}$. $\frac{1 \times 142857}{7 \times 142857} = \frac{142857}{999999} = \frac{1}{7}$. Hence, if the circulate $.\dot{1}4285\dot{7}$ had been given us, and it had been required to find its value in a vulgar Fraction, (that is, to find what vulgar Fraction would produce it,) we might have made the circulate itself the numerator of a Fraction, and just as many nines as it had places, the denominator, thus, $\frac{142857}{999999}$, and this, reduced to its lowest terms, would have given us the original Fraction, $\frac{1}{7}$. $\frac{2}{7}$ would give us twice as great a circulate, $\frac{3}{7}$, three times as great, and so on.

Hence, to change any pure circulate, or repetend, to its equivalent Vulgar Fraction,

MAKE THE GIVEN REPETEND THE NUMERATOR, AND THE DENOMINATOR AS MANY 9s, AS THERE ARE PLACES IN THE REPETEND.

In mixed repetends, the principle is similar. If a repetend begin in the place of hundredths, its value will be the same as before, except that it will be ten times smaller than if it began, like a simple repetend, in the place of tenths. For when it begins in the first, or tenths' place, it is a Fraction of a *unit, or whole number*. But when it begins in the second, or hundredths' place, it is a Fraction of a *tenth*. Its value is therefore decreased ten fold. Thus $.\dot{3}$ is $\frac{3}{9}$ of a *whole* number; but $.0\dot{3}$ is $\frac{3}{9}$ of a *tenth* $= \frac{3}{9}$ of $\frac{1}{10} = \frac{3}{90}$. After finding the Vulgar Fraction, as before, then, we are obliged to divide it by 10, or, (which, (§ LIII.), is the same thing,) multiply its denominator by 10; which is done by annexing a cypher. In like manner, if it had begun a place lower still, we should have been obliged to annex two cyphers, and so on; annexing always as many cyphers as there are places between the separatrix, and the first figure of the repetend, or *as there are finite places in the decimal.* Hence, to find the value of the circulating part of a mixed repetend,

MAKE THE REPETEND THE NUMERATOR; AND FOR THE DENOMINATOR, ANNEX AS MANY CYPHERS AS THERE ARE FINITE PLACES, TO AS MANY 9s AS THERE ARE PLACES IN THE CIRCULATE.

The mode of finding the value of the finite part has already been given. (§ LVIII.) Hence to find the whole value of a mixed repetend, in a Vulgar Fraction,

FIND THE VALUES OF THE FINITE AND CIRCULATING PARTS SEPARATELY, AND ADD THEM TOGETHER.

By the above rule, find the values of $.\dot{1}0\dot{4}$ $.8\dot{3}\dot{9}$ $.61\dot{4}0\dot{7}$ $.9\dot{3}$ $.\dot{8}1\dot{5}$ $.\dot{7}$ $.6\dot{3}1\dot{1}$ $.98\dot{3}\dot{1}$ $.\dot{4}76\dot{2}$ $.8\dot{3}14\dot{1}$ $.9\dot{8}76\dot{4}$ $.309\dot{4}\dot{2}$ $.\dot{1}1133\dot{3}$ $.\dot{7}77888\dot{9}$ $.631\dot{1}543\dot{8}$ $.713\dot{2}428\dot{5}$.

§ LXXI. A single repetend may evidently be regarded as a compound repetend, consisting of as many places as we choose to make it. Thus, $.\dot{3}$ is a single repetend. Giving it two places, thus, $\dot{3}\dot{3}$, or three, thus, $.\dot{3}3\dot{3}$, it becomes a compound repetend. So $.9\dot{7}$ may be made $.9\dot{7}77\dot{7}$ or $9\dot{7}777\dot{7}$, &c. We may also make a single repetend begin later, or at a lower place, reserving its higher figures as finite decimals. Thus, .6 may be made $.6\dot{6}$, or $66\dot{6}$, &c.

We may also make the same changes upon compound repetends; making them begin later, thus, $.\dot{4}\dot{6}$ changed to $.4\dot{6}\dot{4}$ or $.464\dot{6}\dot{4}$, &c. or extending the number of places, thus, $.\dot{3}7\dot{9}$ changed to $.\dot{3}797\dot{9}$, or to $.\dot{3}79797\dot{9}$, &c.; or making both changes at once, thus $.\dot{4}3\dot{2}$ changed to $.4\dot{3}2432\dot{4}$, &c. In changing compound repetends, we must be careful to place the points so that the repeating figures, when extended beyond the last point, shall recur in the same order as before. Thus, $.\dot{8}6\dot{2}$ is inaccurately changed to $.8\dot{6}28\dot{6}$, and $.8\dot{1}\dot{3}$ to $.813\dot{1}\dot{3}$, the latter numbers, in each case, representing different repetends from those given. By moving the points in this manner, we may render any two repetends similar. Thus $.\dot{1}3\dot{5}$ and $.8\dot{6}\dot{1}$ are dissimilar; but $.1\dot{3}5\dot{1}$ and $.8\dot{6}\dot{1}$ are similar. Having made repetends similar, they may also be rendered conterminous, by extending their figures to a number of places, that is a common multiple of the numbers of places in the given repetends. For, on making the number of places in any repetend, twice, three times, or any number of times as great as before, it will still repeat in the same order as at first. The least common multiple is most convenient.

The foregoing examination of the nature of circulating decimals, will afford us some useful rules for conducting arithmetical operations upon them. Single repetends, in their true value, have been seen to be *ninths.* *Hence, for* ADDITION *when there are* FINITE DECIMALS, AND SINGLE REPETENDS, *or* SINGLE REPETENDS ONLY,

I. MAKE THE REPETENDS CONTERMINOUS, EXTENDING THEM ONE PLACE BEYOND THE LONGEST FINITE DECIMAL; CARRY FOR 9 INSTEAD OF 10 FROM THE RIGHT HAND COLUMN; AND IN OTHER RESPECTS ADD AS USUAL. THE RIGHT HAND FIGURE OF THE SUM WILL BE A REPETEND.

In case there are compound repetends, having made them conterminous, it is evident that, if they were extended farther still, (as they might be,) there would often be something to carry from those figures so extended; and this number carried would be the same as that carried forward from the *first* place of the repetends. *Hence, when there are* FINITE DECIMALS, AND CIRCULATES, *or* CIRCULATES ONLY,

II. MAKE THE REPETENDS SIMILAR AND CONTERMINOUS, COMMENCING THEM ONE PLACE BELOW THE LONGEST FINITE DECIMAL; CARRY TO THE RIGHT HAND FIGURE OF THE SUM, THE SAME NUMBER THAT IS CARRIED FORWARD FROM THE FIRST PLACE OF THE REPETENDS; AND IN OTHER RESPECTS ADD AS USUAL. THE SUM OF THE CIRCULATING FIGURES, WILL BE THE CIRCULATE OF THE AMOUNT.

Note. If the circulate found by adding be a series of 9s, it becomes equal to the denominator of its equivalent vulgar Fraction, and of course its value is 1; which may be added to the next higher place, and the repetend neglected.

Multiplication is a repeated addition. *Hence, for* MULTIPLICATION, *when the* MULTIPLIER IS FINITE, *and the* MULTIPLICAND A SINGLE REPETEND,

I. MULTIPLY AS USUAL, CARRYING FOR 9 FROM THE PRODUCT OF THE REPETEND. THE RIGHT HAND FIGURE OF EACH PARTIAL PRODUCT WILL BE A REPETEND. THE TOTAL PRODUCT MUST THEREFORE BE FOUND BY RULE I. FOR ADDITION.

That is, the partial products must be made conterminous, before adding, and, in adding we must carry for 9 from the right hand column. The last figure of the total product will then be the repetend.

When the MULTIPLIER IS FINITE, *and the* MULTIPLICAND A CIRCULATE,

II. MULTIPLY AS USUAL, CARRYING TO THE RIGHT HAND FIGURE OF EACH PARTIAL PRODUCT, THE NUMBER WHICH IS CARRIED FORWARD FROM THE FIRST PLACE OF THE CIRCULATE. EACH PARTIAL PRODUCT WILL HAVE AS MANY CIRCULATING FIGURES AS THE MULTIPLICAND. THE TOTAL PRODUCT MUST THEREFORE BE FOUND BY RULE II. FOR ADDITION.

The case in which the *multiplier* is a repetend, remains to be considered. Here we have no means of proceeding *decimally* with accuracy. *Hence, when the* MULTIPLIER IS A REPETEND,

III. CHANGE THE MULTIPLIER TO ITS EQUIVALENT VULGAR FRACTION; MULTIPLY BY ITS NUMERATOR, AND DIVIDE BY ITS DENOMINATOR.

Note. If the multiplicand be also infinite, this rule anticipates a case of division to be mentioned. (Rule I.) It may however be mentioned here, that, when, in ordinary cases, we should annex cyphers in dividing, we must annex the repeating figures of the number divided instead of them; and so continue to do until the decimal repeats. This will determine the number of figures in the final circulate.

In Subtraction it is evident, that, when the repetends are similar and conterminous, if that of the subtrahend be greater than that of the minuend, we shall be obliged to borrow 1 from the next higher place. And this would also be the case, if the repetends were extended below the last point. This would make the remainder 1 less. Hence, for SUBTRACTION, when EITHER OR BOTH NUMBERS ARE REPETENDS,

PREPARE THE NUMBERS AS IN ADDITION, AND SUBTRACT AS USUAL, OBSERVING TO DIMINISH THE REMAINDER BY 1, WHEN THE REPETEND OF THE SUBTRAHEND IS GREATER THAN THAT OF THE MINUEND.

In division, when the dividend is infinite, it is plain, that since the repeating figures may be extended to any distance, those figures will occupy the places of the cyphers which would otherwise be annexed to continue the division. Hence, for DIVISION, when the DIVISOR IS FINITE, and the DIVIDEND A REPETEND,

I. DIVIDE AS USUAL, ANNEXING THE REPEATING FIGURES OF THE DIVIDEND, IF NECESSARY, INSTEAD OF CYPHERS; AND SO PROCEED UNTIL THE QUOTIENT REPEATS.

If the divisor be infinite, the division cannot be performed *decimally*. Hence, when the DIVISOR IS A REPETEND,

II. CHANGE THE DIVISOR TO ITS EQUIVALENT VULGAR FRACTION; MULTIPLY BY ITS DENOMINATOR, AND DIVIDE BY ITS NUMERATOR.

NOTE. For short repetends, the preceding rules are very useful. But when repetends are long, the processes are tedious, and the value of the lower figures so small, that they may be neglected, without occasioning any important error. The same may be remarked of long finite decimals. We have before mentioned that decimals not carried out in full, are marked with + and —, according as they are too small or too great, and called APPROXIMATES. Particular rules may be given for their calculation, which the limits of this work will not permit us to insert. The common rules are sufficient for ordinary purposes; and those who are curious to examine the subject further, must resort to more extensive works. We subjoin a few examples.

1. Add $3.\dot{6}$; $78.3\dot{4}7\dot{6}$; $735.\dot{3}$; 375; $.\dot{2}\dot{7}$ and $187.\dot{4}$ A. $1{,}380.0\dot{6}4819\dot{3}$

2. Add 5,391.357; $72.3\dot{8}$; $187.\dot{2}\dot{1}$; $4.29\dot{6}\dot{5}$; $217.849\dot{6}$; $42.17\dot{6}$; $.52\dot{3}$; and $58.30\dot{0}4\dot{8}$ A. $5{,}974.1037\dot{1}$

3. Add $9.\dot{8}1\dot{4}$; 1.5; $87.2\dot{6}$; $.08\dot{3}$ and $124.\dot{0}\dot{9}$ A. $222.75\dot{5}7239\dot{0}$

4. Add $162.\dot{1}6\dot{2}$; 1.5; $134.\dot{0}\dot{9}$; $2.\dot{9}\dot{3}$; $97.2\dot{6}$; $3.\dot{7}6923\dot{0}$; $99.08\dot{3}$ and $.\dot{8}1\dot{4}$ A. $501.62\dot{6}5107\dot{7}$

5. Multiply $.13\dot{6}$ by .45 A. .0615

6. Multiply $8.0\dot{3}$ by 3.2 A. $25.70\dot{6}$

7. Multiply $3.\dot{9}7\dot{3}$ by 8. A. $31.\dot{7}9\dot{1}$

8. Multiply $.\dot{3}\dot{6}$ by $.2\dot{5}$ A. $.0\dot{9}2\dot{9}$

9. Multiply $3.1\dot{4}\dot{5}$ by $4.\dot{2}9\dot{7}$ A. $13.51\dot{6}953\dot{3}$

10. Multiply 49,640.54 by $.\dot{7}050\dot{3}$ A. $34{,}998.41\dot{9}900\dot{3}$

NOTE. The pupil will observe, that as either factor may be made the multiplier, questions like the last may be solved, by rule II. or III. at pleasure.

11. From $85.\dot{6}\dot{2}$ take $13.7\dot{6}43\dot{2}$ A. $71.8\dot{6}19\dot{3}$

12. From $26.4\dot{3}$ take $25.\dot{2}$ A. $1.2\dot{1}$

13. From $476.3\dot{2}$ take $84.769\dot{7}$ A. $391.552\dot{4}$.

14. From $3.85\dot{6}\dot{4}$ take $.03\dot{8}\dot{2}$ A. $3.\dot{8}\dot{1}$

15. Divide $319.28007\dot{1}1\dot{2}$ by 764.5 A. $.4176\dot{3}2\dot{5}$

16. Divide $274.\dot{6}$ by $.\dot{7}$ A. $301.\dot{7}1428\dot{5}$

17. Divide $18.\dot{5}\dot{6}$ by $.\dot{3}$ A. $55.\dot{6}\dot{9}$

18. Divide $24.\dot{0}8\dot{1}$ by $.\dot{3}8\dot{6}$ A. $62.3\dot{2}383419689\dot{1}$

19. Divide $.\dot{3}\dot{6}$ by $.2\dot{5}$ A. $1.\dot{4}22924901185770750988\dot{1}$

DUODECIMALS.

§ LXXII. Decimals, we have seen, consist of a series of numbers decreasing towards the right by tens, as far as we choose to carry them. We are now about to speak of a class of numbers, decreasing in a similar manner by *twelves*. Of course, the calculations made upon them must be similar, in many respects, to those upon decimals. They are called DUODECIMALS, from the Latin word *duodecimus*, which signifies *twelfth*. Duodecimals are commonly used only for measuring *length*, *surface*, and *solidity*.

A linear, square or solid foot is, therefore, considered the unit, or whole number. The lower denominations, or orders, are, of course, Fractions of a foot.

The twelfth part of a foot, of any kind, whether a solid, a square, or a linear foot, *is called a* PRIME; the twelfth part of a prime, is called a SECOND; the the twelfth part of a second, a THIRD; the twelfth part of a third, a FOURTH, and so on, as far as we choose to go.

Now, as primes are 12ths, seconds, 12ths of 12ths, or 144ths, thirds, 12ths of 144ths, or 1,728ths; fourths, 12ths of 1,728ths, or 20,736ths, &c. it is plain that we might write these numbers, as vulgar Fractions, with their denominators. But, since we know that they increase and decrease regularly by 12, we may, exactly as in decimals, *omit* the denominators, and write each order in a separate place. This is the mode, in which duodecimals are actually written, and, to distinguish the orders from one-another, primes are marked with an accent, thus $'$: seconds with two accents, thus $''$: thirds with three accents, thus $'''$ and so on. Then 3 feet, 2 primes, 10 seconds, 11 thirds, 5 fourths and 6 fifths are written

$$3 \quad 2' \quad 10'' \quad 11''' \quad 5'''' \quad 6'''''$$

Instead of these marks, the the Roman numerals i, ii, iii, iv,.&c. are sometimes employed. These are generally substituted when the number of marks becomes large. Thus the above example may be written,

$$3 \ 2^{\text{i}} \ 10^{\text{ii}} \ 11^{\text{iii}} \ 5^{\text{iv}} \ 6^{\text{v}} \text{ or } 3 \ 2' \ 10'' \ 11''' \ 5^{\text{iv}} \ 6^{\text{v}}$$

ADDITION and SUBTRACTION of duodecimals may evidently be performed like addition and subtraction of other compound numbers, or like the same operations in decimals, except that we are to *carry* and *borrow* by 12, instead of by 10, from one denomination to another.

MULTIPLICATION of duodecimals may be likewise performed like Multiplication of decimals, observing the same rule in carrying.

Now, in multiplication of decimals, the pupil will recollect, that there must be as many decimal places, (or places below units) in the product, as there are in both factors. For a similar reason, *in multiplication of duodecimals, there must be as many duodecimal places,* (or places below units, or feet,) *in the product as there are in both factors.* This will be evident for the same reasons as in decimals.

The same thing may likewise be proved by writing duodecimals, with their denominators like vulgar Fractions. $\frac{1}{12}\times\frac{1}{12}=\frac{1}{144}$; that is, primes by primes produce seconds, since seconds are 144ths. $\frac{1}{12}\times\frac{1}{144}=\frac{1}{1728}$; that is, primes by seconds, produce thirds, and so on. This principle will enable the pupil to perform multiplication of duodecimals, in any case.

1. In a board 5 ft. 3 inches (5 3′) in length, and 1 ft. 5 inches (1 5′) in breadth, what is the amount of surface? We know, in the beginning, that the product is to have *two* duodecimal places, because there is *one* place in each factor. We therefore know that the right hand place of the product must be seconds. We commence at the right to multiply as usual, and after having multiplied by the 5′, we proceed to multiply by the 1 ft., placing the first denomination of the product one place farther to the left, because we are multiplying by a higher denomination than before. This brings it exactly under the multiplying figure. The process will be seen to be exactly like multiplication of decimals, except that we carry for 12, from denomination to denomination.

$$\begin{array}{lrrr}
\text{Length,} & 5 & 3' & \\
\text{Breadth,} & 1 & 5' & \\
\hline
 & 2 & 3' & 2'' \\
 & 5 & 3' & \\
\hline
\textit{Ans.} & 7 & 5' & 3''
\end{array}$$

2. In a solid block, the base of which contains 9 sq. ft. 6′, 11″, and the height of which is 4 ft. 7′ 2″, what is the solidity?

It will be observed that there are no denominations higher than feet. Therefore, we never carry for 12, after we arrive at feet. We shall always know when this is the case, by observing how many duodecimal places there must be in the answer, and placing the marks, or accents over each denomination of the product as we go on.

$$\begin{array}{lrrrrr}
\text{Base,} & 9 & 6' & 11'' & & \\
\text{Height,} & 4 & 7' & 2'' & & \\
\hline
 & & 1' & 7'' & 1''' & 10'''' \\
 & 5 & 7' & 0'' & 5''' & \\
 & 38 & 3' & 8'' & & \\
\hline
\textit{Ans.} & 44 & 0' & 3'' & 6''' & 10''''
\end{array}$$

NOTE. The number of any denomination, and of course, the number of accents to be placed over it, will be observed always to be equal to the number of accents over *both* the numbers multiplied together. Thus the right hand figure of the above product is *fourths,* which is obtained by multiplying *seconds* by *seconds.* There are two accents over each factor, and *four* over the product.

If the higher duodecimal places be wanting, cyphers must be put in their room.

From the preceding illustrations, we derive the general rule for multiplication of duodecimals.

I. BEGIN ON THE RIGHT AND MULTIPLY THE MULTIPLICAND BY EACH DENOMINATION OF THE MULTIPLIER; CARRYING FOR 12 FROM DENOMINATION TO DENOMINATION, AND PLACING THE FIRST NUMBER IN EACH PARTIAL PRODUCT, EXACTLY UNDER THE MULTIPLYING FIGURE.

II. THERE WILL BE AS MANY DUODECIMAL PLACES IN THE PRODUCT AS IN BOTH THE FACTORS; WHICH MUST BE MARKED ACCORDINGLY.

3. If a floor be 10ft. 4′ 5″ long, and 7ft. 8′ 6″ wide, what is its surface? *Ans.* 79ft. 11′ 0″ 6‴ 6^{iv}

4. What is the surface of a marble slab, 5ft. 7′ long, and 1ft. 10′ wide? *Ans.* 10ft. 2′ 10″

5. How many feet of plastering in a ceiling 43ft. 3′ long, and 25ft. 6′ wide? *Ans.* 1,102ft. 10′ 6″

6. What is the solidity of a wall 53ft. 6′ long, 10ft. 3′ high, and 2 ft. thick? *Ans.* 1,096$\frac{3}{4}$ft.

7. Required the surface of a floor, 48ft. 6′ long, and 24ft. 3′ broad? *Ans.* 1,176$\frac{1}{8}$ft.

8. The length of a room being 20ft., its breadth 14ft. 6′, and its height 10ft. 4′, how many yards of painting in its walls, deducting a fire-place 4ft. 4′ by 4ft. and two windows, each 6ft. by 3ft. 2′? *Ans.* 73$\frac{2}{27}$yds.

9. In a floor 12 8′ by 16 3′ how many sq. ft.? A.205$\frac{5}{6}$

NOTE. Some kinds of work are done by the square yard. Such are painting, paving, plastering, &c.

10. A man paved a court 371 2′ 6″ by 181 1′ 9″ at 2 cts. pr. sq. yd. How many sq. yds. did he pave, and what did he receive?

A. 7,471,$\frac{4427}{10368}$ sq. yds.—149.42\frac{4427}{5184}$

11. How many *cord feet* in a load 8 ft. long, 4 wide and 3 6′ high? A. 7

12. Multiply 4ft. 7′ by 6ft. 4′ *Ans.* 29ft. 0′ 4″

13. Multiply 39 10′ 7″ by 18 8′ 4″

Ans. 745 6′ 10″ 2‴ 4^{iv}

14. Multiply 24 10′ 8″ 7‴ 5^{iv} by 9 4′ 6″

Ans. 233 4′ 5″ 9‴ 6^{iv} 4^{v} 6^{vi}

15. Multiply 44 2′ 9″ 2‴ 4^{iv} by 2 10′ 3″

Ans. 126 2′ 10″ 8″ 10^{iv} 11^{v}

NOTE. It is plain that *division, might be* performed by duodecimals, but the inconvenience of the process renders it of no particular use.

◆

REDUCTION OF CURRENCIES.

§ LXXIII. The term CURRENCY is applied to any thing which is universally received, in any country, for the payment of debts, or for goods, bought and sold. In other words, the *currency* of any country is the *money* of that country. It is often called the *circu-*

lating medium, or the *medium of trade;* because it passes from one individual to another, or is *current* among all persons; and because, by means of it, trade is carried on. Federal Money is the currency established by law in this country. But before the adoption of this currency, in 1786, all accounts were kept in pounds, shillings and pence. Our money was, originally, the same as that of Great Britain; that is, *Sterling Money.* But the legislatures of the different states, or *colonies*, as they were called before our independence, put bills in circulation, which diminished or *depreciated* in value. This depreciation was different in different colonies; so that, while the names remained the same, throughout the country, the values became very different. Thus, a pound in the New-England states, and Virginia, became only $\frac{3}{4}$ of a pound Sterling in value. This was also the value of the currencies used in Kentucky and Tennessee. In New-York and North-Carolina, a pound became $\frac{9}{16}$ of a pound Sterling. This Currency was afterwards used in Ohio. In New-Jersey, Pennsylvania, Delaware and Maryland, a pound became $\frac{3}{5}$, and in S. Carolina and Georgia $\frac{27}{28}$ of a pound Sterling. In Canada and Nova Scotia, a pound is $\frac{9}{10}$ of a pound Sterling; in Scotland $\frac{1}{12}$, and in Ireland $\frac{923}{1000}$, nearly. Hence, in thn United States we have four Currencies besides that established by law. These, with the Canada, Scotch, Irish, and Sterling make eight. The names and values, assigned them are as follows:

Sterling Money,	in which	4s. 6d.	make a dollar.
Georgia Currency	"	4s. 8d.	"
Irish "	"	4s. 10½d. nearly	"
Canada, "	"	5s. 0d.	"
New England	"	6s. 0d.	"
Pennsylvania	"	7s. 6d.	"
New-York	"	8s. 0d.	"
Scotch	"	£2 14s. 0d.	"

We have arranged them in the order of their values, placing highest the currency, whose value is greatest.

NOTE. We have given above the value of a dollar in Sterling Money as all our Arithmetics have it, and as it is estimated throughout the country. It appears, however, by the report in SENATE OF THE U. S., March 29th, 1830, that the value of the Spanish dollar, (generally considered equal to ours,) is only 4s. 1½d. and that of the American dollar 4s. 1d. 1¼qrs. nearly.

It is to be regretted, that these currencies are still retained and employed by our merchants and tradesmen. The Federal Currency is so much more convenient than any of them, that it must ultimately supersede them all; and the sooner this is the case the better. For, in that event, we shall not only be free from the inconvenience of making calculations *in* these currencies, but likewise from the greater evil of being obliged to *change* sums of money *from one to another.* The making of these changes is what is meant by the REDUCTION of Currencies.

The intelligent pupil will perceive, that the table of values given above, is sufficient to enable him to make these changes; and that no processes are necessary but those of common Reduction. But, as more concise methods may be suggested, it seems proper to give the subject a particular consideration. Its comparative unimportance will not justify us in devoting to it much space. By means, therefore, of the following table, we have thought proper to combine all the cases, of which it admits, under a single rule.

TABLE

EXHIBITING THE COMPARATIVE VALUES OF THE SEVERAL CURRENCIES.

ANY SUM EXPRESSED IN										
$ Fed.M.	£ Ster.	£ Geor.	£ Irish	£ Can.	£ N.E.	£ Penn.	£ N.Y.	£ Scotch	IS	
$\frac{10}{27}$	$\frac{1}{12}$	$\frac{7}{81}$	$\frac{250}{2769}$	$\frac{5}{54}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{4}{27}$		£Scot.	OF THE SAME SUM EXPRESSED IN
$\frac{10}{4}$	$\frac{9}{16}$	$\frac{7}{12}$	$\frac{1125}{1846}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{15}{16}$		$\frac{27}{4}$	£N.Y.	
$\frac{8}{3}$	$\frac{3}{5}$	$\frac{28}{45}$	$\frac{600}{923}$	$\frac{2}{3}$	$\frac{4}{5}$		$\frac{16}{15}$	$\frac{36}{5}$	£Pen.	
$\frac{10}{3}$	$\frac{3}{4}$	$\frac{7}{9}$	$\frac{750}{923}$	$\frac{5}{6}$		$\frac{5}{4}$	$\frac{4}{3}$	9times	£ N.E.	
4times	$\frac{9}{10}$	$\frac{14}{15}$	$\frac{900}{923}$		$\frac{6}{5}$	$\frac{3}{2}$	$\frac{8}{5}$	$\frac{54}{5}$	£ Can.	
$\frac{923}{225}$	$\frac{923}{1000}$	$\frac{6461}{6750}$		$\frac{923}{900}$	$\frac{923}{750}$	$\frac{923}{600}$	$\frac{1846}{1125}$	$\frac{2769}{250}$	£Irish.	
$\frac{30}{7}$	$\frac{27}{28}$		$\frac{6750}{6461}$	$\frac{15}{14}$	$\frac{9}{7}$	$\frac{45}{28}$	$\frac{12}{7}$	$\frac{81}{7}$	£ Geo.	
$\frac{40}{9}$		$\frac{28}{27}$	$\frac{1000}{923}$	$\frac{10}{9}$	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{16}{9}$	12 times	£ Ster.	
	$\frac{9}{40}$	$\frac{7}{30}$	$\frac{225}{923}$	$\frac{1}{4}$	$\frac{3}{10}$	$\frac{3}{8}$	$\frac{4}{15}$	$\frac{27}{10}$	$ F.M.	

Suppose I have a sum in £ Sterling, which I wish to reduce to £ New England. I see, by the table, that any sum expressed in £ Ster. is $\frac{3}{4}$, of the same sum, expressed in N. E. This I find, by looking for the given Currency, (which is Sterling,) in the upper line, and for the required Currency, (which is New England,) in the right hand column. Under the one, and opposite the other, I have the Fraction $\frac{3}{4}$. Let the given sum be £6. Then I know that 6 is $\frac{3}{4}$ of the answer. If 6 is $\frac{3}{4}$, 2 is $\frac{1}{4}$. £2 is $\frac{1}{4}$ of 8£. *Ans.*

It will be seen, that, in this case, we divided by 4, and multiplied by 3, which (§ LI.) is dividing by the Fraction $\frac{3}{4}$. Hence, to reduce a sum from one currency to another,

DIVIDE BY THE NUMBER FOUND IN THE TABLE, UNDER THE GIVEN, AND OPPOSITE THE REQUIRED CURRENCY.

For similar reasons, the number *under* the *required* and *opposite* the *given* currency may be used as a *multiplier*: The numbers in the table may often be reduced to decimals with advantage.

NOTE. It will usually be best to reduce shillings, pence, and farthings to a fraction of a pound, either vulgar or a decimal.

It may perhaps be desirable that the pupil should commit to memory the numbers used in reducing the currencies of the U. S. to Fed. Money, and the contrary. The rest is only intended for reference.

EXAMPLES FOR PRACTICE.

1. Reduce £4 N. E. to F. M.	*Ans.* 13.333\frac{1}{3}$.
2. Reduce 2£ 3s. 9d. N. E. to F. M.	*Ans.* 7.291\frac{2}{3}$.
3. Reduce £6 N. Y. to F. M.	*Ans.* $15.00.
4. Reduce £8 ; 4 ; 9 N. Y. to F. M.	*Ans.* 20.593\frac{3}{4}$.
5. Reduce £3 ; 2 ; 3 Penn. to F. M.	*Ans.* $8.30.
6. Reduce $152.60 to N. E.	*Ans.* £45 ; 15 ; 7.2.

7. Reduce $196.00 to N. E. *Ans.* £58 ; 16.
8. Reduce $629.00 to N. Y. *Ans.* 251 ; 12.
9. Reduce £35 ; 6 ; 8 Sterling, to N. E. *Ans.* £47 ; 2 ; 2 ; $2\frac{2}{3}$.
10. Reduce £120 N. E. to Can. *Ans.* £100.
11. Reduce £155 ; 13 N. E. to Sterling. *Ans.* £116 ; 14 ; 9.
12. Reduce £104 ; 10 Can. to N. Y. *Ans.* £167 ; 4.
13. Reduce £300 ; 10 ; 4 ; 2 Can. to Penn. *Ans.* £450 ; 15 ; 6 ; 3.
14. Reduce £937 ; 18 ; 11 ; 1 N.E. to Geo. *Ans.* £721 ; 14 ; 8 ; 3.
15. Reduce $224.60 to Can. *Ans.* £56 ; 3.
16. Reduce 225 ; 6 N. E. to F. M. *Ans.* $752.00.
17. Reduce £880 ; 15 ; 11 ; 1 Penn. to Ster. *Ans.* £528 ; 9 ; 6 ; 3.
18. Reduce £6,750 Irish to Geor. *Ans.* £6,461.
19. Reduce £1,846 Ster. to Irish. *Ans.* £2,000.
20. Reduce £1,722 ; 18 ; 9 ; 3 N. E. to N. Y. *Ans.* £2,298 ; 5 ; 1.
21. Reduce £2,114 ; 1 ; 3 Can to F. M. *Ans.* $8,456.25.
22. Change £784 ; 5 ; 6 ; 2 Penn. to Geor. *A.* £487 ; 19 ; 10 ; $2\frac{38}{45}$.
23. Change £923 Sterling to Irish.
24. Change £4,000 Irish to Sterling.
25. Change £157 ; 8 ; 3 ; 3 N. Y. to N. E.
26. Change £1,654 ; 3 ; 8 : 1 Penn. to N. E.
27. Change £947 ; 9 ; 4 ; 2 N. E. to F. M.
28. Change $1,444.66 to N. E. To N. Y. To Penn.
29. Change $945.32 to N. Y. To Geor. To Can.
30. Change £1,846 ; 15 ; 4 N. E. to F. M. To Penn. To Geor.
31. Change $4,444.444$\frac{4}{9}$ to Sterling.
32. Reduce £1,000,000 Ster. to F. M.
33. What is the value of 1£ Ster. in F. M. ? *Ans.* 4.444\frac{4}{9}$.
34. What is the value of 1£ Irish in F. M. ? *Ans.* 4.10\frac{2}{9}$.
35. What is the value of 1£ N. E. in F. M. ? *Ans.* 3.333$\frac{1}{3}$.
36. What is the value of 1 £ N. Y. in F. M. *Ans.* $2.50.
37. What is the value of 1£ Can. in F. M. ? *Ans.* $4.00.
38. What is the value of 1£ Scotch in F. M. ? *Ans.* 0.370\frac{10}{27}$.
39. What is the value of 1£ Penn. in F. M. *Ans.* 2.666\frac{2}{3}$.
40. What is the value of 1£ Geo. in F. M. *Ans.* 4.285\frac{5}{7}$.

Hence we see that there are but two of the currencies, in which a pound can be expressed in Federal Money by a finite decimal. This is one of the disadvantages in their use.

The following FOREIGN COINS *have assigned to them the values, in* FEDERAL MONEY, *placed opposite to them, respectively.*

Shilling Sterling,	0.22\dot{2}$
Crown 5s.	1.11$\dot{1}$
Sovereign, (a gold coin, =£,)	4.44$\dot{4}$
Guinea, (21s. nearly out of use in England.)	4.66$\dot{6}$
Livre of France,	0.185+
Franc "	0.1875—
Pistole* 10 livres "	1.852—
Louis d'or, "	4,444+
Five franc piece, "	0.937
Real of Plate, of Spain,	0.100
Real of Vellon, "	0.050
Pistole, "	3.60
Dollar, "	1.00

Re, of Portugal,	$0.0012+
Testoon, "	0.125
Milre*, "	1.250
Moidore, "	6.000
Joanese, "	8.000
Marc Banco of Hamburgh,	0.333+
Pistole of Italy,	3.200
Rix Dollar of Austria,	0.778—
Rix Dollar of Denmark and Switzerland,	1.000
Rix Dollar, of Sweden,	1.037
Rix Dollar*, of Prussia	0.778—
Florin, "	0.259+
Ducat, of Sweden and Prussia,	2.074

Piaster of ex, of Spain,	0.80	Ducat, of Denmark,	8.833+
Ducat of ex*, "	1.102—	Ruble, of Russia,	1.000
Stiver of Holland,	0.019+	Zervonitz, "	2.000
Guilder or Florin, "	0.388	Tale, of China,	1.480
Rix Dollar, "	0.970	Pagoda, of India,	1.840
Ducat, "	2.079—	Rupee, of Bengal,	0.500
Gold Ducat. "	8.000	Xeriff of Turkey,	2.222

* Those denominations which have the asterisk, [as the Pistole of France, and the Milre of Portugal,] are merely *nominal;* that is, they are represented by no *real coin.* In this respect, they are like the Mill in Federal Money. We subjoin a very few,

EXAMPLES FOR PRACTICE.

1. In 300 Milres, how many dollars? *Ans.* $375.00.
2. In $900, how many Gold Ducats of Holland? *Ans.* 112½.
3. In 3,000 Francs, how many dollars? *Ans.* $562.50.
4. In 2,876 Pistoles of Spain, how many dollars? *Ans.* $10,353.60.
5. In $600 how many Moidores?
6. In 3,842 Rupees how many dollars?
7. In 1,945 Pagodas how many dollars?
8. In $2,955 how many Francs?

NOTE. The above examples belong to the rule commonly called EXCHANGE.

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CALCULATIONS OF CENTAGE,

INCLUDING STOCKS, PROFIT AND LOSS, INSURANCE, COMMISSION, DUTY, INTEREST AND DISCOUNT.

§ LXXIV. Men in business find it often necessary to calculate, at what *rate* they are gaining, or the contrary; that is, how great a part their profit or loss is, of the sum laid out. For this purpose, they usually calculate how many *hundredths* of the sum expended, they have lost or gained: and this number of hundredths is called the *rate per cent.* of their loss or profit. *Cent.* is a contraction from the Latin word *centum,* which means *hundred.* Rate per cent., then, means *rate by the hundred.* When we speak therefore, of a gain of 5 pr. cent. on any sum, we mean a gain of 5 *hundredths* of that sum.—Thus, 5 pr. ct. on $20, is $\frac{5}{100}$ of $20. So 8 pr. ct. on $312, is $\frac{8}{100}$ of $312, &c. Various calculations are made by *hundredths;* that is, by a rate per cent. The most important are embraced under the following heads.

PROFIT AND LOSS, which terms explain themselves.

STOCKS, which is a general name for the capitals of banking, trading and insurance companies; and for funds established by government. Persons owning stock are called STOCKHOLDERS. It sometimes happens, that *stock rises;* that is, that stockholders can dispose of their right to it, for more than was originally paid. Stocks sometimes, likewise *fall*, and then they will not command their original value. A fund is established by government, when they borrow money, on condition of paying annual *interest*, until they see fit to discharge the debt.

INSURANCE, which is an allowance made to a company, who agree to make up any loss by fire, storms, &c.

COMMISSION, which is an allowance made to a *broker* or *factor*, or an *agent* or *correspondent*, for assisting in procuring and disposing of goods.

DUTY, which is a tax, required by governments to be paid on the importation, exportation, or consumption of goods.

INTEREST, which is an allowance made for the use of borrowed money; or for debts of any kind, remaining unpaid after they are due.

DISCOUNT, which is an allowance made for the payment of money before it is due; or for advancing money on notes, bills, &c., payable at a future day.

These subjects will be severally examined.

RATE PER CENT. IS PROPERLY EXPRESSED BY A DECIMAL FRACTION. For it is always a certain number of hundredths. Thus, when we speak of 3 pr. ct., we mean 3 hundredths, =.03, and so of any other number. Thus,

1. How is 1 pr. ct. written? A. .01. 2 pr. ct.? A. .02. 3 pr. ct.? A. .03. 4 pr. ct.? 5 pr. ct.? 6 pr. ct.? 7 pr. ct.? 8 pr. ct.? 9 pr. ct.?

2. How is 10 pr. ct. written? A. .10 or .1. 11 pr. ct.? A. .11. 12 pr. ct.? 13 pr. ct.? 14 pr. ct.? 15 pr. ct.? 16 pr. ct.? 17 pr. ct.? 18 pr. ct.? 19 pr. ct.? 20 pr. ct.? 21 pr. ct.?

3. How is 99 pr. ct. written? A. .99. 100 pr. ct.? A. 1.00 or 1. 101 pr. ct.? A. 1.01. 102 pr. ct.? A. 1.02. 115 pr. ct.? A. 1.15. 125 pr. ct.? 137 pr. ct.? 149 pr. ct.? 156 pr. ct.? 181 pr. ct.?

We sometimes speak of ½ pr. ct., ¼ pr. ct. &c. These expressions signify *½ a hundredth, ¼ of a hundredth, &c.* Hence, when we speak of a *fraction pr. cent.*, we mean *a fraction of a hundredth*, and the fraction may be reduced, and the result placed next below hundredths, or, in other words, written with two cyphers on the left.

4. Express ⅛ pr. ct., decimally. ⅛=.125. Prefix two cyphers, and it becomes .00125 *Ans.*

5. Express decimally the following rates. $\frac{3}{4}$, $\frac{1}{5}$, $\frac{3}{10}$, $\frac{4}{5}$, $\frac{1}{3}$, $\frac{1}{7}$, $\frac{1}{6}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $9\frac{3}{4}$, $16\frac{1}{8}$, $7\frac{1}{4}$, $8\frac{3}{4}$, $9\frac{1}{10}$, $15\frac{1}{7}$, and $2\frac{2}{6}$ pr. ct. *Ans.* .0075, .002, .003, .008, $.00\dot{3}$, $.00\dot{1}4285\dot{7}$, $.001\dot{6}$, .015, .025, .0975, .16125, .0725, .091, .08375, $.15\dot{1}4285\dot{7}$, and $.02\dot{3}$.

6. Express $\frac{4}{5}$, $\frac{3}{5}$, $\frac{5}{6}$, $\frac{1}{9}$, $\frac{2}{17}$, $\frac{6}{13}$, $\frac{8}{11}$, $\frac{4}{7}$, $25\frac{7}{8}$, $31\frac{4}{5}$, $19\frac{1}{8}$, $1\frac{1}{11}$, $3\frac{4}{13}$, $41\frac{5}{9}$, $187\frac{3}{4}$, $563\frac{8}{13}$, $962\frac{93}{94}$.

In order to find a number of *hundredths* of any sum, we have only to multiply by the decimal expression for those hundredths.

Thus, to find 3 hundredths of $50, we multiply $50 by .03; to find 9 hundredths of $50, we multiply $50 by .09, &c. Hence, *to calculate an allowance of a certain rate per cent. on a given sum,*

MULTIPLY THE SUM BY THE RATE PER CENT. EXPRESSED DECIMALLY.

Thus, what will an allowance of 8 pr. ct. on $543 come to?

$543	On the right, I multiply by 8 pr. ct.=.08,	$543
1.08	and obtain the allowance required. On the	.08
——	left, a unit is added to the rate, and, by this	——
43.44	means the *amount* of the allowance, and of	$43.44
543	the given sum is obtained. This is often	
——	convenient.	
$586.44		

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§ LXXV. STOCK. Stocks are usually bought and sold in *shares*. These shares are some definite amount, as 50 or 100 dollars, &c. usually 100. The amount of a share is its NOMINAL VALUE. The REAL VALUE of stock is the sum for which a share will *actually sell*. When stock sells for exactly its nominal value, it is said to be AT PAR. When it will bring more in market than its nominal value, it is said to be ABOVE PAR; when not so much, BELOW PAR. When stock is above par it is said to be at so much per cent. ADVANCE.

EXAMPLES.

1. What is the value of $350.00 of stock at 105 pr. ct., that is, at 5 pr. ct. advance? A. $367.50

The rate here is 105 pr. ct.=105 *hundredths*. The question, then, is, what is 105 hundredths of 350; or, multiply 350 by 1.05.

2. What is the value of 35 hundred dollar shares of stock, at $\frac{3}{4}$ pr. ct. advance? Rate 1.0075 A. $3,526.25

3. At $112\frac{1}{2}$ pr. ct., what must I pay for $7,564.00 of stock? Rate 1.125 A. 8,509.50

4. What is the value of $615.75 of stock, at 30 pr. ct. advance? A. $800.475

5. What is the value of $7,650.00 of stock at $119\frac{1}{2}$ pr. ct.? A. $9,141.75

6. What is the value of $1,500.00 of stock at 110 pr. ct.? A. $1,650.00

7. What is the value of $2,500 of stock at 20 pr. ct. advance ?
A. $3,000.00

8. What is the value of $6,000.00 of stock at 15 pr. ct. advance ?
A. 6,900.00

9. What is the value of $9,854.25 of stock at 113 pr. ct. ?
A. $11,135.3025

NOTE. Stock is bought and sold by *even shares.* We have given some examples in which the shares are not regarded, as affording a better exercise.

10. What is the value of $823.00 of stock at 4 pr. ct. advance ?
A. $855.92

11. What is the value of $1,976.44 of stock at 111 pr. ct. ?

12. What is the value of $366,000.00 at 114 pr. ct. ?

13. What is the advance on $600.00 at 6 pr. ct. advance ?
A. $36.00

14. What is the advance on $5,460.00 at 116 pr. ct. ?

COMMISSION. 1. If my agent sells goods to the amount of $2,317.46, what is his commission at 3¼ pr. ct. ? A. $75.31745

2. What commission must be allowed for a purchase of goods to the amount of $1,286.00, at 2½ pr. ct. ? A. $32.15

3. What commission shall I allow my correspondent for buying and selling on my account, to the amount of $2,836.23 at 3 pr. ct. ?

4. A merchant paid his correspondent $25.00 commission on sales to the amount of $1,250.00. At what pr. ct. was the commission ?

He paid him $\frac{25}{1250}=\frac{1}{50}=\frac{2}{100}=.02=2$ pr. ct. *Ans.*

5. If $76.25 be paid as commission on $3,900.00, what pr. ct. commission is allowed ?

6. A factor receives $988.00 to be laid out in goods, after deducting his own commission at 4 pr. ct. What sum did he lay out for his employer ?

He evidently ought only to receive commission on the money laid out. Then, as the money laid out is $\frac{100}{100}$ of itself, and the commission $\frac{4}{100}$, the money laid out, together with the commission, must be $\frac{104}{100}$. Then, $988 is 1.04 times the answer. 988÷1.04= $950 *Ans.*

7. I remit $2,963.75 to my correspondent to be laid out for me, directing him first to deduct his own commission at 3½ pr. ct. What was his commission, and what remains to be laid out ?

INSURANCE. The written contract or obligation which binds the parties, is called POLICY. The sum paid for insurance, is called PREMIUM.

1. What premium must be paid for the insurance of a vessel and cargo, valued at $123,425.00, at 15½ pr. ct. ?

15½ pr. ct.=.155, and the question is, what is .155 of 123,425.
A. $19,130.875.

2. What must I pay annually for the insurance of a house worth $3,500.00, at 1¾ pr. ct. ? A. $61.25.

3. What must be paid for the insurance of property, at 6 pr. ct., to the amount of $2,500.00 ? A. $150.00.

4. What insurance must be paid on $375,000.00, at 5 pr. ct.? A. $18,750.00.

5. What premium must be annually paid for the insurance of a house worth $10,650.00, at 3 pr. ct.; and a store worth $15,875.00, at 4 pr. ct.; and out houses worth $3,846.00, at 5 pr. ct.?

6. What premium must be annually paid for the insurance of a Factory worth $30,946.00, at 10 pr. ct.; and 7 dwelling houses, worth $875.00 each, at 8 pr. ct.; and 3 grist mills, worth $1,930.00 apiece, at 7 pr. ct.; and 1 storing house, worth $9,859.00, at 6 pr. ct.? Also, what is the average rate of insurance on the whole?

7. If I pay $930.00 annually for insurance, at 5 pr. ct., what is the value of the property insured?

Here 930 is .05 of the answer; 930÷.05=$18,500 *Ans.*

8. If I pay $6,849.00 for insurance, at 3 pr. ct., what is the value of the property insured? A. $228,300.00.

9. If $978.00 be paid for insurance, at 2 pr. ct., what is the value of the property insured? A. $48,900.00.

10. If $765.00 be paid for insurance annually, at 5 pr. ct., what is the amount of property insured?

11. If $1,000.00 be paid, annually, for insurance on $20,000.00, what is the rate of insurance? A. 5 pr. ct.

12. If $288.00 be paid for the insurance of $48,000.00, what is the rate of insurance? A. 6 pr. ct.

13. If $975.35 be paid for insurance on $19,507.00, what is the rate of insurance?

14. If $1,200.00 be paid for insurance on $12,000.00, what is the rate pr. ct.?

PROFIT AND LOSS. 1. Sold a bale of goods at $735.00 by which I gain at the rate of 6 per cent. What sum do I gain? *Ans.* $44.10

2. In selling 50 hhds. of Molasses at 38 dollars a hhd., I gain 10 per cent. What is my gain. *Ans.* $190.00

3. In selling 25 bales of cloth, each containing 27 pieces and each piece 50 yards, a merchant gained 20 per cent. on the cost, which was 10 dollars a yard. What did he gain, and what did he sell the whole for? *Ans.* Gain $67,500.00. Whole $405,000.00

4. A merchant gained at the rate of 15 per cent, in selling the following articles: 6 hhds. of brandy for which he paid $1.50 per gal.; 7 barrels of flour, cost 11 dollars a barrel; 2 quintals of fish, cost 4 cents a pound; 16 hhds. of molasses, cost 56 cents per gal. and 25 bls. of sugar, containing each 175 lbs., cost 9 cents per lb. What was his gain on the whole, and what did he receive in all? *Ans.* Gain $241.5345. Sold for $1,851.7645.

5. A merchant purchased a large amount of cotton contained in 1,899 bales, each containing 194 lbs. at 13 cts. pr. lb.; but the price suddenly falling he was obliged to lose 8 per cent. on the cost. What did he lose, and how much did he obtain for his cotton?

6. A merchant laid in 634 pieces of broadcloth, each piece containing 85 yds., for which he paid 7 dollars per yd.; and 537 pieces of Russia sheeting, at $10.25 a piece; but they were so much in-

jured by a fire that he was willing to lose 40 per cent on the cost. What was the value of the goods after the fire ?

7. A grocer bought 7 hhds. of wine, at $154.00 a hhd. But in transportation the head of one of them was stove in, so that it was entirely lost. On this account he was willing to lose 12 per cent. on the whole. What ought he to sell the remainder at pr. gal. ?

8. A piece of cloth valued at $1,635.00, and containing 75 yds. was so much damaged by being wet, that the owner offered it at a deduction of 23 per cent. At what price pr. yd. did he offer it ?

9. A merchant in selling a quantity of goods for which he paid $230.00, made a profit of 25 dollars. At what per cent did he gain ? *Ans.* $10\frac{20}{23}$ pr. ct.

10. A grocer gained on 16 hhds. of molasses, each of which cost him 25 dollars, $20.00. At what rate pr. ct. did he gain ?
Ans. 5 pr. ct.

11. A merchant sold 15 pieces of cloth, each containing 64 yds. at 5 dollars a yd. ; also, 25 bls. of flour at 11 dollars per bl. ; also, 130 cwt. of lead, at 8 cents per lb. On the whole he gained 500 dolls. At what pr. ct. did he gain ?

12. Sold 275 bales of cotton at 11.5 cts. pr. lb., each bale containing 362 lbs. ; and 278 hhds. of brandy at $1.25 per gal. ; and 15 tons of iron at 4 cts. pr. lb. On the whole there was a gain of 628 dollars. At what pr. ct. was the gain ?

13. Sold merchandize which cost 1,856 dollars, at $2,784.00. At what pr. ct. was the gain ?
2,784−1,856=928 dolls., amount gained. *Ans.* 50 pr. ct.

14. A certain amount of merchandize which cost $2,940.00 was sold at $4,116.00. At what pr. ct. was the gain ? *Ans.* 40 pr. ct.

15. On a quantity of flour a merchant gained 575 dollars, and his gain was at 5 pr. ct. What did the flour cost him ? *Ans.* $11,500.

16. A man gained $846.00 on 8 pieces of cloth, each containing 282 yds., gaining at the rate of 6 pr. ct. What did the cloth cost him, and what did he sell it at pr. yd. ?
Ans. Cost $6.25. Sold it at $6.625.

17. A man sold 1,722 bls. of sugar, each containing 2 cwt. 2 qrs. 13 lbs. at a profit of 15 pr. ct., and his gain amounted to $6,054.552. What cost the whole, and what did he sell it for. Also, what were the buying and selling prices pr. lb. ?
Ans. Cost $40,363.68. Sold for $46,418.232. Cost pr. lb. 8 cts. Sold pr. lb. at 9.2 cts.

18. On 9,856 bu. of wheat a man gained $2,464.00, gaining at the rate of 12½ pr. ct. What was the whole cost, the cost pr. bu., the whole price of sale, and the price of sale pr. bu. ?

19. A merchant sold 279 cwt. (100 lb,) of rice, at a profit of $69.75, gaining 10 pr. ct. What cost the whole, what was the cost pr. lb., what was the whole price of sale, and the price of sale pr. lb. ?

20. A grocer sold 7,862 boxes Malaga raisins at a profit of 15 pr. ct., and his whole profit was $3,420.00. What cost the whole, and what did he gain pr. lb., supposing each box to contain 56 lbs. ?

21. A merchant sold 500 yards of cloth at $3,500.00, by which he made a gain of 16⅔ pr. ct. What did the cloth cost, and what did he gain pr. yd.? (See COMMISSION, ex. 6.)

22. A merchant sold 17 hhds. of brandy, and 25 hhds. of wine, for $4,600.00. The wine was worth pr. gal. 3 times as much as the brandy, and on the whole he gained 15 pr. ct. What were the buying and selling prices of each pr. gal.?

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§ LXXVI. INTEREST. As the calculation of interest is of great importance to all classes of persons, we propose to examine it somewhat minutely.

MENTAL EXERCISES.

1. If a dollar gain 6 cts. interest in a year, what will 2 dollars gain in the same time?

2. If a dollar gain 6 cts. interest in a year, what will 10 dollars gain in the same time?

3. If a dollar gain 6 cts. in 1 year, how much will it gain in 2 years? in 5 years? in 9 years?

4. If a dollar gain 6 cts. in 1 year, how much will it gain in 11 years? in 7 years? in 4 years? in 10 years?

5. If a dollar gain 6 cts. in a year, what will 50 cts., that is, half a dollar, gain in the same time?

6. If a dollar gain 6 cts. in 1 year, what will it gain in 6 mo., or half a year?

7. If a dollar gain 6 cts. in a year, what will 2 dollars gain in 6 months, or half a year? what will 3? 4? 7? 8?

8. If a dollar gain 6 cts. in one year, what will 2 dollars gain in 1 year and a half; that is in one year and 6 months?

9. If a dollar gain 6 cts. in a year, what will 7 dollars gain in 1 year and 6 months? what will 3? what will 4? 5? 8? 9?

10. If a dollar gain 6 cts. a year, what will it gain in 2 years and 6 months? what will 2 dollars gain in the same time?

11. What will 4 dollars gain in 4 years and 6 months, if 1 dollar gain 6 cts. a year? what will 5? 6? 7? 8? 9? 10? 11? 12?

12. If a dollar gain 5 cts. in a year, what will it gain in 2 years? what will 2 dollars gain in 2 years at that rate? what will 3?

13. If a dollar gain 7 cts. a year, what will it gain in 2 years? what will 3 dollars gain in 1 year? what will 3 dollars gain in 2 years?

14. If a dollar gain 8 cts. in one year, how much will 3 dollars gain in 5 years? 2 dollars in 3 years? 7 dollars in 2 years and 6 months?

INTEREST differs from the calculations of profit and loss, stock, and commission, in taking TIME into consideration. Thus on a note of two years' standing, twice as much interest must be paid, as upon a note of the same amount, of one years' standing. The same is true of insurance, but insurance is generally calculated for a number of *even years;* whereas, interest is estimated for *months* and *days*. A certain rate pr. ct. of interest is established by law, and this rate differs in different countries. This rate is allowed to be taken *yearly*, or, *per annum*, as it is expressed; which means, *by the year*. For *parts of a year*, as *months and days*, a proportional part of the interest per annum is allowed. The rate established by law is called LEGAL INTEREST.

In NEW ENGLAND, the legal rate is 6 pr. ct.; in NEW YORK, 7 pr. ct.; and in other states various. This treatise is equally adapted to all.

By saying that a rate of interest is established by law, we do not mean that *this* rate must always be allowed, but that *no greater rate* shall be required. Any less rate may be taken, which the parties shall agree upon. But if a greater rate be required than the *legal rate*, it is called USURY, and the person requiring it is subject to a penalty. When no mention is made of a particular rate, but the note or agreement says simply *with interest*, the legal interest is always understood, and may be collected by law.

The sum of money on which interest is calculated is called the PRINCIPAL. The sum of the principal and interest is called the AMOUNT.

YEARS. 1. What is the interest on $200.00, for one year at 6 pr. ct. pr. annum? *Ans.* $12.00.

1. What is the interest on $300.00, for one year, at 4 pr. ct. pr. annum? *Ans.* $12.00. At 5 pr. ct. pr. an.? *Ans.* $15.00. At 6? *Ans.* $18.00. At 7? *Ans.* $21.00. At 8? *Ans.* $24.00. At 9? *Ans.* $27.00.

3. What is the interest on $575.00 at 5 pr. ct. pr. an.? At 6 pr. ct.? At 7? At 8? At 9?

4. What is the interest on $834.65 at 2 pr. ct. pr. an.? At 3 pr. ct.? At 4? At 5? At 6? At 7?

NOTE. The words *per annum*, or *pr. an.* are often omitted, but they must always be understood.

When the interest is required for *two* years, you must *double the rate* before multiplying by it; when for *three*, make the rate three times as great, &c. For the interest of $100.00 at 6 pr. ct. for 2 years, is just as much as the interest of the same sum at 12 pr. ct. for one year; it being 12 dols. in each case. Hence, *to find the interest of any sum for a number of even years,*

I. MULTIPLY THE GIVEN RATE IN DECIMALS, BY THE NUMBER OF YEARS, AND THE PRINCIPAL BY THAT PRODUCT. Or,

II. MULTIPLY THE PRINCIPAL BY THE RATE AND TIME SUCCESSIVELY.

5. What is the interest of $325.00 for 3 years at 6 pr. ct.? *Ans.* $58.50.

6. What is the interest of $233.00 at 5 pr. ct. for 7 years? *Ans.* $81.55.

7. What is the interest of $647.33 at 6 pr. ct. for 8 years?

8. Find the interest on $2,345.65 at 7 pr. ct. for 9 years.

9. What is the amount of $500.00 at 6 pr. ct. for 4 years? *Ans.* $620.

NOTE. The amount may be found by adding the principal to the interest, or by adding a unit to the rate before multiplying. (§ LXXIV.)

10. What is the amount of $850.00 for 5 years at 6 pr. ct.? A. $1,105.

11. What is the amount of $1,000 at 7 pr. ct. for 8 years? A. $1,560.

12. What is the amount of $1,275.63 for 9 years at 5 pr. ct.?

13. What is the amount of $9,653.61 for 10 years, at 6 pr. ct.?

14. What is the amount of $11,943.11 for 11 years, at 8 pr. ct.?

§ LXXVII. MONTHS. It will be observed that when the time is a number of *even years*, it is as easy to calculate interest at one rate as at another; since the rule is the same for all. Whatever be the rate pr. cent., then, if the time be *even years*, let the pupil proceed as in the last section. But when there are months and days, 6 pr. ct. is the easiest rate. In the following examples then, 6 *per. ct. may be understood, unless another rate be expressed.* Particular rules will, afterwards, be given, for calculating interest at different rates.

MENTAL EXERCISES.

1. If the rate for a year (12 months) be 6 pr. ct., what ought it to be for half a year, that is for six months?

2. If the rate for 6 months be 3 pr. ct., what ought it to be for 2 months?

3. If the rate for two months be 1 pr. ct., what ought it to be for 4 months? What for 8 months? What for 10 months?

4. If the rate for 2 months be 1 pr. ct. what ought it to be for 1 month? For 3 months? For 5 months? For 7 months? For 9 months? For 11 months?

Here the pupil will observe, that the rate per ct. is always just half the number of months. The same is true if we go above a year, thus,

5. If the rate for 2 months be 1 pr. ct., what is that for 14 months? For 16 months? For 18 months? For 20 months?

6. If the rate for 1 month be ½ pr. ct., what is that for 13 months? For 15 months? For 17 months? For 19 months?

Hence, when interest is at 6 *pr. ct. pr. an.*,

THE RATE FOR ANY NUMBER OF MONTHS, IS HALF THE NUMBER OF MONTHS.

Let the following be written.

1. What is the interest on $98.75 for 2 months ?

Half of 2 months is 1 month. 1 pr. ct. then, is the rate.

98.75×.01=$0.9875 *Ans.*

2. What is the interest on $29.25, for 4 months ?

Half of 4 is 2. Therefore the rate is 2 pr. ct. *Ans.* $0.585.

3. Find the interest on $33.33 for 5 months. A. $0.83325.

NOTE. *In practice*, neglect the decimals of a mill if under .5 or ½ a mill; if equal to or over, add another mill instead of them. Or, rather, men in business usually neglect the *mills*, if under half a cent, if equal to or over, they add another cent instead of them. This is only to be done in the *final result.* If done before multiplying, it will make too great an error. In the examples, thus far given, however, the decimals have been retained, in order to insure perfect accuracy in the operations.

4. What is the interest on 72.05 for 3 months ?

Half of 3 is 1½. Therefore the rate is 1½ pr. ct.=.015. A. $1.08075.

5. What is the interest on $294.63 for 6 months ? A. $8.8389.

6. Find the interest on $765.23 for 7 months. A. $25.78305.

7. Find the interest on $895.64 for 8 months. A. $35,8256.

8. Find the interest on $934.31 for 9 months.

9. Find the interest on $1.853.63 for 10 months.

10. Find the interest on $3.293 for 11 months.

11. Find the interest on $7.86345 for 11 months.

NOTE. The pupil may, in the examples which follow, multiply by the rules given in § LXIII., Multiplication of decimals, retaining decimals only to three places, or mills. The nearest mill is usually given in the answer.

12. What is the interest of $243.23 for 14 months ? A. $17.026

13. What is the interest of $147.96 for 6 months ? A. $4.439

14. Find the interest on $15.125 for 11 months. A. $0.832

15. Find the interest on $28.14 for 5 months. A. $0.703

16. Find the interest on $284.85 for 3 months. A. $4.273

17. Find the interest on $396.27 for 7 months.

18. Find the interest on $19.395 for 9 months.

19. Find the interest on $1,288.91 for 10 months.

20. Find the interest on $2,956.84 for 11 months.

21. What is the interest on $37.00 for 2 years and 4 months ?

In this example, we may either reduce the whole time to months, and find the rate by taking half, as usual; or we may find the rates for the years and months separately, and add them together. The latter mode is best, when the number of years is great.

Rate for 2 years =2×.06=.12 ; for 4 months =.02 .12+.02=.14, rate for the whole time. *Ans.* $5.18.

22. Find the interest on $834.31 for 3 years and 8 months.

Ans. 183.5482.

23. Find the interest on $976.24 for 5 years and 7 months.

24. Find the interest on $631.874 for 9 years and 9 months.

25. What is the amount of $8,600, for 6 yrs. 6 mo. ?

26. What is the amount of $5,837.29, for 19 yrs. 11 mo. ?

§ LXXVIII. **Days.** To a pupil, the calculation of interest for *odd days*, as they are called, is usually perplexing. We hope to make it clear. In calculating interest, 30 days are reckoned to the month, and 12 months to the year. This is slightly erroneous, but custom has rendered it almost universal. It will be seen, that this mode of allowance makes the year to consist of only 360 days. For ordinary purposes, however, it is sufficiently accurate; and its convenience renders it general.

MENTAL EXERCISES.

1. If the rate for 2 months be 1 pr. ct., that is, if the rate for 60 days be 1 pr. ct., what is that for 6 days?

NOTE. 6 days$=60\div10=\frac{1}{10}$ of 60 days. A. $\frac{1}{10}$ pr. ct.

2. If the rate for 6 days be $\frac{1}{10}$ pr. ct., what is that for 12 days? For 18 days? For 24 days? For 30 days? For 36 days? For 42 days? For 48 days? For 54 days.

3. What is the rate pr. ct. for 2 months and 6 days?

NOTE. For 2 months, it is 1 pr. ct., and for 6 days $\frac{1}{10}$ pr. ct. *Ans.* $1\frac{1}{10}$ pr. ct.

4. What is the rate for 2 months, 12 days? For 2 mo. 18 d.? For 2 mo. 24 d.? For 2 mo. 36 d.? For 4 mo. 18 d.? For 4 mo. 24 d.?

5. What is the rate for 6 mo. 6 d.? For 6 mo. 12 d.? For 8 mo. 18 d.?

6. What is the rate for 8 mo. 6 d.? For 12 mo. 18 d.? For 1 yr. 24 d.?

It will be observed, that for *once* 6 days we have $\frac{1}{10}$ pr. ct.; for *twice* 6 days, we have $\frac{2}{10}$ pr. ct.; for *three times* 6 days, we have $\frac{3}{10}$ pr. ct. &c. Now $\frac{1}{10}$ pr. ct. is .001; (§ LXXIV.) $\frac{2}{10}$ is .002; $\frac{3}{10}$ is .003, and so on.

Hence, *for every 6 days we have one thousandth in the rate.*

If there be fewer days than 6, as 3 days, we shall have a part of a thousandth, that is, in this case, $\frac{3}{6}=\frac{1}{2}$ a thousandth $=.0005$. If the days be 2, we shall have $\frac{2}{6}=\frac{1}{3}$ of a thousandth; $=.000\dot{3}$; if 4, $\frac{4}{6}=\frac{2}{3}$ of a thousandth $=.000\dot{6}$; if 5, $\frac{5}{6}$ of a thousandth $=.0008\dot{3}$; if 1, $\frac{1}{6}$ of a thousandth $=.0001\dot{6}$. Hence, to find the rate decimally for any number of days.

DIVIDE THE DAYS BY 6, ANNEXING CYPHERS, IF NECESSARY, AND PUT THE FIRST FIGURE OF THE QUOTIENT IN THE THOUSANDTHS' PLACE, WHEN

THE DAYS ARE 6 OR MORE, IN THE TEN THOUSANDTHS' WHEN THEY ARE UNDER 6.

Thus, for the rate for 23 days; $23 \div 6 = 38\dot{3}$. As the days are over 6, put the first figure 3 in the thousandths' place, thus, $.0038\dot{3}$. The rate for two days is found, thus; $2 \div 6 = \dot{3}$. As the days here are under 6, the quotient must begin in the ten thousandths' place, thus $.000\dot{3}$. In these cases the cyphers, though not written, were supposed to be annexed.

EXAMPLES FOR PRACTICE.

1. What is the interest of $100.00 for 2 mo. 6 d.? A. $1.10.

Rate for 2 mo.=.01 For 6 d.=.001. Rate for the whole time =.011.

2. Find the interest on $600.00 for 4 mo. 18d. A. $13.80.

Rate for 4 mo.=.02. For 18 d.=.003. Rate for whole time =.023.

3. Find the interest of $800.00 for 6 mo. 21 d. A. $26.80.

Rate for 6 mo.=.03. For 21 d.=.0035. Rate for whole time =.0335.

4. Find the interest on $9,000.00 for 8 mo. 4 d. A. $366.00.

Rate for 8 mo.=.04. For 4 d.=$.000\dot{6}$. For whole time=$.040\dot{6}$.

5. Find the interest on $127.47 for 2 mo. 12 d. A. $1.53—

6. Find the interest on $115.42 for 7 mo. 15 d. A. $4.328+

7. Find the interest on $143.18 for 1 yr. 7 mo. 14 d. A $13.936—

Rate for 1 yr.=.06. For 7 mo.=.035. For 14 d.=$.002\dot{3}$. For whole time=$.097\dot{3}$.

NOTE. We would here mention, that when the rate repeats, or is a long decimal, the vulgar Fraction may be retained. This will often be most convenient, and convenience should decide the question. Thus, instead of using the decimal $.097\dot{3}$, we would recommend the use of .097⅓. This is likewise perfectly *accurate*, whereas, when the repetend is used, *perfect* accuracy cannot be obtained, by the common mode of multiplying.

8. Find the interest on $625.00, for 3 yrs. 2 mo. 7 d.

A. 119.479⅙.

Rate for 3 yrs. =.18. For 2 mo. =.01. For 7 d. =.001⅙. For whole time =.191⅙.

9. Find the interest on $930.00 for 6 yrs. 3 mo. 11 d.

***Ans.* $350.455.**

10. Find the interest on $865.25 for 9 yrs. 4 mo. 15 d.

NOTE. When the months are even, and the days are an *aliquot*, or an *even* part of 2 months or 60 days, it is well to make a vulgar Fraction for the days instead of a decimal. For example, if, as in the last question, there are 15 days, this will be ¼ of 60. Now the rate for 60 d. or 2 mo. is .01. Therefore the rate for 15 d. is ¼ of .01=.0¼. This will often save figures. Then, in the last question, the rate for 9 yrs.=.54, for 4 mo.=.02, for 15 d.=.0¼, for the whole .56¼. *Ans.* $486.703⅛.

11. Find the interest of $750.00 for 12 yrs. 3 mo.

NOTE. The learner will readily see that an odd month, being ½ of 2 months, may have .0½ as its rate. Then, in the last example, rate for 12 yrs.=.72, for 3 mo.=.01½, whole rate =.73½. *Ans.* $551.25.

These abbreviations are only suggested, and if they do not seem clear, the pupil is recommended to follow the general mode. Other vulgar Fractions may sometimes be conveniently used, which we leave the pupil to discover. Hence, the general rule for calculating interest seems to be—

FIND THE DECIMAL RATES FOR YEARS, MONTHS AND DAYS SEPARATELY, ADD THEM TOGETHER, AND MULTIPLY THE PRINCIPAL BY THEIR SUM.

12. Find the interest on \$725.34 for 16 yrs. 6 mo. 12 d.

NOTE. 12 d. is $\frac{1}{5}$ of 60 d. Hence the rate is $.99\frac{1}{5}$.

13. Find the interest on \$348.31 for 11 yrs. 8 mo. 10 d.

14. Find the interest on \$795.333 for 13 yrs. 9 mo.

15. Find the interest on \$5,863.63 for 7 yrs. 11 mo.

16. Find the interest on \$325,965.813 for 15 yrs. 8 mo. 20 d.

17. A note was given *with interest*, for \$2,635.00, on the 17th of February, 1827, and paid on the 12th of March, 1830. What was the interest due, and what was the amount of the note ?

A. Int. $\$485.279\frac{1}{6}$. Am. $\$3,120.279\frac{1}{6}$.

NOTE. In this example we are obliged to find the time. In business, we are almost always under the necessity of doing this. It may be done by Subtraction of Compound numbers, (§XXXVII.) The greater number consists of 1830 years, 2 months, (Jan. and Feb.) and 12 days, (in March.) The less, consists of 1827 years, 1 month, (Jan.) and 17 days, (in Feb.) Set down the numbers, thus,

Yrs.	mo.	d.
1830 ;	2 ;	12
1827 ;	1 ;	17
3 ;	0 ;	25

In subtracting, disregard entirely the inequality of the months, and call them all 30 days, allowing 12 months to the year. If interest be required for *days exactly*, the inequality of months must be taken into account, (see § LXXIX.)

18. A note was given with interest, for \$1,143.16 on the 9th of August, 1826, and paid on the 16th of April, 1830. What was the amount of the note, when paid ? A. \$1,395.989—

19. A note was given, with interest, for \$6,325.13 on the 8th of October, 1823, and paid on the 11th of August, 1829. What was then due ?

20. A note was given, with interest, for \$2,647.53, on the 27th of December, 1821, and paid on the 15th of November, 1828. What was then due ?

21. A note was given for \$15,833.75, on the 8th of July, 1819, and paid, with interest, on the 11th of June, 1827. What was then due ?

22. A note was given on the 23d of March, 1813, and paid, with interest, on the 17th of September, 1830. The note was for \$12,981 .375. What was due at the time of payment ?

23. A note continued accumulating interest for 50 years, 9 mo. 3 d. It was given for \$25,864.29. What was the interest ? What was the amount ?

24. A note of \$30,329.18, lay on interest from July 19th, 1763, to August 17th, 1829. What did it amount to ?

◆

§ LXXIX. In BANKS, notes are usually written for a certain *number of days*, as 30 days, 60 days, 95 days, &c. Sometimes, also, but not in Banks, interest is calculated for years and days, without regard to months.

The interest *accurately* for 1 day is, of course, $\frac{1}{365}$ of the interest for 1 year. For 2 days, then, it is $\frac{2}{365}$; for 3 days, $\frac{3}{365}$; for 47 days, $\frac{47}{365}$ of the interest for 1 year, and so on.

Hence, to find the interest accurately for any number of days:

FIND WHAT FRACTIONAL PART OF 365 THE DAYS ARE, AND MULTIPLY THE INTEREST FOR 1 YEAR BY IT.

NOTE. Reduce the Fraction to its lowest terms, before multiplying. This method is so tedious that it is little used. The common mode is usually as accurate as is necessary.

1. Find the interest on $200.00 for 73 days.

$\frac{73}{365}=\frac{1}{5}$. $200\times.06=\$12.00$. $\$12.00\times\frac{1}{5}=\$2.40$ *Ans.*

Ans. By com. meth. $2.433⅓.

2. Find the interest on 600.00 for 55 days.

NOTE. $\frac{55}{365}=\frac{11}{73}$. A. $\$5.424\frac{48}{73}$.

Ans. By com. meth. $5.50.

3. A note lay at interest from the 15th Jan. 1824, to the 19th April, 1827. It was given for $1,025.67. What was its interest at the time of settlement?

A. $\$200.12662\frac{30}{73}$.

Ans. By com. meth. $200.68943.

NOTE. The error of the common mode was here greater on account of the intervention of February.

4. Find by both modes, the amount of $943.61 for 5 years, 47 d.
5. Find the interest, by both modes, on $1,864.00 for 6 yrs. 125 d.
6. What was the amount of a note for $7,684.33½, which had been lying on interest from Dec. 19th, 1811, to May 27th, 1825, by both modes?

◆

§ LXXX. We mentioned (§LXXVII.) that it was easier to find interest at 6 pr. ct. than at any other rate. The pupil is now prepared to understand the reason of this; which is, that 6 is just half the number of months in the year. To find interest at any other rate, for months and days, let it be remembered, that

1 pr. ct. is $\frac{1}{6}$ of 6 pr. ct.
2 pr. ct. is $\frac{2}{6}=\frac{1}{3}$ of 6.
3 pr. ct. is $\frac{3}{6}=\frac{1}{2}$ of 6.
4 pr. ct. is $\frac{4}{6}=\frac{2}{3}$ of 6.
5 pr. ct. is $\frac{5}{6}$ of 6.

7 pr. ct. is $\frac{7}{6}=1\frac{1}{4}$ times 6.
8 pr. ct. is $\frac{8}{6}=\frac{4}{3}=1\frac{1}{3}$ " 6.
9 pr. ct. is $\frac{9}{6}=\frac{3}{2}=1\frac{1}{2}$ " 6.
10 pr. ct. is $\frac{10}{6}=\frac{5}{3}=1\frac{2}{3}$ " 6.
11 pr. ct. is $\frac{11}{6}=1\frac{5}{6}$ " 6.

In this manner, may be found what part any rate is of 6 pr. ct.

Hence, we have a universal rule for interest at *any rate pr. ct.*

1. FIND THE RATE FOR THE WHOLE TIME AT 6 PR. CT., TAKE SUCH PART OF THIS AS THE GIVEN RATE IS OF 6 PR. CT., AND MULTIPLY THE GIVEN PRINCIPAL BY IT. Or the following:

II. FIND THE INTEREST AT 6 PR. CT., AND TAKE SUCH PART OF IT AS THE GIVEN RATE IS OF 6 PR. CT.

We leave abbreviations to the ingenuity of the pupil.

EXAMPLES FOR PRACTICE.

1. What is the interest on $29.25 for 4 months, at 3 pr. ct.?

NOTE. 3 pr. ct. is ½ of 6 pr. ct. Therefore, find the interest at 6 pr. ct. and multiply it by ½. A. $0.292½.

2. Find the interest for 3 months on $72.05, at 2 pr. ct. A. $0.360¼.

3. Find the interest on $600.00, at 4 pr. ct., for 4 months and 18 days? A. $9.20.

4. Find the interest on $9,000.00, at 5 pr. ct., for 8 mo. 4 d.? A. $305.00.

5. Find the interest on $800.00 for 6 mo. 21 d., at 7 pr. ct.? A. $31.266⅔.

6. Find the interest on $1,000.00, at 8 pr. ct., for 3 yrs. 6 mo. 9 d. A. $281.00.

7. Find the interest on $1,200.00, for 1 yr. 3 mo. 15 d. at 9 pr. ct. A. $139.50.

8. Find the interest on $625.00, at 10 pr. ct., for 3 yrs. 2 mo. 7 d. A. $199.131⅙.

9. Find the interest on $930.00, for 6 yrs. 3 mo. 11 d., at 11 pr. ct. A. $642.500⅚.

10. Find the interest on $780.00, for 9 yrs. 7 mo. 16 d. at 12 pr. ct.

11. Find the interest on $1,825.25, for 6 yrs. 8 mo. 13 d., at 13 pr. ct.

12. Find the interest on $3,976.18, for 2 yrs. 4 mo. 8 d., at 8 pr. ct.

13. Find the interest on $1,964.43, for 9 yrs. 5 mo. 13 d. at 7 pr. ct.

14. Find the interest on $2,675.33, for 7 yrs. 7 mo. 7 d. at 5 pr. ct.

15. Find the interest on $3,865.49, for 12 yrs. 3 mo. 5 d. at 7 pr. ct.

Besides the modes given above, there is another, which is often convenient. We have seen how to compute the interest for *even years* at *any rate.* (§ LXXVI.) A rate *for the whole time* is found, by multiplying the rate *pr. an.* by the number of years. Now if we reduce *months and days* to decimals *of a year*, we can treat them in the same way. Thus,

16. Find the interest on $100.00, for 2 yrs. 6 mo., at 7 pr. ct. A. $17.50.

2 yrs. 6 mo.=2½ yrs.=2.5 yrs. Then, 2.5×.07=.175 rate for whole time.

17. Find the interest on $700.00, for 3 yrs. 2 mo. 12 d., at 5 pr. ct. A. $112.

The time is 3.2 yrs. Therefore, the rate for whole time is 3.2×.05=.16.

Hence, the rule is

III. MULTIPLY THE RATE PER CENT. PER ANNUM BY THE TIME IN YEARS AND DECIMALS OF A YEAR; THE PRODUCT WILL BE THE RATE FOR THE WHOLE TIME, WITH WHICH PROCEED AS USUAL. Or,

IV. MULTIPLY THE PRINCIPAL SUCCESSIVELY BY THE TIME AND RATE PER CENT. PER ANNUM. THE LAST PRODUCT IS THE INTEREST REQUIRED.

Some rules for finding the decimal of time, may be found in § LXXXI.

18. Find the interest on $1,875.31, for 7 yrs. 6 mo. 4 d., at 8 pr. ct.

19. A note for $2,633.75, was at interest at 7 pr. ct. from the 28th of September, 1809, to the 15th of March, 1829. What was then due?

VARIOUS CALCULATIONS IN INTEREST.

LXXXI. ART. I. When the TIME, RATE, and INTEREST are given, to find the PRINCIPAL.

When no particular rate is mentioned, 6 pr. ct. must be understood, *throughout* these exercises.

1. What principal will gain 60 dolls. in 2 years?

If the rate be 6 pr. ct. *pr. an.* it is 12 pr. ct. *for two years.* The question then is, 60 is 12 pr. ct. on what principal? *Ans.* $500.00.

2. What principal will acquire $459.45 in 25 years 6 mo. 9 d.?
The rate for the whole time is 1.5315. *Ans.* $300.00.

3. What principal will acquire $934.20 in 17 yrs. 3 mo. 18 d.?
Ans. $900.00.

4. What principal will give an interest of $72.25 in 3 yrs. 5 mo. 25 d.?

5. What principal will gain $965.30 in 18 yrs. 9 mo. 3 d.?

6. What principal will acquire $1,000.00 in 16 yrs. 8 mo.?

Hence, the rule is,

FIND THE DECIMAL RATE FOR THE WHOLE TIME, AND DIVIDE THE GIVEN INTEREST BY IT.

NOTE.—For any other than 6 pr. ct. find the rate for the whole time by decimals. § LXXX, Rule III. Or find the rate at 6 pr. ct., and take such part of this rate as the given rate may be of 6 pr. ct.

7. A note lay 12 yrs. 3 mo. 20 d. and at the end of the time there was an interest on it of $350.27. What was the principal of the note?

8. A note was given April 29th, 1820, and settled Nov. 13th, 1826. At that time the interest was $540.00. What was the principal of the note?

ART. II. When the PRINCIPAL, INTEREST, and TIME are given, to find the RATE.

1 A man has $2,000.00 at interest, and at the end of 3 yrs. the interest is $300. What pr. ct. pr. an. does he receive?

If the rate had been 1 pr. ct. pr. an., the interest on $2,000.00 would in three years have been 2,000×.03=$60.00. As often, then, as $60.00 is contained in the actual interest $300.00, so many times 1 pr. ct. will there be in the rate required.

$300÷60=5 pr. ct. the rate required.

2. $720.00 lay at interest 7 yrs. 2 mo. and 12 d. The interest then was $311.04. What was the rate?

At 1 pr. ct. the principal would have gained $51.84. 311.04÷51.84=6 pr. ct. *Ans.*

3 $300.00 in 2 years gained $42.00. What was the rate of interest allowed? *Ans.* 7 pr. ct.

4. $960.00 gained $180 in 2 yrs. 6 mo. 23 d. What rate of interest was allowed?

Hence, the rule seems to be,

DIVIDE THE GIVEN INTEREST BY THE INTEREST ON THE SAME PRINCIPAL, FOR THE SAME TIME, AT 1 PER CENT.

4. $750.00 in 12 yrs. 3 mo. gained $551.25. What was the rate pr. ct. pr. an.? *Ans.* 6 pr. ct.

5. A note given for $2,365.00, on the 19th of June 1826, was paid on the 14th of July 1829. The interest then due was 435.554\frac{1}{4}$. What rate was allowed? *Ans.* 6 pr. ct.

6. $20,000.00 lay 73 years at interest. The interest then was $58,400.00. What was the rate allowed?

A different mode from the last may be proposed, which in many instances will be better. Since $58,400.00 is the interest on $20,000.00 *for the whole time*, we may find the *rate for the whole time*, thus: $\frac{58400}{20000}$=2.92=rate pr. ct. for 73 yrs. If this be divided by the number of years, we shall have the rate for 1 year.

2.92÷73=4 pr. ct. the rate required. This saves us the trouble of calculating the interest at 1 pr. ct. for a great number of *years*. It is likewise convenient for shorter periods.

7. $300.00 was at interest 5 years, when the interest was $45.00. Required the rate pr. ct. pr. an.

$\frac{45}{300}=\frac{15}{100}$=.15 rate for 5 years. This is 3 pr. ct. for 1 yr. *Ans.*

8. The interest on $761.00 was $98.93 for 2 yrs. 7 mo. 6 d. Required the rate pr. ct. pr. an.

$\frac{9893}{76100}$=.13 rate for whole time. But the time is not *even years*. Therefore we must reduce the months and days to a decimal before dividing. 2 yrs. 7 mo. 6 d.=2.6 (§ LX.)

.13÷2.6=.05=5 pr. ct. *Ans.*

9. A note for $930.25 was given on the 11th March 1823, and on the 29th of May, 1824, its interest was $111.63. What was the rate? *Ans.* 10 pr. ct.

Hence, we have the rule,

II. FIND THE RATE FOR THE WHOLE TIME AND DIVIDE IT BY THE TIME IN YEARS AND DECIMALS.

In determining which rule to use, the judgment of the pupil must be exercised. Some questions are solved by the last, much more expeditiously, than by the other; and, on the other hand the first is often the most convenient. Some concise modes of finding the decimal of a year may be given. In calculating interest we allow 30 days to the month, and 12 months to the year; making 360 days in a year. 1 tenth or .1 of a year, then, is 36 days. To reduce months and days to the decimal of a year, therefore,

BRING THE WHOLE TO DAYS AND DIVIDE BY 36, ANNEXING CYPHERS IF NECESSARY. IF THE DAYS ARE 36 OR MORE, THE FIRST QUOTIENT FIGURE

WILL BE TENTHS; IF NOT, IT WILL BE HUNDREDTHS, OR THOUSANDTHS, ACCORDING AS ONE OR TWO CYPHERS ARE ANNEXED, TO OBTAIN IT.

Thus, 3 mo. 24 d.=114 d. $114 \div 36 = 31\dot{6}$ of a year.

9 d.÷36=.025 of a year. 2 d.÷36=$.00\dot{5}$ of a year.

Another mode may be given for months. 6 months is $\frac{1}{2}$=.5 of a year. Then, .1 of a year is six fifths of a month.

Hence, to reduce months to the decimal of a year,

DIVIDE THE NUMBER OF MONTHS BY SIX FIFTHS ANNEXING CYPHERS IF NECESSARY. THE FIRST QUOTIENT FIGURE, IF FOUND WITHOUT ANNEXING A CYPHER, IS TENTHS; IF A CYPHER IS ANNEXED TO OBTAIN IT, HUNDREDTHS.

Days, with or without months, may be made Fractions of a month, vulgar, or decimal, and treated in the same manner. Or, the decimal for the months may be found by this rule, and that for the odd days, by the last, and the two results added. Thus,

7 mo.÷$\frac{6}{5}$=$.58\dot{3}$ of a year. 1 mo.÷$\frac{6}{5}$=$.08\dot{3}$ of a year.

9 mo. 6 d.=9.2 mo.÷$\frac{6}{5}$=$.7\dot{6}$ of a year. 9 d.=.3 mo.÷$\frac{6}{5}$=.025 of a year.

10. A note was given on the 7th Jan. 1823, for $375.50 and paid on the 16th Oct. 1824, at which time the interest was $53.321. What rate was allowed? *Ans.* 8 pr. ct.

11. A note was given on the 4th March 1818, for $2,000.00, and paid on the 16th May 1829, when the interest was $1,120.00. What rate pr. ct. pr. an. was allowed?

12. A note was given on the 17th June 1827, for $2,000.00, and paid on the 17th Dec. 1829, when the interest was $200.00. What rate was allowed?

ART. III. When the PRINCIPAL, RATE, and INTEREST are given to find the TIME.

1. $300.00 gains $54.00. How long has it been at interest?

In one year $300.00 will gain $18.00. It will gain $54.00 then in as many years, as there are 18s in 54. 54÷18=3 years. *Ans.*

2. $1,000.00 gains $300.00 at 5 pr. ct. How long has it been at interest?

In one year $1000.00 will gain $50.00. 300÷50=6 yrs. *Ans.*

3. How long will it take $500.00 to gain $350.00, at 7 pr. ct.?

Hence, the rule is,

I. DIVIDE THE GIVEN INTEREST, BY THE INTEREST ON THE SAME PRINCIPAL, FOR 1 YEAR.

NOTE.—When a Fraction is obtained by this division, it must be reduced to months and days.

4. How long will $2,000.00 be in gaining $414.00?

A. 3 yrs. 5 mo. 12 d.

5. How long will $200.00 be in gaining $36.00?

A different mode from the last may be employed. Let the rate be found *for the whole time*, as in ART. II. Thus, $\frac{36}{200}=\frac{18}{100}$=.18. Now as .18 is the rate for the *whole time*, if I divide by the rate for 1 year, I shall find the number of years. .18÷.06=3. *Ans.* 3 yrs.

Hence, we have another rule.

II. FIND THE RATE FOR THE WHOLE TIME, AND DIVIDE IT BY THE RATE GIVEN.

Fractions, as before, must be reduced to months and days.

6. How long will $600.00 be in gaining $30.00? *Ans.* 10 mo.

7. A note was given on the 11th of Jan. 1819, for $754.50, and when it was paid the interest was $58.866½. On what day was it paid?

IV. When the AMOUNT, RATE, and TIME are given, to find the PRINCIPAL.

1. A sum of money has been on interest 2 years, and it amounts to $112.00. What is the sum?

6 pr. ct. for 1 year is 12 pr. ct. for 2 years. The question then is, what principal will amount to $112.00 at 12 pr. ct.

In $112 is contained once the principal, and .12 of it besides. To the rate .12 then, if I add a unit, making 1.12, this number will express how often the principal is contained in the amount. Of course I must divide the amount by it. $112÷1.12=$100. *Ans.*

2. A sum of money in 3 yrs. 9 mo. 12 d. amounts to $1,227.00. What is the sum? *Ans.* $1,000.00.

3. A note in 2 yrs. amounted to $275.00 at 5 pr. ct. What was the principal of the note?

Hence, the rule,

FIND THE RATE FOR THE WHOLE TIME, ADD TO IT A UNIT, AND DIVIDE THE GIVEN AMOUNT BY THE SUM.

NOTE. For any other than 6 pr. ct. the rate for the whole time may be found by decimals. § LXXX. Rule III. Or the rate at 6 pr. ct. may be found, and such part of it taken, as the given rate is of 6 pr. ct. The interest may be found by subtracting the principal from the amount.

4. An amount for 3 yrs. 6 mo. 15 d. was $2,515.93¾. What was the principal? *Ans.* 2,075.00

5. A note was given on the 10th Sept. 1825, and paid on the 17th May 1829, at which time $1,375.98883⅓ was due. For what sum was the note given?

6. A note was given for 6 yrs. 8 mo. 27 d. with interest. When paid, it amounted to $30,628.33. What was the principal?

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§ LXXXII. DISCOUNT. 1. I have a note of $318.00 due me one year from the present date without interest; but my debtor is willing to pay me now, if I will allow discount at 6 pr. ct. I agree to this, and he pays me. What sum do I receive?

The pupil should consider that I have $318.00 due me at the end of the year, and *not before*. Of course it makes no difference to me, whether my note is $318.00 exactly, without interest, or whether it is for some other sum, which will amount to $318.00 at the end of the year. For, in either case, I receive, actually, the same amount, at the same time. Hence, the sum which my debtor pays me ought to be such as would amount, at interest, to $318.00, at the end of the year. The question then is; what principal will amount to $318.00 in one year. It therefore belongs to § LXXXI. ART. IV.

Ans. $300.00.

This $300.00 which my debtor should fairly pay me, at the present time, instead of the whole debt at a future day, is called the PRESENT WORTH of the debt. Then, when money is due, at a future day, without interest,

THE SUM WHICH, AT INTEREST, FOR THE GIVEN RATE AND TIME, WOULD AMOUNT TO THE SUM THEN DUE, IS CALLED THE PRESENT WORTH OF THAT SUM.

The DISCOUNT on the debt, or the deduction made, on account of present payment, then, is the interest on the *present worth*, and not on the whole debt, for the given time. Therefore,

The PRESENT WORTH may be considered as a PRINCIPAL, the DISCOUNT, the INTEREST, for the given time, on that principal, and the DEBT itself, the AMOUNT.

2. In 2 yrs. 6 mo. I have $1,000.00 due me. What is the present worth of the debt? A. $850.00.

3. What is the present worth of $725.25 for 3 yrs. 5 mo. 8 d.?

Hence, the rule, *to find the present worth*,

FIND THE RATE FOR THE WHOLE TIME, ADD TO IT A UNIT, AND DIVIDE THE GIVEN SUM BY IT. *The* DISCOUNT *will be found by subtracting the present worth from the whole debt.*

NOTE. For any other rate than 6 pr. ct. it is best to use decimals § LXXX. Rule III. Or we may find the rate for the whole time at 6 pr. ct. and take such part of this rate, as the given rate may be of 6 pr. ct. by the rules in the same §.

NOTE. A decimal *multiplier*, for calculating discount, may be found thus. Any sum is $\frac{100}{106}$ of its amount for 1 year, at 6 pr. ct. And its interest, for the same time, is $\frac{6}{106}$ of the same amount. Hence, the present worth of a sum, for 1 year, at 6 pr. ct., is $\frac{100}{106}$, and the discount, for the same time, $\frac{6}{106}$ of the sum itself. $\frac{100}{106}$, then, reduced to a decimal, will give a multiplier for finding the present worth; and $\frac{6}{106}$ reduced to a decimal will give a multiplier for finding discount. For any other rate and time, the process is similar. When the decimal multiplier is obtained, discount may be calculated in the same manner as interest.

4. What is the present worth of $600.00 due 2 yrs. 6 mo. hence at 5 pr. ct. discount?

2 yrs. 6 mo.=2.5. yrs. 2.5×.05=.125 rate for whole time.

Or 15=rate for whole time at 6 pr. ct. (§ LXXVIII.) 5 pr. ct. is $\frac{5}{6}$ of 6 pr. ct. .15×$\frac{5}{6}$=.125 and 600÷1.125=$533.33\dot{3}$ *Ans.*

5. What is the discount of $100.00 for 1 year? A. 5.66$\frac{2}{53}$.

.06=rate for 1 yr. Then 1+.06=1.06 and 100÷1.06=93.33$\frac{51}{53}$.

6. What is the difference between the discount of \$100.00 for 1 year, and the interest of the same sum, for the same time?

Ans. \$0.339$\frac{33}{53}$.

7. A note was given without interest for 13 months, but at the end of 5 months the debtor paid the note at a discount of 6 pr. ct. How much did he pay, the note being for \$847.00. A. \$822.33$\frac{1}{103}$.

8. I pay a debt of \$394.00 1 year and 8 mo. before it is due. What ought I to pay? A. \$358.181$\frac{9}{11}$.

9. If a debt of \$137.90 be paid 2 yrs. 6 mo. before it is due what ought to be paid? A. \$114.16$\dot{6}$.

10. What is the difference between the discount and the interest of \$500.00 for 4 yrs.? A. 23.225$\frac{25}{31}$.

11. What is the discount on \$100.00 for 1 yr. 6 mo. at 5 pr. ct.? Rate at 6 pr. ct.=.09$\frac{5}{6}$ of .09 (§ LXXX.)=.075 rate at 5 pr. ct.

A. \$93.023$\frac{11}{43}$.

12. What is the present worth of \$227.26 for 4 yrs. 6 mo. at 7 pr. ct. A. \$172.821$\frac{77}{263}$.

13. What is the present worth of \$1,000.00. for 9 yrs., 9 mo., and 18 d., at 9 pr. ct.?

14. What is the present worth of \$7,963.88 for 13 yrs. 7 mo. 6 d. at 4 pr. ct.?

15. What is the present worth of \$1,759.40, for 10 yrs. at 11 pr. ct.?

The principles of discount, as we have exhibited them, have their foundation in common sense. It is perfectly manifest, that the discount on a sum of money ought never to be equal to the interest on the same sum for the same time. For the discount is the interest on the *present worth*, which is, of course, less than the sum itself. Yet, it is customary, in banks, to take the *full interest* on a note in discounting. This is evidently *usury*. For example, if I present a note for \$100.00, due 6 months hence, for discount, the bank allows me only \$97.00 taking \$3.00, the full interest on the note for 6 months, as discount. Now if I put \$97.00 at interest for 6 months, it only amounts to \$99.91, i. e. 9 cts. less than the note. If the time were a year, I should receive only \$94.00, 6 dollars, being reserved for discount. \$94.00 in a year, amounts to \$99.64, being 36 cents less than the note. If it were 2 years, the error would be \$1.44, and so on, increasing as the *square* of the time. [See § XCVIII. for the meaning of the word *square*.] Banks do not, however usually discount for so long a period. For \$1,000.00, this last error would become \$14.40. For \$100,000.00, it would be \$1,440.00.

When discounts are made for 6 months in this manner, the rate per cent. is actually, 6$\frac{18}{97}$=more than 6$\frac{2}{11}$ pr. ct. When for 1 year, the rate is 6$\frac{18}{47}$ pr. ct.=more than 6$\frac{1}{3}$, and almost 6$\frac{2}{5}$ pr. ct. When for 2 years, the rate is 6$\frac{18}{22}$ pr. ct.=more than 6$\frac{3}{4}$, and almost 6$\frac{1}{2}$ pr. ct.

Yet, notwithstanding these facts, the supreme courts have decided, that this mode of calculating discount *is not usury*. Banks are, therefore, authorized by judicial authority to discount at a higher rate than that established by law. We know not on what principles this decision has been made.

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§ LXXXIII. It will often be found convenient, even when the rate is 6 pr. ct. to find the interest on a sum for 12 pr. ct. pr. an., and to take such part of the interest, thus found, as the given rate may be of 12 pr. ct. The reason of this is, that there are 12 months in the year, and therefore the rate will be 1 pr. ct. a month. In finding the rate for odd numbers of months, at 6 pr. ct., we get two figures; one in the hundredths', and one in the thousandths' place. Thus for 7 months, at 6 pr. ct., the rate is .035; for 9 months, .045, &c. But at 12 pr. ct. these same rates will be .07 and .09. Thus, in the latter case, we have but one figure in the multiplier.

1. Find the interest on $962.54 for 9 mo. at 6 pr. ct.

.09=rate at 12 pr. ct. .09×962.54=86.6286. 86.6286÷2= 43.3143 *Ans.*

2. Find the interest on $862.00 for 7 mo. at 4 pr. ct.

.07=rate at 12 pr. ct. .07×862=60.34 60.34÷3=20.11$\dot{3}$ *Ans.*

3. Find the interest on $365.25 at 3 pr. ct. for 5 months.

In many cases where days occur, we obtain, by this method, a similar advantage.

4. Find the interest on $324.00 for 7 mo. 15 d.

15 days are half a month. At 12 pr. ct., then, the rate is .075. It would be .0375 at 6 pr. ct., which is not so convenient, .075×324 =24.30 24.30÷2=$12.15 *Ans.*

5. Find the interest on $800.00, at 8 pr. ct. for 5 mo. 21 d.

The rate at 12 pr. ct. is .057 .057×800=45.60×⅔=$30.40 *Ans.*

6. Find the interest on $9,823, for 11 mo. 27 d. at 9 pr. ct.

7. Find the interest on $8,763.25 at 2 pr. ct. for 3 mo. 9 d.

8. Find the interest on $11,864 for 13 mo. 3 d.

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§ LXXXIV. In calculating interest on Sterling Money, or upon the U. S. currencies, it is best to reduce the lower denominations to decimals of a pound. For this purpose, it is sufficiently correct, and much the most convenient mode to employ the contraction in § LX. After the calculation is made, reduce the resulting decimal to shillings, &c. again, by the contraction § LXIV.

1. Find the interest on £13; 3; 6 for 1 year. A. 15s. 9d. 2 qrs.
2. Find the interest on 13£ 15s. 3d. 2 qrs. for 1 yr. 6 mo. A. £1; 4; 9; 1.
3. Find the interest on £75; 8; 4 for 5 yrs. 2 mo. A £23; 7; 7.
4. Find the interest on £174; 10; 6 for 3 yrs. 6 mo. A. £36; 13.
5. Find the interest on £325; 12; 3 for 5 yrs. A. £97; 13; 8.
6. Find the interest on £150; 16; 8 for 4 yrs. 7 mo. A. £41; 9; 7.
7. Find the amount of £3,000, for 12 yrs. 10 mo. A. £5,310.
8. Find the amount of £279; 13; 8 for 3 yrs. 6 mo. at 5¼ per ct. A. £331; 1; 6.

9. Find the interest on £137 ; 17 ; 2 from Jan. 11th, 1822, to Aug. 15th, 1822. A. £4 ; 18 ; 4.

10. Find the interest on £43 ; 16 for 9 mo. 13 d. at 8 pr. ct. A. £2 ; 15 ; 1.

11. Find the interest on £87 ; 15 ; 4 for 2 yrs. 11 mo. 3 d, at 7½ pr. ct. A. £19 ; 5 ; 2.

12. Find the interest of £137 ; 11 for 11 d. at 9 per ct. A. 7s. 6d. 3 qrs.

13. Find the interest on £16 ; 7 ; 8 ; for 2 mo. at 12 per ct. A. 6s. 6d. 3 qrs.

14. Find the interest on 15s. for 3 mo. at 8 per ct. A. 3d. 2 qrs.

15. Find the interest on £193 ; 2 ; 6 ; 1, for 9 years..

16. Find the interest on £967 ; 18 ; 3, for 16 yrs. 4 mo. 11 d.

17. Find the interest on £1,000, for 5 yrs. 3 mo. 4 d.

18. Find the interest on £2,030 ; 17 ; 6 ; 1, for 6 yrs. 4 mo. 3 d.

19. Find the interest on £1,894 ; 0 ; 0 ; 1, for 19 d.

20. Find the interest on £6,872 ; 0 ; 3, for 17 d.

21. Find the interest on £15 ; 15 ; 5, for 8 hours.

22. Find the interest on £17 ; 13 ; 9 ; 2, for 3 yrs. 2 d. at 11 per ct.

23. Find the interest on £225 ; 17 ; 9 : 1 for 15 d. at 7 per ct.

24. Find the interest on £ 8,857 ; 16 ; 3 ; 2 for 27 d. 15 hours.

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§ LXXXV. In the calculation of profit and loss, where credit is given, it is plain, that we must *discount* for the time that intervenes between the bargain and payment, in order to form a correct conclusion. To be perfectly accurate, we should likewise calculate the interest on the *cost*, for the time the goods have lain useless in the hands of the seller.

1. Sold goods to the amount of $725.00 at 6 months credit. By the bargain I gain 25 dolls. What is my true profit?

Ans. 3.883\frac{51}{163}$

2. I make sales to the amount of $654.37½. The goods cost me $600.00, and have been lying a year on my shelves. I give credit for 8 months. Do I gain or lose, and how much?

I apparently gain $54.37½. But I must deduct 25.168\frac{7}{26}$ for discount, and likewise $36.00 for interest on the cost. These deductions amount to 61.168\frac{7}{26}$, so that in fact I lose 6.793\frac{7}{26}$, instead of gaining as I supposed.

As money will not always command the legal interest, however, the deductions ought to be made at the rate pr. ct. which can be obtained for it.

3. I have merchandize on hand for 6 months, and at last am able to sell it at an advance of 25 pr. ct. on the cost, giving 4 months credit. The amount of my sales is $1,000.00. What is my gain supposing that money will command 5 pr. ct. interest?

Ans. 163.606\frac{34}{61}$,

4. If I sell goods for 15 pr. ct. advance on the cost, ready cash, to to the amount of $1,680.00, when money will command only 4 pr. ct., what is my gain, supposing I have had the goods on hand 1 yr. 6 mo.?

5. If merchandize is sold at 5 per ct. deduction on the cost, to the amount of $1,380.00 when money will bring 5 per ct.; what is the total loss, supposing it has been 5 months on hand, and 3 months' credit is given?

6. A merchant sold 3 pieces of broadcloth, each piece containing 27 yards, at 7 dollars a yard, making an advance of 12 per ct. on the cost; it had been 3 months on hand, and 2 months credit was given: 7 pipes of wine at $4.50 per gallon, at an advance of 18 per ct. on the cost, which had been 7 months on hand, and for which he gave 3 months credit; and 7 bales of cotton at 11½ cents a pound, each bale containing 230 pounds, which had been on hand 1 mo. 15 days, at an advance of 20 per ct. on the cost, and giving 6 months credit. What was the whole amount of his profit, and his profit on each article, supposing that money will command 4 per ct.? Also, what was his real gain per ct. on each article, and his real gain per cent. on the whole?

Ans. Whole gain $\$566.049\frac{225182177}{367122072}$.—On each art. $\$51.932\frac{161}{302}$.–$\$487.660\frac{3560}{5959}$.–$\$26.456\frac{197}{408}$.—Rates $.10\frac{39}{151}$.—$.14\frac{151}{303}$.—$.17\frac{5}{34}$.—Average rate $.14\frac{196445181689}{2954672585818}$.

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§ LXXXVI. DUTIES. When a duty is said to be AD VALOREM, it is meant that it is at a certain rate *on the value* of the articles. The term is used to distinguish this class of duties from those imposed *on the quantity*; as, a duty upon the gallon, pound, barrel, cwt., ton, &c.

A written account of articles, sent to a purchaser, factor, or consignee, with the prices and charges annexed, is called an INVOICE.

In computing duties, *ad valorem*, (or *ad val.* as it is commonly written,) it is usual in custom houses to add one tenth to the invoice value, before casting the duty. This makes the *real* duty one tenth greater than the *nominal* duty. It will be equally well to make the *rate* one tenth greater, instead of increasing the invoice.

1. Find the duty on a quantity of tea, of which the invoice is $215.17, at 50 pr. ct. *Ans.* $118.3435=$118.343½.

In this example we may add, as directed above, one tenth of 215.17 to 215.17. Thus, 215.17+21.517=236.687. Then, 236.687 ×50=$118.3435. Or we may add to the rate .50, one tenth of itself =.05: thus, .50+.05=.55. Then, 215.17×.55=$118.3435, as before.

2. Find the duty on a quantity of hemp, at 13½ pr. ct., of which the invoice is $654.59. The second of the above modes is recommended. Another might be used—viz.: to find, first, the duty on the invoice at the given rate, and add to it one tenth of itself. Thus, 654.59×13½=$88.36965. *Ans.* $97.206615.

3. What is the duty on a quantity of books, of which the invoice is $1,670.33, at 20 pr. ct.? A. $367.4726.

4. Find the duty on a quantity of wine, of which the invoice is $2,964.666, at 45 pr. ct. A. $1,467.50967.

5. Find the duty on a quantity of merchandize, of which the invoice is $6,954.73, at 18 pr. ct.

6. Find the duty on a quantity of goods, of which the invoice is $7,458.133, at 15 pr. ct.

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§ LXXXVII. EQUATION OF PAYMENTS. It is sometimes required to know at what time several debts, which fall due at different times, may be paid, so that neither creditor nor debter shall lose. In this case, it is plain, that those due latest, must be paid before they are due, while those due earliest, may be kept after they are due.

MENTAL EXERCISES.

1. How long will $1.00 be in gaining as much as $3.00 gains in one year?

2. How long will it take $1.00 to gain as much as $5.00 in 6 mo.?

3. How long will $5.00 be in gaining as much as $1.00 gains in 50 yrs.?

4. How many months will it take $7.00 to gain as much as $1.00 in 35 mo.?

5. How long will $2.00 be in gaining as much as $3.00 in 4 mo.?

Ans. $3.00 in 4 mo. would gain as much as $1.00 in 3×4=12 mo. $2.00 would be half as long as $1.00. 12÷2=6 mo.

6. How long will $3.00 be in gaining as much as $5.00 in 6 yrs.?

7. How long will $4.00 be in gaining as much as $3.00 in 8 yrs.?

8. How long will $9.00 be in gaining as much as $6.00 in 6 mo.?

9. How long will $6.00 be in gaining as much as $1.00 in 1 yr.?

10. How long will $7.00 be in gaining as much as $1.00 in 3 yrs.?

11. How long will $4.00 be in gaining as much as $1.00 in 4 yrs. 4 mo.?

12. How long will $1.00 be in gaining as much as $9.00 in 9 yrs.?

Let the following be written.

13. How long will $1.00 [illegible] in gaining as much as $742.00 in 3 mo. A. 2,226 mo.

14. How long will \$1.00 be in gaining as much as \$895.00 in 5 mo.? A. 4,475 mo.

15. How long will \$1.00 be in gaining as much as \$272.00 in 7 mo., and \$336.00 in 6 mo.?

Ans. \$272.00 for 7 mo.=\$1.00 for 1,904 mo., and \$336.00 for 6 mo.=\$1.00 for 2,016 mo. Then, \$272.00 for 7 mo.+\$336.00 for 6 mo.=\$1.00 for 1,904 mo. +2,016 mo.=3,920 mo. *Ans.*

16. How long will it take \$1.00 to gain as much as \$700.00 in 3 mo., \$800.00 in 4 mo., \$900.00 in 6 mo., and \$500.00 in 12 mo.? A. 16,700 mo.

Then, it is plain that if I keep \$1.00, 16,700 mo., I gain as much interest as I should by keeping the several sums in the example, the times specified. The amount of those sums is \$2,900, for 700+800+900+500=2,900. Then,

17. How long may I keep \$2,900.00 so as to gain as much interest as I gain on \$1.00 in 16,700 mo.? I may keep it $\frac{1}{2900}$ as long. A. 5 mo. 22$\frac{22}{29}$ d.

Now, as \$2,900.00 is the amount of all the sums in ex. 16, if we consider them debts due at the times mentioned, it is plain that the whole might be paid in 5 mo. 22$\frac{22}{29}$ d., without loss on either side. Hence, the two questions might have been expressed in one, as follows:

18. If I owe several debts, due as in example 16, at what time may I pay the whole together, without loss to me or to my creditor? A. In 5 mo. 22$\frac{22}{29}$ d.

For \$700 for 3 months is the same as \$1 for 3×700=2,100 mo.
\$800 for 4 months is the same as \$1 for 4×800=3,200 mo.
\$900 for 6 months is the same as \$1 for 6×900=5,400 mo.
\$500 for 12 months is the same as \$1 for 12×500=6,000 mo.

Then, \$2,900 for these several times, is the same as \$1 for 16,700 mo.

But \$1 for 16,700 mo. is the same as \$2,900 for $\frac{1}{2900}$ of the time=5 mo. 22$\frac{22}{29}$ d.

Hence, to find the time when several payments, due at different times, may fairly be made at once,

MULTIPLY EACH PAYMENT BY ITS TIME, AND DIVIDE THE SUM OF THE PRODUCTS BY THE SUM OF THE PAYMENTS.

The time thus found is called the EQUATED TIME.

When there are years and months, or years, months and days, it is best to reduce the months, or months and days, to a vulgar or decimal Fraction. A decimal is to be preferred.

19. Find the equated time for the following payments; viz. $100 due in 6 mo., $120 in 7 mo., and $160 in 10 mo. A. 8 mo.

20. Find the equated time for the following payments; viz. $50 in 2 mo., $100 in 5 mo., and $150 in 8 mo. A. 8 mo.

21. Find the equated time for paying $900 due in 4 mo., $700 due in 1 yr. 3 mo., $1,200 due in 2 yrs., and $600 due in 3 yrs. 6 mo.

22. Find the equated time for paying $900 due in 9 mo., $1,500 due in 6 yrs., $1,800 due in 7 yrs. 9 mo., and $1,000 due in 3 yrs. 8 mo., 12 d.

It will be seen that the above rule supposes discount and interest to be equal, which is not true. But the error is too trifling to be regarded, in the common concerns of business. If any instance should occur, where the debts are very large, and the times very unequal, a rule for discovering the equated time may be found by an algebraic operation, but it would not be understood here, if given, and is therefore omitted.

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§ LXXXVIII. COMPOUND INTEREST. 1. A owes B $600, on which he is under obligation to pay interest at the end of every year, until a final settlement is made. At the end of the first year, he finds it inconvenient to pay the interest, and B allows him to keep it, on condition that he will pay yearly interest on this likewise. For the second year, then, he is to pay interest on the original debt of $600, and also on the interest which he should have paid at the end of the first. What was the sum at interest the second year? A. $636.

For $600\times.06=36$ the interest for 1st year, and $600+36=636$.

2. At the end of the second year, A is again unable to pay the interest, and the same agreement is made as before. What was the principal for the third year. A. $674.16.

For $636\times.06=38.16$ int. for 2nd year, and $636+38.16=674.16$.

3. A settlement was made at the end of the 3rd year. What did A pay; and how much for interest?

Ans. He paid $714.6096, for int. $114.6096.

Interest, calculated thus on both principal and interest, is called COMPOUND INTEREST. Interest, calculated only on the principal, is called SIMPLE INTEREST. Though the only just interest is compound interest, it is not allowed by law. From the preceding examples, we derive the rule,

I. I. FIND THE AMOUNT FOR THE FIRST YEAR, AS IN SIMPLE INTEREST, AND MAKE IT THE PRINCIPAL FOR THE SECOND; ON THIS, FIND THE AMOUNT FOR THE SECOND, AND MAKE IT THE PRINCIPAL FOR THE THIRD, AND SO ON, UP TO THE NUMBER OF YEARS GIVEN. THE LAST AMOUNT IS THE AMOUNT AT COMPOUND INTEREST.

II. SUBTRACT THE PRINCIPAL FROM THE AMOUNT, AND THE REMAINDER WILL BE THE INTEREST.

NOTE.—When there are months and days, calculate first for the years, and on the amount thus found, compute the interest for the months and days, which add to the amount previously obtained.

4. Required the amount of $100.00 for 3 years, at compound interest. A. $119.1016.

5. Required the compound interest on $300.00 for 4 years. A. $64.6519+

In obtaining amounts, we may multiply the principal by the rate, with a unit added. (§ LXXIV. AND NOTE EX. 9. § LXXVI.) Hence, to find an amount at Compound interest, we may multiply the given principal, by the rate with a unit added, as many times successively as there are years. But this, (§ XII.) is the same as though we should multiply the principal, at once, by the product of all these successive multipliers; and as all the multipliers are alike, being the rate + a unit, we have the following rule.

II. I. ADD A UNIT TO THE RATE, AND MULTIPLY THE SUM BY ITSELF CONTINUALLY, UNTIL IT IS TAKEN AS A FACTOR, AS OFTEN AS THERE ARE YEARS.

II. MULTIPLY THE PRINCIPAL BY THE LAST PRODUCT, AND THE RESULT WILL BE THE AMOUNT AT COMPOUND INTEREST.

6. Find the amount of $256, at compound interest, for 3 yrs. A. $304.900+

7. Find the amount of $1,000.00 for 4 yrs. at 7 per ct. compound interest. A. $1,310.796+

8. Find the amount of $425.00 for 4 yrs. at 5 per ct. compound interest. A. $516.590+

9. Find the amount of $2,000.00 for 3 yrs. 6 mo. at compound interest.

10. Find the amount of $1,900.25 for 9 yrs. 7 mo. 7d.

On the principles of the last rule, the following table has been prepared.

TABLE OF MULTIPLIERS,

FOR FINDING COMPOUND INTEREST, ON ANY PRINCIPAL, FOR ANY NUMBER OF YEARS, FROM 1 TO 24 INCLUSIVE, AT 5 AND 6 PER CENT.

Years.	5 pr. ct.	6 pr. ct.	Years.	5 pr. ct.	6 pr. ct.
1	1.05000	1.06000	13	1.88564	2.13292
2	1.10250	1.12360	14	1.97993	2.26090
3	1.15762	1.19101	15	2.07892	2.39655
4	1.21550	1.26247	16	2.18287	2.54034
5	1.27628	1.33822	17	2.29201	2.69277
6	1.34009	1.41851	18	2.40661	2.85433
7	1.40710	1.50363	19	2.52695	3.02559
8	1.47745	1.59384	20	2.65329	3.20713
9	1.55132	1.68947	21	2.78596	3.39956
10	1.62889	1.79084	22	2.92526	3.60353
11	1.71033	1.89829	23	3.07152	3.81974
12	1.79585	2.01219	24	3.22509	4.04893

In calculating compound interest by this table, take the multiplier opposite the number of years and under the given rate, multiply the principal by it, and the product will be the amount. For most cases it will be sufficient to employ four decimal places in the multiplier, increasing the last figure by 1, if the figure omitted is 5 or more.

NOTE. If the time be greater than the extent of the table, take any two or more multipliers, whose times, added together, will equal the given time, and multiply by them *successively*; that is, multiply the principal by one of them, and that product by another, and so on—thus,

11. Find the amount of $1,000.00 at 5 pr. ct. Compound interest, for 36 years.

Multiplier, by table 3.22509, for 24 years. Then 1,000×3.22509 =3,225.09 and 3,225.09×1.79585 (multiplier for 12 years)=$5,791. 7778+ *Ans.*

NOTE. The pupil will understand why we multiply *successively* by these multipliers, if he considers that the product $3,225.09 is the *amount for 24 years.* This, by the rule for compound interest, must be made the principal for the succeeding time; and, as 12 years remain, the question becomes, what is the amount of $3,225.09 for 12 years. We must therefore multiply $3,225.09 by the multiplier corresponding to 12 years.

12. Find the amount of $7,500.00 at 6 pr. ct. Compound interest for 4 years. *Ans.* $9,468.529½.

NOTE. This answer is strictly correct, as will be seen y performing the example by rule I. The table will give $9,468.536. Thus it will be seen that, in ordinary cases, the error by table is of no importance.

13. Find the amount of $5,000.00 for 18 yrs. 6 mo. at 5 pr. ct. Compound interest. *Ans.* $12,333.876¼.

14. Find the amount of $1,108.092 at 6 pr. ct. Compound interest for 11 years.

15. Find the amount of $63,700.00 for 19 years, at 5 pr. ct. Compound interest.

16. Find the amount of $47,753.18 for 17 yrs. 6 mo. 9 d. at 5 per ct. Compound interest.

If a principal be multiplied by a number taken from the table, an *amount* is obtained. Of course, if this amount be *divided* by the same number, the quotient will be the principal again. In calculating *present worths,* therefore, at compound interest, on sums due at a future day, we may divide the debt by the number in the table, corresponding to the given time. The present worth, thus obtained, being subtracted from the whole debt will give the discount.

17. What is the present worth of $304.900096 due 3 years from the present time, when compound interest is allowed at 6 per ct.? A. $256.

18. Find the present worth of $1,593.30, for 20 yrs., compound interest being allowed at 5 per cent. A. $600.50— Of $1,925.881, for the same time, at 6 per cent. A. $600.50—

19. Find the discount at 5 per cent. on $607.753⅛, for 4 years. A. $107.753⅛.

20. Find the discount on $1,642.992 for 21 years, at 4 per cent. A. $921.992. This divisor must be calculated, as it is not in the table.

21. Find the present worth of $875.00 for 6 years at 5 per cent. Of $947.00 at 6 per cent. for 9 years. Of $1,785.00 for 16 years at 5 per cent. Of $8,963.25 at 5 per cent. for 11 years.

At simple interest, a sum is 16 yrs. 8 mo. in doubling itself. At compound, it is 11 yrs. 10 mo. and between 21 and 22 days. But as simple interest only doubles the *original principal*, every 16 yrs. 8 mo., and compound interest doubles *its amount*, every 11 yrs. 10 mo. and 21 or 22 d., it is evident that a sum will increase vastly more rapidly at compound, than at simple interest. In a few centuries the increase of a sum of money at compound interest becomes almost incredible.

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§ LXXXIX. TO CALCULATE INTEREST ON NOTES WHEN THERE ARE ENDORSEMENTS. We have hitherto used the term NOTE without defining it. A NOTE is a written promise to pay a sum of money, either *on demand* (that is, when it is required,) or after a certain space of time has elapsed. Notes are given with or without interest. Unless the words, "*with interest*" are expressed, a note is understood to be without interest. If a note without interest given for a specified time, be not paid when it is due, it draws interest afterwards, till paid.

Payments in part, or partial payments are sometimes made, and, in this case, a written acknowledgement is made on the note, called an ENDORSEMENT.

As the debtor is not obliged to make payments in part, before payment is *demanded*, or before the note falls due, it is evident that he ought to be allowed interest on the payment, up to the time of settlement, if the note is given on demand, or till it is due, if given for a specified time. For if he had kept the money in his own hands, he might have had the use of it for that time. And, after he has paid it, the creditor has the same advantage. Hence, the following rule,

I. FIND THE AMOUNT OF THE PRINCIPAL OF THE NOTE FOR THE WHOLE TIME.

II. FIND THE AMOUNT OF EACH PAYMENT, FROM THE TIME IT WAS PAID, TILL THE TIME OF SETTLEMENT.

III. ADD THE AMOUNTS OF THE PAYMENTS, AND DEDUCT THE SUM FROM THE AMOUNT OF THE PRINCIPAL.

NOTE. When the note is for a specified time, it should be recollected that the payments ought to draw interest no longer than till the note is due.

EXAMPLES.

$500.00 Hartford April 1, 1828.

1. On demand I promise to pay PACKARD AND BUTLER, or order, five hundred dollars, with interest; value received.

TIMOTHY BOOKWORM.

On this note were the following endorsements.

Jan. 16, 1829, rec'd $150.00
April 1, 1829, " $150.00
July 13, 1830, " $100.00

What was due Aug. 1, 1830.

Whole time, 2 yrs. 4 mo.; time for 1st payment, 1 yr. 6 mo. 15 d.; for 2d, 1 yr. 4 mo.; for 3d, 18 d. Hence we have the following Rates, .14; .0925; .08; .003—Then, to find amounts, 500×1.14=570. 150×1.0925=163.875. 150×1.08=162. 100×1.003=100.3.

Then, amt. of prin. $570.000 and
163.875+162+100.3= sum of amts. of payments, $426.175

Hence there remains due $143.825

2. For value received, I promise to pay E. P. Barrows, Jr. or bearer, one thousand dollars with interest. Hartford, March 10, 1827.

$1,000.00 William Morris.

On this note were the following endorsements.

June 28, 1827, rec'd $500.00
Aug. 4, 1827, " $200.00
Nov. 22, 1828, " $100.00
Jan. 16, 1829, " $100.00

What is due on the note Aug. 19, 1830.

Amount of prin. $1,206.50
Sum of amts. of payments, $1,050.75

Due $155.75

3. For value received, I promise to pay Selden W. Skinner, or order, six hundred and seventy five dollars. Hartford, April 17, 1816.

$675.00 Simeon Faithful.

On this note were the following endorsements.

May 7, 1817, rec'd $148.00
Aug.17,1819, " $341.00
Jan. 2, 1821, " $99.00

What is due June 17, 1821? Ans. $219.5108+

4. On demand, I promise to pay Geo. D. Prentice, or order, nine hundred and ninety nine dollars, ninety nine cents and nine mills; for value received. Hartford, May 19, 1823.

$999.999 Thomas Punctual.

On this note were the following endorsements.

June 1, 1824, rec'd $300.00
Aug.15,1824, " $400.00
Sept. 8, 1824, " $60.00
Jan.11, 1825, " $100.00

What is due June 1, 1826?

Note. This rule has been commonly considered as incorrect. Accordingly we are told that "though a very common rule it is a very *bad* one," and that "it may answer for short periods of time, but, in a long course of years will be found very erroneous." The objection made, is, that the interest allowed on the payments, [illegible] in time "expunge," or pay off the principal itself. For, it is said, that if a [illegible] be given for $100.00, and $6.00 be annually paid and endorsed on the note, then, as each of these $6.00 endorsements begins to draw interest from the time of its date till the settlement, it will be found in 10 years, that only $89.80 is due the creditor, according to the above rule. Now, it is said, these several annual payments were justly due the creditor on account of interest, since $6.00 is the annual interest of $100.00. It is therefore unfair that they should "eat up" the principal, "on which not one cent has been paid, but only its annual interest." We are further told, that, in 20 years, there would be due the creditor only $37.60, "not the least fraction of the $100.00" principal having been paid. "Extend it to 28 years and the creditor would become the debtor, without receiving a cent of the $100.00."

The fact that the debt is thus "expunged" we grant, and this will take place, allowing 365 days to the year, in 23 years, 27 days, 13 hours, 37 minutes, 53.1—seconds. But we maintain that on the principles of *simple interest*, this *ought* to be the case. Let us take the instance mentioned above. Let the note for $100.00 run for 10 years. The amount at Simple Interest is $160.00. Then, if the note is settled at the end of 10 years, the debtor is only bound in law to pay $160.00, and this is all the creditor can claim. But suppose the annual 6 dollar payments to have been made, and suppose the creditor has put them at interest as soon as received. Then, on the first $6.00 he will gain 9 years interest; on the second, 8 years interest; on the third, 7 years interest, &c. For the nine payments, then, preceding settlement, he will receive the sum of these several amounts, which is found to be $70.20. But, as the legal interest is but $60.00, it is evident that he has received $10.20 *more than the interest*, and, consequently, the debtor must either *give* this amount to the creditor, or it must be deducted from the principal, leaving $89.80. If he *gives* it, nothing is more evident than that he gives 6 pr. ct. *Compound Interest.* Indeed, *Simple* Interest, *annually* paid, *is Compound* Interest. For, the creditor, on receiving his interest, may employ it in any profitable adventure, or he may instantly put it out at interest. Thus, the interest itself *may be made* to draw interest, and that is all that is necessary to constitute Compound Interest. It is the *creditor's own fault* if he does not improve his advantage. Of course, on the ground of Simple Interest, when the principal vanishes, by the above rule, it has been actually and really paid off to the "least fraction." And when the creditor falls in debt, it is because he has received, in fact, more than both principal and interest of the note. That is, although the sum of the payments *themselves* may be less, the *real advantage*, derived from them, and the *actual sum which they might have been made to produce*, will be found to exceed the amount of the note.

On the other hand, suppose that the debtor, instead of making these annual payments, should put $6.00, yearly, at interest. In 10 years he would have, as before, $70.20, and this he might apply towards paying the note. The result is the same as before; for $160.00—$70.20=89.80 which he has still to pay. Now, if he be allowed no interest on his payments, it is best for him to make no payment at all till the time of settlement; but to keep his money rather at interest than to give it to his creditor. For in this way he obtains the interest, which in the other case he loses, or rather gives away to his creditor.

The case we have been considering has been the strong ground of objection against this rule, and the argument drawn from it is very plausible, because at first thought it would seem, that the annual payments were nothing more than legal interest. But the pupil will by this time perceive, that 6 per cent. Simple Interest, for several years, is not worth as much as an *annual payment* of 6 dollars on the hundred. And he will perceive that the value of Simple Interest diminishes, as the number of years increases. This, then, is the evil, if there be any, and it is an evil which results from the nature of Simple Interest itself. Hence, the argument drawn from the case above, may be a very good one *against Simple, and in favour of Compound Interest*, but instead of invalidating the rule, on the ground of Simple Interest, it establishes, most completely, its correctness.

When the payments are larger, the case is clearer still. Any one will grant, that when a part, *manifestly, of the principal* is paid, allowance ought to be made for it, so that the creditor shall not still draw interest on the whole debt. And for the reason given above, viz. that the payment may be made to produce, in the hands either of creditor or debtor, its legal interest, it is evident that the whole payment should be deducted from the whole principal, and interest thenceforward, calculated on the remainder: or, which is the same thing in effect, the process in the rule should be employed. If it be granted, then, that the above rule is correct, on the ground of Simple Interest, it is plain that one which produces different results, must be incorrect. Hence, the rules established by law in Connecticut and Massachusetts are incorrect, since they seem to be constructed for the special purpose of *preventing the "eating up" of the principal.* Of course they both allow more than Simple Interest, that is, they allow Compound Interest, which is contrary to the statute laws of both states. Hence, we have one statute *enjoining* a practice, which is made *penal* by another. It will be necessary for the pupil to be acquainted with these rules.

MASSACHUSETTS RULE. FIND THE AMOUNT OF THE PRINCIPAL TO THE FIRST TIME WHEN A PAYMENT WAS MADE, WHICH, EITHER ALONE, OR TOGETHER WITH THE PRECEDING PAYMENTS, (IF ANY,) EXCEEDS THE INTEREST THEN DUE. FROM THIS, SUBTRACT THE PAYMENT, OR THE SUM OF THE PAYMENTS, MADE WITHIN THE TIME FOR WHICH INTEREST WAS COMPUTED, AND THE REMAINDER WILL BE A NEW PRINCIPAL, WITH WHICH, PROCEED AS BEFORE; AND SO ON, TO THE TIME OF SETTLEMENT.

EXAMPLES.

1. On demand, I promise to pay WILLIAM CARTER, or order, five hundred dollars, with interest; value received. Hartford, May 7, 1821.

$500.00. THOMAS TRUSTY.

On this note were the following endorsements.

April 29, 1822,	received	$23.29
July 13, 1822,	"	11.70
Sept. 19, 1822,	"	125.51
Jan. 7, 1823,	"	125.50
May 25, 1823,	"	5.50
Nov. 25, 1823,	"	100.63

What is due June 1, 1824?

Principal on interest from May 7, 1821,		$500.00
Interest to Sept. 19, 1822, (1 yr. 4 mo. 12 d.)		41.00
Amount,		$541.00
Payment, April 29, 1822, less than int. due,	$23.29	
Do. July 13, 1822, do.	11.70	
Do. Sept. 19, 1822, greater than int. due,	125.51	
Sum of payments,		$160.50
Due Sept. 19, 1822, forming a new principal,		$380.50
Interest to Jan. 7, 1823, (3 mo. 18 d.)		6.849
Amount,		$387.349
Payment, Jan. 7, 1823, greater than int. due,		125.50
Due Jan. 7, 1823, forming a new principal,		$261.849
Interest to Nov. 25, 1823, (10 mo. 18 d.)		13.878—
Amount,		$275.727
Payment, May 25, 1823, less than int. due,	$5.50	
Do. Nov. 25, 1823, greater than do.	100.63	
Sum of payments,		$106.13
Due Nov. 25, 1823, forming a new principal,		**$169.597**
Interest to settlement, June 1, 1824, (6 mo. 6 d.)=		**5.257+**
Balance due June 1, 1824,		**$174.854**

2. Value received, I promise to pay Robert F. Barnard, or order, five hundred dollars, with interest. Sheffield, Feb. 1, 1820.

$500. Joseph Paxwell.

On this note were the following endorsements.

May 1, 1820,	received	$40.00
Nov. 14, 1820,	"	8.00
April 1, 1821,	"	12.00
May 1, 1821,	"	30.00

How much remains due Sept. 16, 1821? *Ans.* $455.5748¼.

3. For value received, I promise to pay D. E. Bartlett, one thousand dollars, with interest. New-Haven, May 9, 1825.

$1,000.00. David Gamut.

On this note were the following endorsements.

June 21, 1826,	received	$60.00
Nov. 17, 1826,	"	28.00
Aug. 9, 1827,	"	200.00
Jan. 27, 1828,	"	15.00
Aug. 9, 1828,	"	300.00

How much remains due May 21, 1829?

This rule is very erroneous in principle, for, though it never suffers any part of the interest of the note to become incorporated with the principal, and thus *openly* to draw interest, yet it "applies the payments to *keep down* the interest," without suffering them to infringe, in the least, on the principal, until they exceed the interest due. In this way, not only does the debtor lose that interest on his payment which he might have had, had he retained the money, but the creditor gains the same interest, while the full principal of the note is still drawing interest. The creditor, therefore, draws interest on the interest paid, and on the principal likewise, while the debtor is in fact the person from whom this interest ultimately comes. This is obviously Compound Interest. Very similar but somewhat better, is the following,

CONNECTICUT RULE. I. FIND THE AMOUNT OF THE PRINCIPAL TO THE TIME OF THE FIRST PAYMENT, (IF IT BE A YEAR OR MORE AFTER INTEREST COMMENCED) AND SUBTRACT THIS PAYMENT FROM IT.

II. THE REMAINDER WILL BE A NEW PRINCIPAL; ON WHICH CALCULATE THE AMOUNT TO THE NEXT PAYMENT, AND SUBTRACT; AND SO ON, FROM PAYMENT TO PAYMENT, UNTIL THEY ARE ALL EMPLOYED, PROVIDED A YEAR OR MORE INTERVENES, BETWEEN EACH TWO PAYMENTS.

III. BUT, IF THE INTERVENING TIME BE LESS THAN A YEAR, FIND THE AMOUNT OF THE LAST PRINCIPAL FOR A YEAR, AND OF THE PAYMENT UP TO THE SAME TIME, AND SUBTRACT THE LATTER FROM THE FORMER. IF, HOWEVER, A YEAR OVERRUNS THE TIME OF SETTLEMENT, FIND THE AMOUNTS UP TO THAT TIME, INSTEAD OF FOR A YEAR.

IV. IF ANY REMAINDER, AFTER SUBTRACTION, BE GREATER THAN THE PRECEDING PRINCIPAL, THEN THE PRECEDING PRINCIPAL IS TO BE CONTINUED AS THE PRINCIPAL FOR THE SUCCEEDING TIME, INSTEAD OF THE REMAINDER; AND THE DIFFERENCE TO BE REGARDED AS SO MUCH UNPAID INTEREST.

Note. Of course, this "unpaid interest" must be added to the principal, at the time of the next payment.

$1,000.00 Hartford, Jan. 4, 1826.

On demand, I promise to pay H. ALDEN, or order, one thousand dollars, with interest; value received.

THOMAS FAIRWEATHER.

On this note were endorsements as follows.

Feb. 19, 1827,	received	$200.00
June 29, 1828,	"	500.00
Nov. 14, 1828,	"	260.00
Dec. 29, 1831,	"	25.00

What is the balance June 14, 1832?

Principal on interest from Jan. 4, 1826,	$1,000.00
Interest to Feb. 19, 1827, (13½ mo.)	67.50
Amount,	$1067.50
First payment, Feb. 19, 1827,	200.00
Due Feb. 19, 1827, forming a new principal,	$867.50
Interest to June 29, 1828, (16⅓ mo.)	70.846—
Amount,	$938.346
Second payment, June 29, 1828,	500.000
Due June 29, 1828, forming a new principal,	$438.346
Interest for 1 year,	26.300+
Amount for 1 year,	$464.646
Amount of $260.00 from Nov. 14, 1828, to June 29, 1829, (7½ mo.)	269.750
Due June 29, 1829, forming a new principal,	$194.896
Interest to Dec. 29, 1831, (30 mo.)	29.234+
Amount,	$224.130
Fourth payment, Dec. 29, 1831,	25.00
Remainder due, greater than last principal,	$199.13
Int. on $194.896 (prin. continued) to June 14, 1832, (5½ mo.)	5.36—
Whole amount due June 14, 1832,	$204.49

2. For value received, I promise to pay WILSON WHITON, Jr., or order, eight hundred and seventy-five dollars, with interest. Hingham, Jan. 10, 1821.

$875.00 JAMES CAREFUL.

On this note were the following endorsements.

Aug. 10, 1824,	received	$260.00
Dec. 16, 1825,	"	300.00
March 1, 1826,	"	50.00
July 1, 1827,	"	150.00

What was due Sept. 1, 1828? *Ans.* $474.95.

3. By each of the preceding rules, find the balance due on the following note.

On demand, I promise to pay GEORGE F. OLMSTED, twenty thousand dollars, with interest; value received. Hartford, July 1, 1825.

$20,000.

THOMAS THRIFTY.

ENDORSEMENTS.

Jan. 1, 1826,	received	$400.00
Jan. 1, 1827,	"	2,000.00
Sept. 1, 1827,	"	5,000.00
Feb. 16, 1829,	"	6,000.00

Settlement, July 1, 1830.

ANSWERS.

By common rule,	$10,727.00.
By Connecticut rule,	$11,332.566848.
By Massachusetts rule,	$11,370.3635.

In this example, it will be seen, that there is an error in the solution by the Connecticut rule, of $605.567— ; and in that by the Massachusetts rule, of $643.363+. By the Massachusetts rule, likewise, the error is $37.797— greatest. This comparison of results is sufficient to show which of the rules is preferable. If we examine the rules themselves, we see that this difference arises from the fact that, in certain cases, interest is allowed on payments for a time less than a year, by the Connecticut rule, whereas, it is never allowed at all by that of Massachusetts.

The subject of endorsements has been an unfortunate one. Besides the erroneous reasoning, of which we have been speaking on the subject of the common rule, a writer has asserted of the Connecticut mode, that when the payment is smaller than the interest due, the excess of interest becomes united with the principal, and draws interest as a part of it. To illustrate which objection, the following example has been instanced.

"*Messrs. Brown and Ives, Providence, loan to government* $200,000 *at* 6 *pr. ct. interest, The avails of the post-office, in said town, being about* $1,000, are *to be annually applied to the payment of the debt, and to operate as endorsements. The interest of* $200,000 *for one year is* $12,000, *and the amount*, $212,000. *Then, by the Connecticut mode, the payment of* $1,000 *is to be deducted from* $212,000, *which leaves* $211,000 *on interest for the second year ; that is*, $11,000 MORE THAN THE ORIGINAL DEBT IMMEDIATELY ACCUMULATING INTEREST, *besides the* $1,000, *which is at their disposal.*"

A slight inspection of the rule will convince any one that this objection is groundless. It is there particularly directed that the $200,000 shall be the *continued principal*, and that the $11,000 shall not draw interest. A clause of the same import, though in different language, and the language, we believe of the Superior Court of the State, may be found attached to this rule in the very book from which the above example is taken, which renders the writer's error the more remarkable. Messrs. Brown and Ives in the above example only derive advantage from the $1,000, the remaining $11,000 of interest, lying unproductive.

We have been thus full on this subject, because, by the generality of people, very incorrect notions are entertained with respect to it.

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RATIO.

MENTAL EXERCISES.

§ XC. 1. What part of 2 is 1 ? (See § XXVIII.) is 3 ? is 5 ? is 7 ? is 9 ?

2. What part of 3 is 2 ? of 7 is 5 ? of 17 is 16 ? of 19 is 18 ?

3. What part of 4 is 5 ? of 7 is 8 ? of 6 is 9 ? is 12 ? is 18 ?

We have already mentioned (§ XXVIII.) that there is another name for the Fraction expressing what part of one number, another is ; and that this name is RATIO. The Fraction is said to express *the*

Ratio, which exists between its denominator and its numerator ; or ***the Ratio of its denominator to its numerator.***

The meaning of the word ratio, is RELATION. When we speak of a ratio between two numbers, we mean, the *relation existing between them, as it respects quantity,* or *size.* RATIO, then, IS A MEASURE OF THE COMPARATIVE SIZE OF TWO NUMBERS, OR QUANTITIES.

In the Fraction expressing ratio, one of the numbers is the denominator, and the other the numerator. These two numbers instead of being written as a Fraction, are often written *one after the other,* with a *colon* between them, thus ; $\frac{4}{3}$, which expresses the ratio of 3 to 4, is written 3 : 4. When ratio is expressed thus, *the denominator of the Fraction is written first, and the numerator last.* On this account,

THE DENOMINATOR IS CALLED THE ANTECEDENT OF THE RATIO, and

THE NUMERATOR IS CALLED THE CONSEQUENT OF THE RATIO.

EITHER NUMBER TAKEN SEPARATELY IS CALLED A TERM OF THE RATIO, AND BOTH TOGETHER ARE CALLED A COUPLET.

Form ratios from the following questions,

4. What part of 8 is 4 ? Fractional ratio $\frac{4}{8}$=8 : 4

5. What part of 6 is 12 ? Frac. ratio $\frac{12}{6}$ = 6 : 12

6. What part of 3 is 18 ? of 7 is 5 ? of 9 is 3 ? of 9 is 18 ? of 7 is 14 ? of 6 is 24 ? of 9 is 36 ? of 15 is 30 ? of 11 is 33 ? of 8 is 2 ? of 6 is 3 ? of 9 is 12 ? of 3 is 2 ? of 4 is 7 ? of 16 is 32 ? of 2 is 16 ? of 8 is 64 ? of 15 is 75 ? of 5 is 30 ?

Many of the Fractional ratios obtained above, may be reduced to lower terms, or to whole or mixed numbers. This may evidently be done, without affecting the ratio, since ratio is only a measure of *comparative* size, and has nothing to do with the *actual* magnitude of the quantities, between which it exists. Thus a bullet may have the *same ratio* to a small particle of matter, which the planet Jupiter has to our Earth, notwithstanding that the *actual size* of these quantities is so different. Hence,

THE RATIO BETWEEN TWO NUMBERS, OR QUANTITIES, MAY BE EXPRESSED BY THE QUOTIENT OF ONE DIVIDED BY THE OTHER.

Perhaps the pupil need not be informed, that, RATIO CAN ONLY EXIST BETWEEN QUANTITIES OF THE SAME KIND. For if things be not of the same kind, we cannot have a measure of comparative size between them. Thus, though we may speak of one weight as being twice as great as another, we cannot say that one body weighs twice as much as another costs.

It may be here remarked that English mathematicians have usually made the Antecedent the Numerator, and the Consequent the Denominator of the Fractional ratio, while the French have employed the mode given above. The English mode is obviously most in accordance with reason, but the popularity of the French mathematical writings in this country, renders it almost necessary for us to follow them. It is not of essential importance *to our calculations,* which is used ; but of considerable consequence, as it respects the clearness of our conceptions, that the principles of ratio should be thoroughly understood. Now when 2 is compared with 5 *in respect to quantity,* a relation of 2 to 5 is discovered ; and this is, that the quantity 2, considered in relation to the quantity 5, is only two fifths as great. $\frac{2}{5}$ then expresses the true ratio, between 2 and 5, considering ratio to be, as defined, a measure of the comparative size of quantities. This is the English mode. By the French mode 2 is to 5 in the ratio $\frac{5}{2}$. No argument is necessary to show that 2 does not bear the relation, (in respect to quantity,) of *five halves*, to 5; or, in other words, that 2 is not five halves of 5. But this argument is derived from the meaning of the word *ratio*. When this is clearly understood, it matters not which term is put first, nor which last. The French mode has some advantages in practice, which will, perhaps, make it universally prevalent.

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PROPORTION.

MENTAL EXERCISES.

§ XCI. 1. What is the ratio of 2 to 4? (Reduce to its lowest terms.) *Ans.* $\frac{2}{1}$ or 1 : 2.

2. What is the ratio of 7 to 14? *Ans.* $\frac{2}{1}$ or 1 : 2.
3. Which is the greatest, the ratio 2 : 4 or the ratio 7 : 14?
4. Which is the greatest, the ratio 6 : 3 or the ratio 10 : 5?
5. Which is the greatest, the ratio 9 : 12 or the ratio 15 : 20?
6. Which is the greatest, the ratio 18 : 36 or the ratio 3 : 6?
7. Which is greatest, the ratio 7 : 8 or 14 : 16?
8. Which is greatest, 2 : 3 or 200 : 300? 5 : 6 or 125 : 150?
9. Which is greatest, 1 : 3 or 18 : 54? 9 : 2 or 117 : 26?
10. Which is greatest, 2 : 5 or 150 : 375? 3 : 27 or 64 : 576?

In the preceding examples, we have several instances of equal ratios, between very different numbers. When two ratios, thus brought together, are equal, they are said to constitute a PROPORTION. Hence,

A PROPORTION IS AN EXPRESSION DENOTING THE EQUALITY BETWEEN RATIOS.

THE NUMBERS, OR QUANTITIES, WHICH MAKE UP A PROPORTION, ARE CALLED PROPORTIONALS.

THE FIRST AND LAST TERMS ARE CALLED THE EXTREME TERMS, OR EXTREMES; AND THE SECOND AND THIRD, OR MIDDLE TERMS, ARE CALLED MEAN TERMS, OR MEANS.

As two numbers are necessary to a ratio, and two ratios to a proportion, it is evident, that, *under ordinary circumstances, there must be* FOUR *numbers, or proportionals, in every proportion*. As, however, the consequent of the first couplet may sometimes be the same with the antecedent of the second, *a proportion may exist with only* THREE *numbers*. Thus,

3 : 6 = 9 : 18 and 3 : 6 = 6 : 12 are two proportions, in the first of which, we have four proportionals, and in the second, three, because the middle term, 6, is used twice. Three numbers thus forming a proportion, are said to be in CONTINUED PROPORTION. Instead of = the usual sign of equality, *two colons*, or *four points*, thus, :: are used to signify equality of ratios. Thus, instead of 3 : 6 = 9 : 18,

we write 3 : 6 : : 9 : 18. This proportion is read thus ; 3 has the same ratio to 6, that 9 has to 18 ; or more briefly, 3 is to 6 as 9 to 18.

These ratios expressed fractionally are $\frac{6}{3}=\frac{18}{9}$. As these two Fractions are equal to one another, it is evident that if I were to reduce them to a common denominator, their numerators would become equal. Now (§ XLIV.) their numerators would be 6×9 and 3×18. For "each numerator is multiplied into every denominator, except its own," in reducing Fractions to a common denominator. Then 6×9 ought to equal 3×18. On trial, we find this the case, for 6×9=54, and 3×18=54. And, as this will always be true, we may conclude that,

IN A PROPORTION, THE PRODUCT OF THE EXTREMES, IS EQUAL TO THE PRODUCT OF THE MEANS.

This principle, we shall see, is of great use in solving many questions. Thus,

11. If 2 yards of cloth cost 5 dollars, what will 6 yards cost ? 6 yards are 3 times as much as 2 yards. Therefore the *price of* 6 *yards* will be 3 times as much as the *price of* 2 *yards*. That is, the ratio *between the prices*, is the same as that *between the quantities*, or numbers of yards. If we knew *both prices*, then, we should have a complete proportion. As it is, one term is wanting. The proportion is,

2 yards : 6 yards : : price of 2=\$5 : price of 6=\$. . . . or
2 : 6 : : 5 : . . .

The term wanting is an extreme. Now, by the principle just laid down, we know that the two means multiplied together, thus 6×5, produce just as great a product as the two extremes multiplied together, thus, 2×*the wanting term*. But 6×5=30. Therefore, 2×*the wanting term*=30. In other words 30 is *twice* the wanting term. Hence $\frac{30}{2}$=15=the price of 6 yards. The proportion is now rendered complete, by writing the 15 with it, thus,

yds. yds. \$ \$
2 : 6 : : 5 : 15

The same method might be pursued to find any other term that should be wanting. Thus,

12. If \$5 will buy 2 yards, what will \$15 buy ?

As 15 is 3 times 5, it will buy 3 times as many yards. That is, there is the same ratio between the *yards bought* as between *the prices*. Thus,
\$ \$ yds. yds.
5 : 15 : : 2 : . . .

But, by the principle above, 15×2=5× *the wanting term*. But 15×2=30. Therefore, 5× *the wanting term*=30. In other words, 30 is *five times* the wanting term. Hence $\frac{30}{5}$=6=the number of yards required. Then the completed proportion is

\$ \$ yds. yds.
5 : 15 : : 2 : 6

which is the same as in the last example, except that the ratios are differently arranged.

So, if the 5 were not known, in the same proportion, I should multiply 2 into 15, and divide by 6 to obtain it; and if the 2 were wanting, I should multiply 5 into 6 and divide by 15. Hence, *to find an unknown term in a proportion,*

IF IT BE AN EXTREME DIVIDE THE PRODUCT OF THE MEANS BY THE OTHER EXTREME; IF A MEAN, DIVIDE THE PRODUCT OF THE EXTREMES BY THE OTHER MEAN.

This rule, for finding, on the principles of proportion, a fourth number, from three given numbers, is commonly called the RULE OF THREE. It is distinguished into two kinds, both comprehended in the above general principle. The difference will be best understood after comparing the following examples.

13. If 3 men dig 15 rods of ditch in a day, how much would 6 men dig in the same time?

It is evident that the rods will increase in the same ratio as the men; that is, that 16 rods has to the answer the same ratio, that 3 has to 6, or $3 : 6 :: 16 : 6\times16\div3=32$. *Ans.*

In this question *the more men* there are, *the more rods* are dug. It is therefore called *the case in which more requires more*, and belongs to the RULE OF THREE DIRECT.

WHEN ONE RATIO INCREASES AS ANOTHER INCREASES, OR DIMINISHES AS ANOTHER DIMINISHES, THE TWO ARE SAID TO CONSTITUTE A DIRECT PROPORTION.

14. If 6 men dig 32 rods of ditch in a day, how much would 3 men dig in the same time?

The rods diminish in the same ratio as the men. Therefore 32 has to the answer the same ratio as 6 to 3, or

$$6 : 3 :: 32 : 3\times32\div6=16. \text{ Ans.}$$

In this question, *the fewer men, the fewer rods.* It is therefore called *the case in which less requires less,* and belongs to the RULE OF THREE DIRECT.

15. If 3 men can dig a ditch in 16 days, how long will it take 6 men?

The more men the quicker the work will be done; that is, *the more men the less time.* Then if the number of *men* be *doubled*, the number of *days* will be *halved*, or if the number of *men* be *increased in any ratio*, the number of *days* will be *diminished in the same ratio;* or, as the ratio of the men increases, the ratio of the days diminishes. The ratio, then, of 3 to 6, is directly the *reverse* of the ratio of 16 to the answer. Then, if I *turn about*, or *invert* this ratio of 3 to 6, making it 6 : 3, it will be the same which 16 has to the answer. Thus,

$$6 : 3 :: 16 : 3\times16\div6=8 \text{ Ans.}$$

In this case, *the more men the fewer days.* It is therefore called the case in which *more requires less,* and on account of the inversion above mentioned, the rule by which it is solved is called the RULE OF THREE INVERSE.

WHEN ONE RATIO INCREASES AS ANOTHER DIMINISHES, OR DIMINISHES AS ANOTHER INCREASES, THE TWO ARE SAID TO CONSTITUTE AN INVERSE PROPORTION.

16. If 6 men can dig a ditch in 8 days, in how many days will 3 men do it?

The fewer men, the slower the work will go on; that is, *the fewer men, the more time* required. Then if but *half* the number of *men* be employed, the number of *days* will be *doubled;* or if the number of *men* be *diminished in any ratio,* the number of *days* will be *increased in the same ratio;* or as the ratio of men diminishes, the ratio of days increases. The ratio then of 6 to 4 is exactly the *reverse* of the ratio of 8 to the answer. Then, if I invert the ratio 6 : 3, making it 3 : 6, it will be the same which 16 has to the answer. Thus, $3 : 6 :: 8 : 6\times8\div3=16$ *Ans.*

In this case, *the fewer men, the more days.* It is therefore called the case in which *less requires more,* and belongs, like the last, to the Rule of Three Inverse.

Two rules are unnecessary for the two cases, but we have explained them minutely in order to render the pupil's ideas clear on the subject of two classes of questions, which he would otherwise perceive to differ, without, perhaps, understanding why.

The only difficulty which the pupil will now have, in the solution of problems by proportion, will consist in arranging his three given numbers in their proper order. He will see that there are two ratios, in one of which both terms are given, and in the other of which, only one term is given; *the wanting term being the answer.* Of this *incomplete* ratio, then, let him set down the given term in the third place, to stand as one of the *means* of the proportion. He will always know this term because it will be, of course, of the *same nature with the answer.* The remaining two numbers form the *complete* ratio; which is to be set down *before the last mentioned number,* making the *smaller term* the *consequent,* when the answer is to be *less* than that number, and the *antecedent,* when the answer is to be *greater* than that number.

[We have omitted the common rule of three as unnecessary, and as calculated to draw off the mind of the pupil from the principles of proportion. If there be any advantage in exhibiting this mode, when we have the simpler and more natural one of analysis, it must surely consist in keeping these principles in view.]

17. If 73 men consume 235 barrels of provisions in a certain time, how many barrels would 365 men consume in the same time?
Proportion, 73 : 365 :: 235 : .... *Ans.* 1,175.

Since the answer is found by multiplying 235 by 365, and dividing the product by 73, the pupil will perceive that the process is the same as it would be to multiply 235 by the fractional ratio $\frac{365}{73}$ (§ xlviii.) Now this fractional ratio may be reduced to the whole number 5. So that 5×235 will produce the answer.

18. If 18 barrels of flour cost 162 dollars what cost 12 barrels?
Proportion, 18 : 12 :: 162 : .... *Ans.* 108.

The fractional ratio here would be $\frac{12}{18}=\frac{2}{3}$, and $\frac{2}{3}\times 162=$ 108 *Ans.*

Hence, it will often abbreviate the process, to

EXPRESS THE GIVEN COMPLETE RATIO FRACTIONALLY, REDUCE IT TO ITS LOWEST TERMS, AND MULTIPLY THE REMAINING TERM BY IT.

19. If 12 loads of wood cost 51 dollars, what cost 14? Proportion 12 : 14 : : 51 : .... *Ans.* $59\frac{1}{2}$ dolls.

Ratio $\frac{14}{12}=\frac{7}{6}$, and $\frac{7}{6}\times 51=\frac{7\times 51}{6}=\frac{7\times 17}{2}=59\frac{1}{2}$.

Hence the pupil will perceive, that he may

REJECT COMMON FACTORS FROM THE FIRST AND THIRD TERMS, BEFORE PROCEEDING TO FIND THE ANSWER.

20. If an army of 66,153 men require 22,051 barrels of provisions for two months, how much would a company of 100 men require?

Thus, ratio $\frac{100}{66153}$, and $\frac{100\times 22051}{66153}=\frac{100\times 1}{3}=33\frac{1}{3}$.

21. If a man's wages are 75£. 10s. pr. year, what is that for two (calendar) months? *Ans.* 12£. 11s. 8d.

NOTE. When there are compound numbers, the lower must be made fractions (vulgar or decimal) of the higher, or the higher must be reduced to the denomination of the lower. Also, both the terms of the first ratio must be of the same denomination. The answer will be of the denomination of the third term.

22. A goldsmith sold a tankard for £8; 12, at 5s. 5d. pr. oz. What did it weigh? A. 2 lb. 8 oz. 5 dwt.

23. Bought 8 chests of sugar, each weighing 9 cwt. 2 qrs., at 2£ 5s. pr. cwt. What did they come to? A. £171.

24. How long can I board for 32£ 32; 10, at 12s. 6d. per week? A. 1 year.

25. If 2 cwt. 3 qrs. 21 lb. of sugar cost 6£ 1s. 8d., what cost 35 cwt. 1 qr.? A. £73.

26. A man failing was only able to pay 13s. 4d, on a pound. How much could he pay on a debt of £3,475? A £2,316; 13; 4.

27. If a man's yearly income be 208£. 1s., what is that per day? A. 11s. 4d. $3\frac{1}{5}$ qrs.

28. A man failing in trade finds that he can pay only $2.00 on on every $3.00 of his debts. How much can he pay on a debt of $25.00. A. $15.75.

29. A man failing can only pay 13£ in 15£ of his debts. How much can he pay on a debt of £17; 15. A. £15; 7; 8.

30. A man failing, can only pay 17s. on a pound. How much can he pay on 147£. 14s. A. £125; 10; 10; $3\frac{1}{5}$.

31. A man failing in trade, gives up his property, amounting to $1,020.20 to his creditors, to whom he owes the following sums: viz., to A, $1,600.60; to B, $500.00; to C, $750.20; to D, $1,000; to E, $230.00. How much does he pay on a dollar, what does he pay each creditor, and what was the loss to the whole together?

Ans. $0.25 on a dollar. To A, $400.15; to B, $125.00; to C, $187.55; to D, $250.00; to E, $57.50. Total loss, $3,060.60.

32. If $7,200.00 will buy 3,600 bu. of wheat, what will 600 bu. cost? A $1,200.00.

33. If a staff 6 ft. long cast a shadow 9 ft., what is the height of a tower, which, at the same time casts a shadow 198 ft.
A. 132 ft. high.

34. If 12 horses eat up 30 bu. of oats in a week, how many bu. will 45 horses eat in the same time. A. 112½ bu.

35. Bought a piece of cloth for $48.27, at the rate of $2.38 for every 2 yds. What was the number of yds.? A. 40 yds. 2 qrs. 1⅕ na.

36. If the wages of 15 weeks amount to $64.19, what is a year's wages at that rate? A. $222.525.

37. If a man is 5 days on a journey, travelling 12 hours a day; how long would he be going the same distance, travelling 10 hours a day? A. 6 days.

38. If 12 men build a wall in 20 days, how many will do it in 3 days? A. 80.

39. A building was finished in 8 months by 120 workmen; but being destroyed by fire, it was necessary to rebuild it in 2 months. How many men did it take? A. 480.

40. If a field will pasture 6 cows 91 days, how long will it pasture 21 cows? A. 26 days.

41. If a man performs a piece of work in 15 days, when the days are 12 hours long, how many days will it employ him, when they are 10 hours long? A. 18.

42. 2,000 soldiers, having provision to last them, at an allowance of 14 oz. a day, 12 weeks, find on examination that 105 barrels, containing 200 lb each are wholly spoiled. What must be the daily allowance to each man, that the remainder may last the same time?
A. 12 oz.

43. 65 men are in garrison, with provision enough for 2 months. How many must leave the garrison that the same provision may last those who remain, 5 mo.? A. 390.

44. If $100.00 gain $6.00 interest in a year, what principal will gain the same in 8 mo.? A. $150.00.

45. If $5.00 will pay the freight of 2 cwt. 150 miles, how far may 15 cwt. be carried for the same money? A. 20 miles.

46. How many men will finish in 3 days, what would take 15 men 20 days. A. 100 men.

47. If $\frac{2}{9}$ of a yard of cloth cost $\frac{7}{25}$ of a dollar, what cost $\frac{3}{8}$ of a yard? A. $\frac{189}{400}$ of a dollar=$0.472½.

NOTE. When Fractions occur they are to be treated according to the rules for multiplying and dividing that class of numbers. Perhaps, however, the mode of solving questions of this kind by analysis is best. By analysis, the above question is thus solved. If $\frac{2}{9}$ cost $\frac{7}{25}$, $\frac{1}{9}$ will cost half as much, that is $\frac{7}{50}$; and $\frac{9}{9}$, or a whole yard, $\frac{63}{50}$. Then, if a yard cost $\frac{63}{50}$, $\frac{1}{8}$ of a yard will cost an eighth as much, that is, $\frac{63}{400}$ and $\frac{3}{8}$, three times as much, that is $\frac{189}{400}$.

48. If $\frac{1}{2}$ a pound of sugar cost $\frac{11}{12}$ of a dollar, what cost $17\frac{3}{5}$ lb.? A. $\$32\frac{4}{15}=\$32.266\frac{2}{3}$.

49. If $\frac{11}{12}$ of a lb. of sugar, cost $\frac{7}{15}$ of a shilling, what will $\frac{3}{4}\frac{2}{3}$ of a lb. cost? A. 4d. $3\frac{1657}{2365}$ qrs.

50. If $\frac{5}{8}$ yd. cost $\frac{5}{7}$£., what cost $\frac{9}{15}$ of an Ell English? A. 17s. 1d. $2\frac{6}{7}$ qrs.

51. If 1 cwt. cost $3\frac{5}{8}$£., what cost $9\frac{2}{3}$ lbs.? A. 6s. $3\frac{5}{56}$d.

52. A man owning $\frac{2}{3}$ of a vessel, sold $\frac{4}{5}$ of his share for $956.00 what was the vessel worth? A. $1,794.375.

53. If $3\frac{4}{7}$ cwt. of iron cost $\$14\frac{5}{9}$ what is that pr. cwt.? A. $\$4\frac{17}{225}=\$4.07\frac{5}{9}$.

54. If $\frac{5}{7}$ of a yd. cost $\frac{3}{5}$£., what cost $\frac{9}{11}$ of an Ell English. A. £$\frac{189}{220}$=17s. $2\frac{2}{11}$d.

55. If $7\frac{5}{8}$ lb. of butter cost $\$1\frac{6}{11}$, what cost $27\frac{3}{5}$ lbs.? A. $5\$\frac{1017}{3355}=\$5.30\frac{219}{671}$.

56. If by travelling $12\frac{1}{3}$ hours in a day, a man performs a journey in $7\frac{2}{3}$ days, how long will it take him, at the rate of $9\frac{1}{2}$ hours a day? A. $9\frac{173}{286}$ days.

57. If $\frac{3}{7}$ of a yd. cost $\$4\frac{2}{5}$, what cost $17\frac{3}{8}$ yds. at that rate? A. $\$178.38\frac{1}{3}$.

58. A man sold $51\frac{8}{15}$ cwt. of sugar for $587.00 What would $17\frac{2}{5}$ cwt. cost? A. $\$198.19\frac{613}{773}$.

59. If $\frac{7}{8}$ bu. of corn cost $\$\frac{5}{9}$, what cost $1\frac{1}{2}$ bu. at that rate. A. $\$0.95\frac{5}{21}$.

60. If $\frac{3}{5}$ of a barrel of flour cost $1\frac{2}{7}$£., what cost $43\frac{5}{8}$ barrels? A. 93£. 9s. 7d. $2\frac{6}{7}$ qrs.

NOTE. When examples containing Fractions are solved by proportion, and the first term, or that by which it is *necessary to divide*, is a Fraction this term may be *inverted* and *used as a multiplier* (§ LI. Rule II.)

61. If $\frac{1}{3}$ of a bushel cost $\frac{3}{16}$ of a dollar, what cost 1 bushel. A. $\$0.56\frac{1}{4}$.

62. If $\frac{5}{112}$ cwt. cost $\$\frac{75}{192}$, what cost 1 cwt.? A. $\$93\frac{3}{4}$.

63. If $\frac{3}{4}$ yd. of silk cost $\$\frac{15}{32}$ what cost 50 yds.? A. $31.25.

64. If $\frac{7}{16}$ cwt. of sugar cost $\$\frac{9}{10}$, what cost 40 cwt.? A. $\$82.27\frac{4}{7}$.

◆

§ XCII. FELLOWSHIP, by analysis, has been already explained. (§ XLIX.) Examples in Simple Fellowship may also be solved by Simple Proportion. No particular rule is necessary.

1. A, and B purchase goods in company, A laying out $350.00, and B, $470.00. They gain $250. How much should each have? *Ans.* A, $106.707+ B, $143.293—.

2. Three merchants trade, A furnishing $240.00, B $360.00, and C, 600.00. They gain $325.00. What is each one's share?

Ans. A's $65.00. B's $97.50. C's $162.50.

3. Four men trade, A furnishing $140.00, B, 260.00, C, $300, and D, $100. They gain $307. What is each one's share?

Ans. A's, $53.725, B's $99.775, C's $115.125, D's $38.375.

4. A and B trade and gain $100.00. A furnished $640.00 stock, and B, so much that he must have $60.00 for his share of gain. What amount of stock did B furnish? *Ans.* $960.00.

5. A, B, and C traded. A furnishes $140.00, B, $250.00, and C, 120 yds. of cloth at cost. They gained $230.00, of which $100 fell to C as his share. What was the cost of C's cloth per yd.?

Ans. $2.50 per yd.

6. In a trading company, A puts in $175.00 capital, B, $200.00, and C, $500.00. They lose $250.00. What is each one's loss?

Ans. A's $50.00, B's 57.142\frac{6}{7}$, C's 142.857\frac{1}{7}$.

In the same manner, money or property, which is to be shared in a given ratio by several individuals, may be divided. This comprehends the division of an insolvent debtor's estate, the distribution of prize money, dividing an inheritance among heirs in a given ratio, the assessment of taxes, &c. &c. It will be observed that all these calculations may be made *analytically*, as well as by proportion.

7. A captain, mate and 20 seamen took a prize worth $3,501.00, of which the captain takes 11 shares, the mate 5 shares and each sailor 1 share. What must each have?

Ans. Capt. $1,069.75. Mate $486.25. Each sailor $97.25.

8. Divide the number 360 into parts which shall be to each other as 2, 3 and 4. *Ans.* 80; 120; 160.

9. Three persons share $600.00, so that the second has twice as much as the first; and the third, twice as much as both first and second. How much has each?

Ans. First 66.666\frac{2}{3}$. Second 133.333\frac{1}{3}$. Third $400.00.

10. A bankrupt gives up his whole property $600.00, to his creditors, to whom he stands indebted as follows: to A, $780.00 to B, $460.00, and to C, $760.00. What is each creditor's share?

Ans. A's, $234.00, B's $138.00, C's $228.00.

NOTE. In practice, it is customary to find first how much the bankrupt can pay on a dollar, and in speaking of the amount of the deficiency, it is customary to say *he pays so much on a dollar*, or sometimes *so much per cent.* Finding what he pays on 1 dollar, and afterwards on a larger debt, is evidently not *proportion*, but *analysis*.

11. An insolvent gives up property to the amount of $1,557.50, to be divided among the following creditors; A, to whom he owes $300.00; B, to whom $1,100.00; C, to whom $1,150.00; D, to whom $505.00; E, to whom $25.00; and F, to whom $35.00. At what per ct. does he pay, how much on a dollar, and how much on each debt? *Ans.* 50 per ct.=50 cts. on a dollar. To A, $150.00; to B, $550.00; to C, $575.00; to D, $252.50; to E, 12.50, and to F, $17.50.

12. A certain rich man left $243,000 to be distributed among 7 children, of the following ages, 3, 5, 7, 9, 11, 13 and 15 years re-

spectively. The third and fifth are sons, the rest daughters. They are to share in the ratio of their ages, except that the males are to share in twice as great a ratio as their ages would allow. What is the share of each? *Ans.* The shares arranged in the same order are, $9,000 $15,000 $42,000 $27,000 $66,000 $39,000 $45,000.

13. Frederick, William, Thomas and Henry were to share an estate of $54,000 in the ratio of the numbers of the letters in their first names. How much had each? *Ans.* Frederick, $18,00; William, $14,000; Thomas, $12,000; Henry, $10,000.

14. But it was proved in court, that Frederick used to spell his name without a K. So his share was diminished, and the others increased. How much did he lose by spelling his name without a K, and how much did the others gain?

Ans. Frederick lost $\$1,384.615\frac{5}{13}$; William gained $\$538.461\frac{7}{13}$; Thomas gained $\$461.538\frac{6}{13}$; and Henry gained $\$384\ 615\frac{5}{13}$.

A TAX is a sum of money required of individuals by a government, for the use of the nation; or by a town, county, society, or corporation, for the purpose of defraying the expenses of the whole, in their collective capacity. A tax is sometimes imposed by government, of a small sum upon every male citizen above a certain age, and this being the same to all, and not varying with the amount of an individual's property, is called, *a tax of so much a head*, or, *of so much a poll*; the word *poll*, meaning *head*. This is commonly abbreviated into the more concise expression POLL-TAX. In some States no poll-tax is allowed. Other taxes are usually rated or *assessed* on property. Property is of two kinds, *real* and *personal*. Real property, or, as it is usually called, *real estate*, consists of possessions, which are *fixed* and *immoveable*; as *lands* and *buildings*. Personal property, comprehends *all other possessions*, which, of course, are *moveables*; as *furniture, jewels, money, cattle, &c.* Taxes are assessed on property, both real and personal. When, therefore, *no particular kind* of property is mentioned, property *generally*, or, *of all kinds*, is understood. In order to *assess* a tax of this kind, then, or, in other words, *to apportion or rate it justly* among the individuals of a town or society, we must have a list, or *inventory* of all the property, both real and personal, on which it is to be assessed. If there is likewise a poll-tax included in the whole amount given, we must also have the whole number of *polls*, or rateable *heads*, or *persons*. We must then find the amount of all the poll-taxes, or the *whole poll-tax*, which we must subtract from the *whole given tax*, and the remainder is to be assessed on property. It is the most expeditious method of making this assessment, to find the taxes on $1.00, $2.00, $3.00, &c., up to $9.00. Then the taxes on $10.00, $20.00, $30.00, &c., up to $90.00 Then on $100.00, $200.00, &c., up to $1,000.00, and if the town or society be very wealthy, on $2,000.00, $3,000.00, &c., up to $10,000.00. Arrange these taxes in a table, and then from the inventory, knowing each one's property, his tax may be taken out of the table, without the trouble of calculation. It is convenient, likewise, to arrange the polls, from 1 to 10, on one side of the table. This is obviously an analytic process, as, in fact, are most of the practical operations of men of business. We will illustrate this method.

15. A town valued, by inventory, at $97,595.50, raises a tax of $4,155.82. There are 630 polls, and the individual poll-tax is $0.40. Find the tax on a dollar, and form a table as above directed.

630 × 40 = $252.00, amount of poll-taxes. Hence, $\$4,155.82-\$252.00=\$3,903.82$ is to be assessed on property. $\frac{390382}{9759550}=.04$ = tax on $1.00. From this we easily form the

TABLE.

Polls.	Tax.	Property.	Tax.	Property.	Tax.	Property.	Tax	Property.	Tax.
1	$0.40	$1.00	$0 04	$10.00	$0.40	$100.00	$4.00	$1,000.00	$40.00
2	0.80	2. "	0.08	20. "	0.80	200. "	8. "	2,000. "	80. "
3	1.20	3. "	0.12	30. "	1.20	300. "	12. "	3,000. "	120. "
4	1.60	4. "	0.16	40. "	1.60	400. "	16. "	4,000. "	160. "
5	2.00	5. "	0.20	50. "	2.00	500. "	20. "	5,000. "	200. "
6	2.40	6. "	0.24	60. "	2.40	600. "	24 "	6,000. "	240. "
7	2.80	7. "	0.28	70. "	2.80	700. "	28. "	7,000. "	280. "
8	3.20	8. "	0.32	80. "	3.20	800. "	32. "	8,000. "	320. "
9	3.60	9. "	0.36	90. "	3.60	900. "	36. "	9,000. "	360. "
10	4.00							10,000. "	400. "

16. From this table, find the taxes on the following persons. A, property, $7,000.00 *real*, $800.00 personal; pays for 3 *polls* : *B*, property, $9,000.00 real, $7,00.00 personal; pays for 4 polls : C, property, $6,000.00 real, $900.00 personal; pays for 1 poll : and D, property, $15,000.00 real, $1,100.00 personal; pays for 6 polls.

A's whole property=$7,800.

Tax on $7,000.00 - - - - - - - -	$280.00
" 800.00 - - - - - - - -	32.00
A's whole tax on property, - - - - - -	$312.00
3 polls at $0.40 each, - - - - - - - -	1.20
A's whole tax, - - - - - - - - - -	$313.20

In like manner, B's=$389.60; C's $276.40; and D's $646.40.

17. Assess the tax on the following persons, by the last table. E, whole property, $900.00, pays for 2 polls ; F, $800.00, pays for 1 poll ; G, $750.00, pays for 3 polls ; H, $323.50, pays for 1 poll ; I, $270.25, 2 polls ; J. $300.75, 2 polls ; K, $175.25, 1 poll ; L, $643.00, 1 poll ; M, $471.50, 2 polls ; N, $533.75, 3 polls ; O, $543.75, 2 polls ; and P, $975.00, 4 polls.

18. A town is valued at $298,769.50, and is taxed $6,177.89. There are 675 polls, taxed $0.30 each. What is the tax on a dollar?

Ans. $0.02 on $1.00.

19. Form a table from the last example, and find the taxes of the following persons. A, property, $3,125.75, 2 polls ; B, $936.00, 2 polls ; C, $540.00, 3 polls ; D, $900.00, 3 polls ; E, $1,500.25, 3 polls ; F, $763.00, 4 polls ; G, $300.00, 3 polls ; H, $984.00, 3 polls ; I, $1,700.00, 4 polls ; J, $1,125.25, 6 polls ; K, $727.50, 5 polls ; and L, $1,384.00, 5 polls.

COMPOUND PROPORTION.

MENTAL EXERCISES.

§ XCIII. 1. If 2 men can mow 16 acres in 4 days, how many acres can 5 men mow in 3 days?

2. If 2 barrels of provisions will last 2 persons 4 months, how long will 5 barrels last 9 persons?

3. If a man can travel 72 miles in 6 days, travelling 6 hours a day, how far can he travel in 8 days, travelling 2 miles a day.

4. If a man can shingle 5 houses in 5 months, labouring 10 hours a day, how many houses can he shingle in 2 months, labouring 15 hours a day?

5. If 3 men build 12 rods of wall in 5 days, how many men will be able to build 16 rods in 2 days?

6. If 7 men dig 21 rods of a ditch in 3 days, how many men will it take to dig 27 rods in 9 days?

7. If a box of a certain depth, and 4 ft. long and 3 ft. wide, contain 24 pecks, how long must a box of the same depth, and 4 ft. wide be, to contain 40 pecks?

8. If it take 3 yards of cloth that is 7 quarters wide to make a suit, how much that is 3 quarters wide will it take to line it?

9. If 5 men can do a piece of work in 8 days, working 5 hours a day, how many hours a day must 10 men work, to do it in 10 days?

Let the following be written.

10. If 20 laborers will clear 60 acres of woodland in 2 months, how many acres will 35 laborers clear in 6 months?

ANALYTIC SOLUTION. If 20 men clear 60 acres, 1 man, in the same time, will clear $\frac{1}{20}$ as much $=\frac{60}{20}=3$ acres. If he clear 3 acres in 2 months, he will clear $\frac{1}{2}$ as much in 1 month $=\frac{3}{2}$ acres. 35 men would clear 35 times as much in a month $=35\times\frac{3}{2}=\frac{105}{2}$, and in 6 months, they would clear 6 times as much $=6\times\frac{105}{2}=\frac{630}{2}=315$ acres, *Ans.*

By proportion, the process is as follows:

It will be observed, that the answer is affected, both by the number of the men, and by the length of time they work. Suppose we find what the answer would be supposing they worked the same length of time, that is 2 months, in each case. The number of men will, then, be the only circumstance on which the answer depends, and the question becomes one of Simple Proportion, stated thus,

men men acres acres
20 : 35 :: 60 : ***$=60\times35\div20=105$ acres.

It seems, then, that the 35 men will clear 105 acres, while the 20 are clearing 60 acres. But they have, likewise, more time to work in, and the question now is, if they will clear 105 acres in 2 months, what will they clear in 6 months; stated thus,

months months acres acres
2 : 6 :: 105 : *****$=105\times6\div2=315$ acres.

This answer, it will be perceived, is the same as that found by analysis. A little observation will show that the original third term 60, has been multiplied by both the second terms, 35 and 6, and divided by both the first terms, 20 and 2. Both the multiplications were not, it is true, performed at the same time, nor both the divisions; but first came a multiplication by 35, and then a division by 20, and then another multiplication by 6, followed by another division by 2. But, by the rules for composite numbers, it is evident, that the effect would have been just the same, if we had multiplied at once, by the product of the two multipliers, and then divided by the product of the two divisors. Thus,

$$\frac{35\times6\times60}{20\times2}=\frac{12600}{40}=315.$$

This expression may be simplified, by rejecting equal factors from both divisor and dividend, which is, in fact, reducing the Fraction to lower terms. It will then be

$$\frac{35\times3\times3}{1\times1}=35\times3\times3=315,$$

The two proportions, above, may therefore be combined into one, and 60 made the *common third* term to both the ratios, of time and of men—Thus,

$$\left.\begin{array}{l}20 \text{ men} : 35 \text{ men} \\ 2 \text{ months} : 6 \text{ months}\end{array}\right\} :: 60 \text{ acres} : *** \text{ acres}.$$

We see, that the ratio of 60 acres to the answer, is dependent on two other given complete ratios. The question, then, might be solved differently still, *by expressing each of the complete given ratios fractionally, and multiplying the remaining term by them successively*. Thus,

$\frac{35}{20}=\frac{7}{4}$, and $\frac{6}{2}=3$. Therefore,

$$\frac{7\times3\times60}{4}=3$$

The ratio of the third term to the answer, is made up of several *simple ratios*, multiplied together, or *compounded*, and hence it is called a COMPOUND RATIO. Thus, in the last example, 60 has to the answer, *a compound ratio of the times and the men*. A proportion, in which compound ratios occur is called a COMPOUND PROPORTION. A proportion consisting of *Simple ratios*, is called a SIMPLE PROPORTION.

11. If 64 men can dig 27 cellars, which are 24 ft. long, 18 ft. wide, and 16 ft. deep, in 36 days, working 8 hours a day; how many men will it take to dig 48 cellars, which are 54 ft. long, and 30 ft. wide, and 13 ft. deep, in 10 days, if they only work 6 hours in the day?

We will exhibit in detail the manner of stating, and solving this question by proportion. There are a number of circumstances, affecting the answer. But we will *first* inquire how many men it would take, *if nothing but the* NUMBER *of cellars were concerned.* The question, then, would be, if 64 men can dig 27 cellars, how many men can dig 48 cellars in the same time?

More cellars will require *more* men. By Simple Proportion

27 cellars : 48 cellars : : 64 men : *** men.

Next, we will find how many it would take, *if nothing but the* LENGTH *of the cellars were concerned.* The question, then is, if 64 men can dig cellars, 24 ft. long, in a certain time, how many men can dig similar cellars, 54 ft. long, in the same time?

More length will require *more* men. By Simple Proportion

24 feet : 54 feet : : 64 men : *** men.

Next, we will find how many, *if nothing but the* WIDTH *were concerned.* The question, then is, if 64 men dig cellars 18 ft. wide, how many men will dig similar cellars 30 ft. wide?

More width will require *more* men. By Simple Proportion

18 feet : 30 feet : : 64 men : *** men.

Next, we will find how many, *if nothing but* DEPTH *were concerned.* The question, then is, if 64 men dig cellars 16 feet deep, how many men will dig similar cellars 13 ft. deep?

Less depth will require *less* men. By Simple Proportion

16 feet : 13 feet : : 64 men : *** men.

Next, we will find how many, *if nothing but the* NUMBER OF DAYS *were concerned.* The question, then is, if 64 men dig cellars in 36 days, how many men will dig similar cellars in 10 days?

Less days will require *more* men. By Simple Proportion

10 days : 36 days : : 64 men : *** men.

Next, we will find how many, *if nothing but the* HOURS IN A DAY *were concerned.* The question, then is, if 64 men dig cellars by working 8 hours a day, how many men will dig similar cellars, working 6 hours a day. *Less* hours will require *more* men.

By Simple Proportion 6 hours : 8 hours : : 64 men : *** men.

All which may be collected, as in the last example, into one statement, thus :

$$\left.\begin{array}{l} 27 \text{ cellars} : 48 \text{ cellars} \\ 24 \text{ ft. length} : 54 \text{ ft. length} \\ 18 \text{ ft. width} : 30 \text{ ft. width,} \\ 16 \text{ ft. depth} : 13 \text{ ft. depth} \\ 10 \text{ days} : 36 \text{ days} \\ 6 \text{ hours} : 8 \text{ hours} \end{array}\right\} :: 64 \text{ men} : {*}{*}{*} \text{ men.}$$

Multiply together all the consequents, in the complete ratios, as before, and 64 by the product. $48 \times 54 \times 30 \times 13 \times 36 \times 8 \times 64 = 18,632,540,160$. This must be divided by the product of the antecedents. $27 \times 24 \times 18 \times 16 \times 10 \times 6 = 11,197,440$. Then $\frac{18632540160}{11197440} = 1,664$ *Ans.*

This is the method by proportion. Its principal disadvantage consists in the prolixity of the operation, when there are many ratios to be compounded. The process becomes shorter if we express each ratio fractionally, reduce it to its lowest terms, and finally multiply them all together with the third term 64.

$$\begin{array}{l|l} 27 : 48 = \frac{48}{27} = \frac{16}{9} & 16 : 13 = \frac{13}{16} \\ 24 : 54 = \frac{54}{24} = \frac{9}{4} & 10 : 36 = \frac{36}{10} = \frac{18}{5} \\ 18 : 30 = \frac{30}{18} = \frac{5}{3} & 6 : 8 = \frac{8}{6} = \frac{4}{3} \end{array}$$

Multiplying all these we have

$$\frac{16}{9} \times \frac{9}{4} \times \frac{5}{3} \times \frac{13}{16} \times \frac{18}{5} \times \frac{4}{3} \times 64 = \frac{43130880}{25920} = 1,664$$

This is sometimes called the method *by Ratios*. It is recommended for practice, in preference to *any other*, particularly with the following modifications.

Let the third term, (or antecedent of the incomplete ratio,) be written down, with all the second terms, (or consequents of the complete ratios,) as factors in the numerator of a Fraction, connected by the sign of Multiplication ; and all the first terms, (or antecedents of the complete ratios,) in the same manner, for a denominator, thus,

$$\frac{\overset{4}{64}\times\overset{2}{48}\times\overset{\overset{1}{2}}{54}\times\overset{\overset{1}{3}}{30}\times13\times\overset{2}{36}\times8}{\underset{1}{37}\times\underset{1}{24}\times\underset{1}{18}\times\underset{1}{16}\times\underset{1}{10}\times\underset{\underset{1}{2}}{6}}$$

Now observe whether any term in the Numerator has a common factor with one in the denominator, and, if so, divide both by that factor, and place the quotients directly under or over the terms divided. Thus, 27 and 54 may each be divided by 27. I accordingly divide and place the quotient, 1, under 27, and 2, over 54. In the same manner, I proceed with 24 and 48; 18 and 36; 16 and 64; and 10 and 30. I then see that the factor 6, in the denominator, and the quotient 3 over 30, may be divided by 3. I, therefore, divide and place the quotient 1, a step higher still, and the quotient 2, under 6, like the others. I then see a 2 over 54, and also a 2 under 6. I divide these, and put down the quotients, (1, in each case,) like the 1 over 30.

The whole Denominator is now exhausted. Each of its factors has become 1 by division, and consequently will not affect the Quotient, or value of the Fraction. Taking, therefore, the last obtained factors in the Numerator, (which are of course, those which stand highest,) and multiplying them (rejecting 1s) with the terms unaltered by the process, (viz. in this case, the 13 and 8) we shall obtain the answer. Thus,

4×2×13×2×8=1,664. *Ans.* as before.

We have been particular in explaining this mode, because it is usually *much the shortest*, and is *never longer* than any other. It also saves the trouble of a regular statement, for each ratio may be taken, (after the third term is written in the Numerator,) and written by itself, one term in the Numerator and the other in the Denominator, according to the direction given in Simple Proportion.

ANALYTIC SOLUTION. If it take 64 men to dig 27 cellars, it will take $\frac{1}{27}$ as many to dig 1 cellar; that is, it will take $\frac{64}{27}$ men. If it take $\frac{64}{27}$ to dig one 24 ft. long, it will take $\frac{1}{24}$ as many to dig one 1 ft. long; that is, it will take $\frac{64}{648}=\frac{8}{81}$ men, If it take $\frac{8}{81}$ to dig one 18 ft. wide it will take $\frac{1}{18}$ as many to dig one 1 ft. wide; that is, it will take $\frac{8}{1458}=\frac{4}{729}$ men. If it take $\frac{4}{729}$ men to dig one 16 ft. deep, it will take $\frac{1}{16}$ as many to dig one 1 ft. deep; that is, it will take $\frac{4}{11664}=\frac{1}{2916}$ men. If $\frac{1}{2916}$

will do it in 36 days, it will take 36 times as many to do it in one day; that is, it will take, $\frac{36}{2916}=\frac{1}{81}$ men. If $\frac{1}{81}$ will do it in a day of 8 hours long, it will take 8 times as many to do it in a day of 1 hour long; that is, it will take $\frac{8}{81}$ men.

Now if $\frac{8}{81}$ dig 1 cellar, it will take 48 times as many to dig 48 cellars; that is it will take $\frac{384}{81}=\frac{128}{27}$ men. If it take $\frac{128}{27}$ when the length is 1 ft. it will take 54 times as many, when the length is 54 ft.; that is, it will take $\frac{6912}{27}=256$. If it take 256 men, when the width is 1 ft. it will take 30 times as many, when the width is 30 ft; that is, it will take 7,680 men. If it take 7,680, when the depth is 1 ft., it will take 13 times as many, when the depth is 13 ft.; that is, it will take 99,840 men. But if 99,840 do it in 1 day, it will require only the tenth as many for 10 days; that is, it will require 9,984 men. If it require 9,984 men, when the days are 1 hour long, it will require but the $\frac{1}{6}$ part as many, when the days are 6 hours long; that is it will require 1,664 men. *Ans.* as before.

By the analytic method, the pupil will perceive that we take each term in the given or supposed case, and find what the answer would be, if that term were made 1. We then, consider what the answer would be, if we increase each term separately, from 1 to the number in the case required to be solved. The teacher should make this clear by illustration and he should require his pupil to find many answers by analysis, for the purpose of mental discipline. For common practice the mode recommended before is best. The pupil will observe, that, by the analytic method we sometimes obtain, in the course of the process, expressions for things which can, actually, have no existence; as, for instance, $\frac{1}{81}$ of a man, in the last example. But, he should recollect that by this expression is only meant $\frac{1}{81}$ of the *labour* of a man, and not of a man's person. Similar explanations may be given in all cases, where these quantities occur.

As we were speaking just now, of *supposed* and *required* cases, contained in a question, it may be well to notice the distinction. It will be observed, that, in the last example, is contained a case *complete in every term;* viz. that "64 men can dig 27 cellars, 24 ft. long, 18 ft. wide and 16 ft. deep in 36 days, working 8 hours a day." This is *complete in every term*, because every number concerned is *given*. This is the *supposed case*, or *supposition* of the question. In the same question is contained *another* case, *incomplete in one term*, which term is *required to be supplied;* and hence, the case is called the *required case*, or the *requisition*, or the *demand* of the question. In the last example, the requisition or demand is, "*how many men* can dig 48 cellars, that are 54 feet long, and 30 feet wide, and 13 feet deep, in 10 days, working 6 hours a day?". Every term in this case is complete except the *number of men*, which is *required*, or *demanded.*

The terms of the supposed case are called TERMS OF SUPPOSITION.

The terms of the required case are called TERMS OF DEMAND.

NOTE. The expression "required *case*," is not strictly proper, since but one *term* is required. Its conciseness, however, recommends it to use, and if it is understood, nothing more is necessary.

From the preceding we derive the following rules :

I. BY PROPORTION. ARRANGE EACH COMPLETE RATIO, BY THE DIRECTIONS IN SIMPLE PROPORTION. COMPOUND THESE RATIOS, BY MULTIPLYING ANTECEDENTS TOGETHER, AND CONSEQUENTS TOGETHER. THE RESULT WILL BE THE RATIO OF THE REMAINING TERM TO THE ANSWER ; WHICH OBTAIN AS IN SIMPLE PROPORTION.

Of this rule we have the following abbreviations.

I. EXPRESS EACH COMPLETE RATIO FRACTIONALLY, AND REDUCE IT TO ITS LOWEST TERMS. FIND THE CONTINUED PRODUCT OF ALL THE REDUCED RATIOS AND THE REMAINING TERM. Or,

II. WRITE THE THIRD TERM, AND ALL THE CONSEQUENTS OF THE COMPLETE RATIOS AS FACTORS IN THE NUMERATOR OF A FRACTION ; AND UNDER THEM ALL THE ANTECEDENTS, AS FACTORS IN THE DENOMINATOR. REJECT COMMON FACTORS FROM THE NUMERATOR AND DENOMINATOR UNTIL NO MORE CAN BE FOUND, WRITING THE QUOTIENTS, INSTEAD OF THEM. REDUCE THE FRACTION THUS OBTAINED, IF POSSIBLE, TO A WHOLE OR MIXED NUMBER. This method is best.

II. BY ANALYSIS. CONSIDER WHAT THE ANSWER WOULD BE, SUPPOSING EACH TERM OF SUPPOSITION, SEPARATELY, CHANGED TO A UNIT. THEN SUPPOSE EACH TERM SEPARATELY CHANGED AGAIN, FROM A UNIT TO THE CORRESPONDING TERM OF DEMAND, AND CONSIDER WHAT THE ANSWER WOULD BE. THIS LAST ANSWER IS THE TERM REQUIRED.

The above rule by proportion is sometimes called the DOUBLE RULE OF THREE.

EXAMPLES FOR PRACTICE.

12. If 20 bushels of wheat are sufficient for 8 persons, 5 months, how many will be sufficient for 4 persons, 12 months ? A. 24.

RULE I. PROPORTION, $\left.\begin{array}{r}\text{8 persons : 4 persons}\\ \text{5 months : 12 months}\end{array}\right\}$ :: 20 bu. : *** bu.

Compound ratio 40 : 48 :: 20 : $\frac{20\times48}{40}=24$

Or,

I. Ratio of persons $\frac{4}{8}=\frac{1}{2}$ Ratio of mo. $\frac{12}{5}$. Then $20\times\frac{12}{5}\times\frac{1}{2}=24$

Or,

II. Fraction $\dfrac{\overset{4}{20}\times\overset{2}{4}\times\overset{3}{12}}{\underset{\underset{1}{2}}{8}\times\underset{1}{5}}=\frac{24}{1}=24$

RULE II. ANALYTIC SOLUTION. If 20 bushels are enough for 8 persons, $\frac{1}{8}$ of 20 will answer for 1 person, that is, $\frac{20}{8}=\frac{5}{2}$ bushels. If $\frac{5}{2}$ answer for 5 months, $\frac{1}{5}$ as much will answer for 1 month, that is $\frac{1}{2}$ bushel. If $\frac{1}{2}$ answer 1 person, 4 persons will need 4 times as much, that is $\frac{4}{2}=2$ bushels. If they need 2 bu. in 1 month, they will need 24 bu. in 12 mo. *Ans.*

13. If 7 men build 36 rods of wall in 3 days, how many rods can 20 men build in 14 days. A. 480 rods.

14. If $100 will gain $6 interest in 1 year. what will $400 gain in 7 months? *Ans.* $14.

15. If 20 men perform a piece of work in 30 days, how many men will it take to accomplish a piece 4 times as large in 4 days? *Ans.* 600 men.

16. If 56 lb. of bread will last 7 men, 14 days, how much will last 3 men, 21 days? *Ans.* 36.

17. If 7 men can reap 84 acres of wheat in 12 days, how many men can reap 100 acres in 5 days? *Ans.* 20 men.

18. If a family of 9 persons spend $305 in 4 months, how much would they spend in 8 months if 5 persons were added? *Ans.* 948.88\frac{8}{9}$.

19. A hare is 50 leaps before a grey-hound, and takes 4 leaps, while the grey-hound takes 3. But 2 of the grey-hound's leaps are equal 3 of the hare's. How many leaps must the grey-hound make to overtake the hare? *Ans.* 300.

20. If $2,000 will support a garrison of 150 men, 3 months, how long will $6,000 support 600 men? *Ans.* 2¼ mo.

21. If the transportation of 12 cwt. 3 qrs. 400 mls. cost $57.12, what will the transportation of 10 T. 75 mls. cost? *Ans.* $168.

22. If 8 men can build a wall 20 ft. long, 6 ft. high, and 4 ft. thick, in 12 days; how long will it take 24 men to build one 200 ft. long, 8 ft. high and 6 ft. thick? *Ans.* 80 days.

23. If 18 men can build a wall 40 rds. long, 5 ft. high and 4 ft. thick in 15 days; in what time will 20 men build one 87 rds. long, 8 ft. high, and 5 ft. thick? *Ans.* 58$\frac{29}{40}$ days.

24. If 180 men, working 6 days, each day 10 hours, can dig a trench 200 yards long, 3 yds. wide, and 2 yds. deep; how many days will 100 men be occupied in digging a trench 360 yds. long, 4 wide, and 3 deep, working 8 hours a day? *Ans.* 48$\frac{3}{5}$ days.

25. If 100 lb. weight English make 95 lb. Flemish, and 19 lb. Flemish, make 25 lb. at Bologna; how many lbs. English are equal to 50 lb. at Bologna? *Ans.* 40 lb. English.

26. If 24 lb. at New York make 20 lb. at Amsterdam, and 50 lb. at Amsterdam, are equal to 60 lb. at Paris; how many lbs. at Paris, are equal to 100 lb. at New York? *Ans.* 100 lb.

This comparison of the weights, measures and coin of two countries, through the medium of those of other countries, is sometimes called CONJOINED PROPORTION. Merchants are accustomed to call it the CHAIN RULE. The process belongs to Compound Proportion.

27. If 70 braces at Venice are equal to 75 braces at Leghorn, and 7 braces at Leghorn be equal to 4 yards in Hartford; how many braces at Venice are equal to 64 yards in Hartford? *Ans.* 104$\frac{8}{15}$.

28. If 40 lbs. at Boston make 48 lb. at Antwerp, and 30 lb. at Antwerp make 36 lb. at Leghorn; how many lbs. at Leghorn are equal to 100 lb. at Boston. *Ans.* 144 lb.

29. A merchant in Petersburgh has to pay 1,000 ducats in Berlin, which he wishes to pay in rubles by the way of Holland; and

he is acquainted with the following proportional values of moneys, viz. 2 rubles equal 95 stivers; 40 stivers make 2 florins; 5 florins make 2 rix-dollars, Hollandish; 100 rix-dollars Hollandish make 142 rix-dollars Prussian; and finally 3 ducats in Berlin equal 9 rix-dollars Prussian. How many rubles must he pay?

Ans. 2,223.87—rubles.

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§ XCIV. Compound Fellowship, by Analysis, has already been explained. (§ XLIX.) Questions in Compound Fellowship may be solved also, by Compound Proportion.

1. A and B traded, A furnishing $215 for 6 months, and B $390 for 9 months. A's share of gain was then $53.75. What was B's, and what the whole gain? *Ans.* B's $146.25. Whole $200.

Proportion, $\left.\begin{array}{r}\$215 : \$390\\ 6\text{ mo.} : 9\text{ mo.}\end{array}\right\} :: \$53.75 : \$***$

2. A and B traded, A funishing $1,000 for 12 mo. His gain was then $200, while B, whose capital was in trade only 8 mo. gained $300. What was B's capital? *Ans.* $2,250.

Proportion, $\left.\begin{array}{r}\$200 : \$300\\ 8\text{ mo.} : 12\text{ mo.}\end{array}\right\} :: \$1{,}000 : \$***$

3. A furnished a common stock with $4,000 for 12 mo.; B, with $3,000 for 15 mo.; C, with $5,000 for 8 mo. C's gain is $200. What is the whole gain and what the gain of the other partners?

Ans. Whole $665. A's $240. B's $225.

4. A has $500 capital for 12 mo. B $600 for 10 mo. and C $800 for 6 mo. B's gain is $250. What is the whole gain, and what the gain of the other partners?

Ans. Whole gain $700. A's 250. B's 200.

5. A has $200 capital for 2 mo.; and B has $100 for 7 mo. Their joint gain is $55. What is that for each?

Ans. A $20. B $35.

6. A, B and C trade in company, and gain $480. A's capital is $800 for 4 mo.; B's $600 for 2 mo.; and C's $720 for 5 mo.—What is each man's share of the gain?

Ans. A's $192, B's $72, and C's $216.

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ALLIGATION.

MENTAL EXERCISES.

§ XCV. 1. A grocer mixed 8 gals. of water with 8 gals. of brandy worth $1 pr. gal. What was the mixture worth pr. gal.?

Note. The 8 gals. of brandy were worth $8; but, after the mixture, there were 16 gals. worth no more than before; that is, worth only $8.

2. A merchant mixed 6 lb. of tea at $1 pr. lb. with 4 lb. worth 3\frac{1}{2}$ pr. lb. What was the mixture worth pr. lb.?

Note. There were $6+4=10$ lb., and the whole price was $6\times1=\$6+4\times3\frac{1}{2}=\$14$, amounting to $20. The price of 1 lb. is, therefore, $\frac{20}{10}=\$2$. *Ans.*

3. If 4 bu. of wheat, worth \$1½ be mixed with 5 bu. worth \$2⅗; what is the mixture worth pr. bu.?

4. If 6 boxes of raisins, worth \$4 pr. box, be mixed with 4 boxes, worth \$1½ a box; what will the mixture be worth pr. box.?

Let the following be written.

5. A grocer mixed 81 lbs. of sugar at 19 cts. pr. lb. with 54 lbs. at 14 cts. What pr. lb. was the mixture worth?

81 lbs. at 19 cts., are worth	\$15.39
54 lbs. at 14 cts., are worth	7.56
The whole 135 lbs. are worth	\$22.95

It needs no reasoning to show that if I divide \$22.95, the worth of 135 lbs. by 135, I shall obtain the price of 1 lb. Hence, $\frac{2295}{135}$ = 17 cts. *Ans.*

This process is called ALLIGATION MEDIAL.

ALLIGATION is a general name given to the mixing of simple things of different qualities, so as to form a compound of a medium, or mean quality.

It will be observed that ALLIGATION MEDIAL *embraces those instances in which the quantities and prices of the simples are given, and the price of the compound required.* Hence, its rule is,

DIVIDE THE WHOLE COST OF THE SIMPLES, BY THE WHOLE QUANTITY.

6. If 20 bu. of wheat worth \$1.35 pr. bu., be mixed with 10 bu. of wheat worth 90 cts. pr. bu., what will the mixture be worth pr. bu.? A. \$1.20.

7. A grocer mixed equal quantities of sugar at 8 cts. pr. lb.; at 9 cts. pr. lb; at 10 cts. pr. lb.; and at 12 cts. pr. lb. What was 5½ cwt. worth? A. \$55.965.

8. If 4 lbs. of gold, 23 carats fine, be mingled with 2 lbs. 17 carats fine, what will be the fineness of the mixture? A. 21 carats.

9. If I mingle 3 kegs of raisins worth 11 cts. pr. lb., and each containing 230 lbs., with 5 kegs worth 13 cts. pr. lb., and each containing 175 lbs., what is the value pr. lb. of the mixture?

10. If 3 barrels of brandy, containing each 29 gals. be mixed, what will be the price pr. gal. of the mixture, supposing the first worth 95 cts. pr. gal., the second \$1.10, and the third \$1.25?

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§ XCVI. 1. A merchant has teas at \$1 pr. lb., and at \$3 pr. lb. He wishes to make such a mixture of them, that the price may be \$2 pr. lb. What quantity must he take of each?

Since \$3 exceeds \$2 just as much as \$1 falls short of \$2, the following will be evident.

A pound of the first simple is worth \$1, which is a dollar too small for the man's purpose; and a pound of the second is worth \$3, which is a dollar too great for his purpose. But if he puts these two together, it is plain that the \$1 *deficiency* of the one, *will be balanced* by the \$1 *excess* of the other, so that the price will be exactly what he wants, that is, \$2. The answer, then, is 1 lb. of each, or, generally, *any equal quantities of each.*

2. A merchant has sugar at 8 cts. and at 11 cts. a pound. He wishes to make a mixture, worth 10 cts. a pound. Required the necessary proportional quantity of each kind.

Here 8 cents falls short of 10 cts. just twice as much as 11 cts. exceeds 10 cts. Hence, it will take *two* lbs. at 11 cts. to exceed the same number of lbs. at 10 cts. as much as *one* lb. at 8 cts. falls short of one at 10 cts. Hence, if 2 lbs. at 11 cts. be mixed with 1 lb. at 8 cts. the *excess* of the one will be exactly *balanced* by the *deficiency* of the other, and the price will be 10 cts. as required.

Ans. 2 lb. at 11 cts. and 1 lb. at 8 cts., or, *any other quantities having the same ratio.*

3. A man has cider at $2 and at $7 a barrel. He mixes in such a manner that the mixture is worth $5 a barrel. What quantities does he take of each?

Here $7 exceeds $5 by $2, and $2 falls short of $5 by $3. If he were to take *three* barrels, then, at the greater price, (viz. $7) he would have 3×$2=$6 too much. But if he were to take *two* barrels, at the less price, (viz. $2) he would have 2×$3=$6 too little. If, then, he were to put these together, the $6 *excess* would be exactly *balanced* by the $6 *deficiency*, and the medium price would be $5, as required.

Ans. 2 bls. at $2, and 3 bls. at $7, or *any other quantities having the same ratio.*

The correctness of all these conclusions may be *verified*, or proved, by reversing the question, and proceeding as in the last §. Thus, in the last instance, the inquiry reversed would be, if 2 bls. at $2 be mixed with *3 bls. at $7, what will* be the price of the mixture pr. bl.?

By last § $\frac{2\times2+3\times7}{2+3}=\frac{25}{5}=$ $5, the given price above.

From the preceding examples, we see, that

THE QUANTITY OF A SIMPLE, TO BE TAKEN, VARIES WITH THE DIFFERENCE BETWEEN ITS PRICE AND THE MIXTURE PRICE; INCREASING AS THIS DIFFERENCE DIMINISHES, AND DIMINISHING AS IT INCREASES. It is therefore *inversely* as this difference.

It will be observed of all the questions, in this section, that they require the *proportional quantities, which are to be mixed* when the prices of the different simples, and the mixture price are given. This is called ALLIGATION ALTERNATE. When but *two* Simples are given, the process is easy, as shown above. When there are more than two, the case requires further illustration.

4. A merchant mixed grain at the following prices, viz. 70 cts. 95 cts. and $1.00 pr. bu. He made the mixture worth 85 cts. What quantities of each did he take?

First, suppose he had but two kinds to be mixed, at the prices 70 cts. and 95 cts. The differences between these prices and and 85 cts., the mixture price, are 15 and 10. And, as the quantities required are inversely as these differences, we have 10 bu. at 70 cts., and 15 bu. at 95 cts.; or any other quantities, having the same ratio.

Next, suppose he had only the kinds at 70 cts., and $1. The differences are 15 and 15; and, of course, the quantities required are 15 bu. at 70, and 15 bu. at $1.00, or any other quantities having the same ratio.

In both of the cases above, the compound is worth 85 cts. pr. bu. If then the two *compounds be themselves mixed*, the price will not be altered, since the quantities mixed, are of the same value per bu.

Hence, he might have mixed 10+15=25 bu. at 70 cts., 15 bu. at 95 cts., and 15 bu. at $1,00, or any other quantities, having the same ratio, and still had his mixture price 85 cts.

The common mode of performing examples of this kind is as follows,

WRITE THE PRICES OF THE SIMPLES, ONE BELOW ANOTHER, PROCEEDING FROM THE SMALLEST TO THE GREATEST, AND WRITING THE MIXTURE PRICE ON ONE SIDE. THEN CONNECT BY A LINE

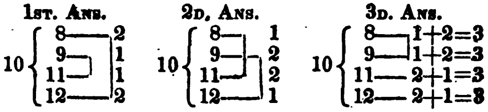
$$85\left\{\begin{array}{lcl}70 & \text{———} & 10+15\\ 95 & \text{—} & 15\\ 100 & \text{———} & 15\end{array}\right\}\textit{Ans.}$$

EACH PRICE GREATER THAN THE MIXTURE PRICE WITH AT LEAST ONE WHICH IS LESS; AND, ON THE OTHER HAND, EACH ONE THAT IS LESS THAN THE MIXTURE PRICE, WITH ONE, AT LEAST, THAT IS GREATER. TAKE THE DIFFERENCE BETWEEN EACH SIMPLE PRICE AND THE MIXTURE PRICE, AND PLACE IT OPPOSITE EVERY NUMBER, WITH WHICH THAT SIMPLE PRICE IS LINKED. THE NUMBER, OR SUM OF THE NUMBERS STANDING OPPOSITE EACH SIMPLE PRICE, WILL BE THE RELATIVE QUANTITY AT THAT PRICE.

Thus, in the above example, 70 being less than 85; and 95 and 100 being both greater, 70 is linked with both the others. Then 15 = 85—70 is placed opposite both 95 and 100, and 95—85 = 10, and 100—85 = 15 are both placed opposite 70. Then 10+15 = 25 is the quantity at 70, 15 is the quantity at 75, and 15 the quantity at 100.

It will be perceived that we may have, in many cases, a variety of answers all corresponding with the requisition of the question. This results from the various modes in which we may link the simples. Where the simples are numerous, and about as many above the mixture price, as below it, the variety is very great. Thus,

5. A grocer wished to mix four qualities of sugar, worth 12 cts., 11 cts., 9 cts. and and 8 cts pr. lb. respectively; so that the mixture should be worth 10 cts. pr. lb. What quantities could be have taken of each ?



Here we have three answers; and as the pair of quantities resulting from each simple linking consists of *relative* numbers, for which any others bearing the same ratio to each other may be substituted, it follows that the variety of answers is indefinite. Several other answers might be found in this example, directly, by linking.

6. A merchant would mix gold of 17 carats fine, with some of 19, 21, and 24 carats fine, so that the compound may be 22 carats fine. What quantity of each must he take ? *Ans.* 2 parts of each of the first three kinds, and 9 of the last.

7. It is required to mix liquors at 75, 88 and 90 cts. pr. qt. with water, so that the mixture shall be worth 60 cts. pr. qt. What quantities must be taken ? *Ans.* 73 qts of water, and 60 of each other liquor.

8. I wish to mix wheat at $2.25 per bu., with some that is $3.00 pr. bu. and some that is $1.75 per bu., so that the compound shall be worth $2.50 per bu. Required the quantities of each.

9. A grocer wishes to mix raisins at 25 cts. pr. lb., with others at 18, 23, 27, 30, and 40 cts. respectively, so that the compound may be worth 29 cts. Required the quantities of each.

We have seen that the quantities obtained by this process are merely *relative*, and that any others might be substituted in their places, which have to each other the same ratio. Hence, if it is desired that a *particular quantity* of one of the ingredients, either greater or less than that found by linking, should be employed, it is only necessary to *vary the others, so that they may still bear to it the same ratio.* This is done by Simple Proportion. The teacher may also exhibit the *method by analysis.*

§ XCVII. 1. I wish to mix 2 gallons of brandy, at \$1.50 pr. gal. with rum at 80 cts. pr. gal., so that the mixture may be worth \$1.00 pr. gal. What quantity of rum must I take?

$$100\left\{\begin{array}{lr} 80 & 50 \\ & \\ 1.50 & 20 \end{array}\right.$$

Here I obtain 20 gals. of brandy by linking. This, by the terms of the question, must be diminished down to 2, and therefore, it is necessary that the 50 gals. of rum should be diminished *in the same ratio.*

(§ XCI.) $20 : 2 :: 50 : 50\times2\div20=5$ gals. rum *Ans.*

The proportion then seems to be,

AS THE RELATIVE QUANTITY OF THE LIMITED INGREDIENT, IS TO THE GIVEN QUANTITY; SO IS THE RELATIVE QUANTITY OF EACH OTHER INGREDIENT TO THE ABSOLUTE QUANTITY OF THAT INGREDIENT REQUIRED. And, hence, the rule,

MULTIPLY EACH RELATIVE QUANTITY BY THE GIVEN QUANTITY OF THE LIMITED INGREDIENT; AND DIVIDE THE PRODUCT BY THE RELATIVE QUANTITY OF THE SAME INGREDIENT. THE SEVERAL QUOTIENTS WILL EXPRESS THE QUANTITIES REQUIRED.

2. A merchant has spices at 32 cts. 40 cts. and 64 cts. pr. lb. He wishes to mix 5 lbs. of the first with the others, so that the compound may be worth 48 cts. How much of each must he use?

Ans. 5 lb. of the 2d, and 7 lb. 8 oz. of the third.

3. A farmer wishes to mix 16 bu. of rye worth 50 cts. pr. bu. with corn at 40 cts. and oats at 30 cts. pr. bu. so that the mixture may be worth 37 cts. pr. bu. What quantities must he take of each?

When the whole compound is limited; that is, when it is desired that the whole mixture should amount to a certain quantity, the process is somewhat similar. For, since each individual quantity found by linking, is a relative quantity, it is evident, that the sum of the whole can only be a *relative sum.* Hence, *if this sum be increased or diminished, the quantity of each ingredient must be increased or diminished in the same ratio.* This is done by Simple Proportion, or by analysis.

4. A flour merchant, having flour at \$4, \$6, and \$8, pr. bar. sold 120 bar. at the *average* price of \$6.50 pr. bar. How many barrels of each kind must he have sold, in order not to have gained nor lost?

NOTE. This case is evidently the same as if the flour had been mixed, as far as calculation is concerned.

$$650\left\{\begin{array}{ll} 400 & 150 \\ 600 & 150 \\ 800 & 250+50=300 \end{array}\right. \qquad 150+150+300=600 \text{ sum.}$$

Here, by linking, I obtain 150 & 150 & 300, the several quantities of the Simples. The sum is 600. Now, by the terms of the question, this must be diminished down to 120, and of course, the quantity of each Simple, must be diminished *in the same ratio.*

(§ XCI.) $600 : 120 :: 150 : 120\times150\div600=30$ at \$4, and also at \$6, since its relative quantity is likewise 150. Also

$$600 : 120 :: 300 : 120\times300\div600=60 \text{ at } \$8.$$

The proportion, then, is,

AS THE SUM OF THE RELATIVE QUANTITIES, IS TO THE GIVEN QUANTITY; SO IS EACH RELATIVE QUANTITY, TO THE ABSOLUTE QUANTITY OF THAT INGREDIENT REQUIRED. And the rule, MULTIPLY EACH RELATIVE QUANTITY BY THE GIVEN QUANTITY, AND DIVIDE THE PRODUCT BY THE SUM OF THE RELATIVE QUANTITIES. THE SEVERAL QUOTIENTS WILL EXPRESS THE QUANTITIES REQUIRED.

It may be mentioned, though perhaps the pupil has always made the discovery himself, that this process is precisely the same with that employed in Simple Fellowship.

5. How much gold, that is 15, 17, 18, and 22 carats fine, must be mixed together to form a compound of 40 lb. 20 carats fine ?

Ans. 5 lb. of 15, 17, and 18, and 25 lb. of 22.

6. A man would mix 100 lbs. of sugar, at 8 cts. 10 cts. and at 14 cts. pr. lb. so that the compound may be worth 12 cts. pr. lb. What quantity of each must he use ?

Ans. 20 lb. at 8, and at 10 cts., and 60 lb. at 14 cts.

7. A grocer has currants at 5 cts. 7 cts. 10 cts. and 12 cts., and he wishes to mix them so as to sell them at 8 cts. pr. lb. How many lb. of each sort must he take ?

This and some of the following admit of a great variety of answers.

8. A farmer mixed meal of the value of 25 cts. pr. bu. with other kinds of the values of 30 cts., 35 cts. and 18 cts. The compound contained 4 bu. and was worth 28 cts. pr. bu. What quantity did he take of each?

9. If a grocer make a mixture of teas of the following prices, pr. lb. viz. $1.25, $1.40, $1.63, and $1.75, so that the mixture may be sold at $1.50 ; how much does he take of each kind, supposing the whole mixture to contain 120 lb. ?

10. If a grocer fill 3 wine hogsheads with water and liquors at $1.20, $1.30, and $1.40 a gallon ; how much does he take of each, supposing the compound worth $1.35 pr. gal. ?

11. In a brewery there are in an upper room 3 vats, each capable of containing 120 gals., and in a lower room 1 vat capable of holding 192 gals. The lower vat is empty and the three upper ones are filled with beer, worth respectively 12 cts. 20 cts. and 30 cts. pr. gal. Pipes are set running at the same moment, from each upper vat to the lower, and in 1 hour, exactly, it is filled. The owner then finds, on mixing what remains in the upper vats, that the compound is worth 25 cts pr. gallon. What is the value of the mixture in the lower vat, and in what time would each pipe separately have filled it ?

A. Price, $0.168⅞.—Times, for first two, 1 h. 8 m.—for last, 16 h.

12. Suppose there are vats situated as the above, the upper ones being of the same dimensions, and containing beer of the values 14 cts. 18 cts. and 32 cts. pr. gal. Suppose the lower one filled as before, in an hour, and that there remains of the first kind of beer, 40 gals., the price of the mixture in the room above, being 25 cts., as before. Required the dimensions of the lower vat, the value pr. gal. of the mixture contained in it, and the time in which each pipe will fill it.

INVOLUTION.

MENTAL EXECRISES.

XCVIII. 1. How much is 3 times 3? 2 times 2? 4 times 4?

2. How much is 5 times 5? 6 times 6? 7 times 7? 8 times 8? 9 times 9? 10 times 10? 11 times 11? 12 times 12?

3. If you multiply 2 into 2, and that product, again by 2, what result do you obtain?

4. How much is 3 multiplied by 3, and the product again by 3?

5. How much is 4×4×4?

6. How much is 2×2×2×2?

7. How much is 3×3×3×3?

8. How much is 2×2×2×2×2?

9. How much is 2×2×2×2×2×2?

Here you perceive that you have been making continued multiplications of numbers into themselves, or, into the same numbers.

THE PRODUCT, WHICH ARISES FROM MULTIPLYING A NUMBER, ONE OR MORE TIMES BY ITSELF, IS CALLED A POWER.

THE PROCESS OF FINDING POWERS IS CALLED INVOLUTION.

Involution, of course, consists in the continued multiplication of a number by itself. This number, and in fact, any number is called the FIRST POWER, of itself. It is likewise called the ROOT *of the other powers;* since they seem to *grow up* out of it.

Different powers have different names. Thus we have the *second, third, fourth, fifth,* &c. powers. 2×2=4 is the *second* power of 2, *because 2 is used as a factor, twice,* in finding it. 3×3×3=27 is the *third* power of 3, *because 3 is used as a factor three times* in finding it. 3×3×3×3=81 is the *fourth* power of three, *because 3 is used as a factor, four times,* in finding it. Hence,

A POWER TAKES ITS NAME, OR NUMBER, FROM THE NUMBER OF TIMES THE ROOT IS CONTAINED IN IT AS A FACTOR.

The pupil must carefully distinguish between this number, and the number of *multiplications,* necessary in finding a power. For in finding the second power of 2, for instance, we multiply 2 into 2 and call the product the second power, because 2 is used twice *as a factor* in obtaining it. But it will be seen that but one multiplication takes place. So also the expressions 3×3, 5×5×5, 7×7×7×7 denote, respectively the second power of 3, the third power of 5, and the fourth power of 7. The numbers, themselves, of course, are the *factors,* and the crosses represent the number of *multiplications.* In every instance the latter number is *a unit* less than the former. Hence,

THE NUMBER OF A POWER IS ALWAYS A UNIT GREATER THAN THE NUMBER OF MULTIPLICATIONS NECESSARY TO PRODUCE IT.

A short mode of expressing powers is often employed for the sake of convenience. For instance, instead of writing out the words, *fourth power of 7,* in full, we write the root, 7, simply, and the number of the power, at the right of it a little elevated, thus, 7^4

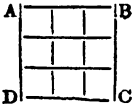
So, the third power of 8 is written, 8^3

The eighth power of 5, 5^8 &c.

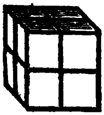
This mode of writing is called *indicating the power.*

The small figure at the right is called the INDEX, or EXPONENT, of the power. Thus ^{8} is the *index* of the eighth power of 5, in the last example; and shows, that, in order to obtain the power, 5 must be employed 8 times *as a factor.* Hence,

THE INDEX, OR EXPONENT OF A POWER IS THE NUMBER, FROM WHICH THE POWER DERIVES ITS NAME, AND SHOWS HOW MANY TIMES THE ROOT MUST BE USED AS A FACTOR TO PRODUCE THAT POWER.



The second power is likewise called the SQUARE ; because, if the root be one side of a figure of that form, the power will be the surface contained within it. Thus the figure A B C D represents a square, whose side is 3. The surface, it will be seen, is 9, which is 3×3 or 3^2.



The third power, is, also, called the CUBE ; because, if the root be one side of a solid of that description, the power will express the solid contents. Thus, in the figure we have a cube, whose side is 2. It will be readily seen, that the solidity is $8 = 2 \times 2 \times 2 = 2^3$.

The fourth power is likewise called the BIQUADRATE ; which means *twice-squared*, and is applied to this power, because it is the same as the square of the square; the original root being a factor 4 times in each case.

The fifth power is called the SURSOLID ; which means *beyond a solid*, and is perhaps applied in distinction from the cube, which has a solid to represent it.

The sixth power is called the SQUARE-CUBED ; which term explains itself.

All these powers are likewise called by their *numbers* ; and except the square and cube, the above names are little used.

We have seen that powers may be indicated by exponents. When a power is found by *actual multiplication*, involution is said to be *performed*, and the number is said to be *involved*.

Hence, to involve a number,

MULTIPLY IT BY ITSELF UNTIL IT IS EMPLOYED AS A FACTOR, AS OFTEN AS THERE ARE UNITS IN THE INDEX OF THE POWER, TO WHICH IT IS TO BE RAISED.

Involution may be abbreviated, *by multiplying together any two or more powers of the same root, already found, whose indices, together, make up the required index.* For the sum of the indices, shows how often the root is contained as a factor in the product.

EXAMPLES FOR PRACTICE.

10. What is the square of 13? Ans. $13 \times 13 = 169$.
11. What is the cube of 5? Ans. $5 \times 5 \times 5 = 125$.
12. What is the 4th power of 4? Ans. 256.
13. What is the square of 14? A. 196.
14. What is the cube of 6. A. 216.
15. What is the fifth power of 2? A. 32.
16. What is the seventh power of 2? A. 128.
17. What is the square of $\frac{1}{2}$? A. $\frac{1}{4}$.
18. What is the cube of $\frac{2}{3}$? A. $\frac{8}{27}$.

Hence, a Fraction is involved, by involving both numerator and denominator.

19. What is the fourth power of $\frac{3}{5}$? A. $\frac{81}{625}$.
20. What is the square of $5\frac{1}{2}$? A. $30\frac{1}{4}$.
21. What is the square of $30\frac{1}{4}$? A. $915\frac{1}{16}$.
22. Perform the involution of 8^5. A. 32,768.
23. Involve $\frac{4}{39}$, $\frac{11}{12}$ and $\frac{8}{9}$ to the third power each.
A. $\frac{64}{59319}$; $\frac{1331}{1728}$; $\frac{512}{729}$.
24. Involve 211^3 A. 9,393,931.
25. Raise 25 to the fourth power. A. 390,625.
26. Find the sixth power of 1.2. A. 2.985984.

27. Find the 11th power of 67. The 6th power of 41.
28. Find the 4th power of 821. The 7th power of 13.
29. Involve 35^5. Involve 72^3. Involve 91^2.
30. Involve 76^4. Involve 85^9. Involve 311^{11}.

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EVOLUTION.

MENTAL EXERCISES.

§ XCIX. 1. 4 is a square or 2d power; what is its root? that is, what number, multiplied by itself, will produce 4?

2. 9 is a square; what is its root? that is, what number, multiplied by itself, will produce 9?

3. 16 is a square; what is its root? 36 is a square; what is its root?

4. 8 is a cube, or 3d power; what is its root? that is, what number, taken 3 times as a factor, will produce 8?

5. 27 is a cube; what is its root? 81 is a square; what is its root?

6. $\frac{1}{9}$ is a square; what is its root? $\frac{16}{81}$ is a 4th power; what is its root?

Hence, the root of a Fraction is found by taking the roots both of the numerator and denominator

7. $\frac{4}{16}$ is a square; what is its root? 64 is a cube; what is its root?

8. $\frac{8}{27}$ is a cube; what is its root? 25 is a square; what is its root?

9. Find the square root of 64. The square root of 49.

10. Find the square root of $\frac{49}{64}$. Of $\frac{36}{49}$. Of $\frac{64}{81}$.

11. Find the 4th root of 16. Of 81. Of $\frac{81}{10000}$.

12. Find the square root of $\frac{4}{36}$. Of $\frac{25}{100}$. Of $\frac{100}{144}$. Of $\frac{9}{121}$.

The requisition, in the above examples, will be seen to be exactly the reverse of that, contained in the questions in the last §. There, *a root was given to find a power.* Here, *a power is given to find a root.* It may be well to give a more particular definition of a root, than we have yet done.

A ROOT OF ANY NUMBER IS A FACTOR, WHICH, MULTIPLIED ONE OR MORE TIMES INTO ITSELF, WILL PRODUCE THAT NUMBER.

THE PROCESS OF FINDING ROOTS IS CALLED EVOLUTION.

It consists of course is ascertaining the factor, which multiplied into itself, one or more times, will produce the given number. Of course it is the opposite of Involution. It may briefly be defined, *the resolving of a number into two or more* EQUAL FACTORS.

Different roots have different names, corresponding to those of the different powers. 2 is the *second or square root* of 4, because 2 must be taken twice as a factor to produce 4. Thus 2×2=4. 3 is the *third or cube* root of 27 because it must be used as a factor three times to produce 27. Thus, 3×3×3=27. 3 is also the *fourth* or *biquadrate* root of 81 because it must be used four times as a factor to produce 81. Thus, 3×3×3×3=81. Hence, A ROOT TAKES ITS NAME OR NUMBER, FROM THE NUMBER OF TIMES IT IS CONTAINED AS A FACTOR, IN THE CORRESPONDING POWER.

Hence also,

The words POWER and ROOT are *correlative terms*. That is, if one number be a certain *power* of another; the latter is the same *root* of the former. Thus, 16 is the fourth power of 2, and 2 is the fourth root of 16.

We have seen that when a root is given, there is no difficulty in involving it to any power. For Involution is performed by multiplication: and any two or more numbers, whatever, may be multiplied together, and the exact product obtained. But there are many numbers, whose *roots* cannot be exactly ascertained. Thus, the square root of 2 cannot be exactly expressed. It is nearly 1.41421356237+

A Root, which cannot be exactly expressed, is called a SURD or IRRATIONAL number. On the other hand,

A Root which can be exactly ascertained, is called a RATIONAL number. A power of a rational number, is called a PERFECT power. Thus 9 is a perfect square, its root being 3; but it is an imperfect cube, its root being 2.0800865+ Also, the square root of 9 is a rational number, and its cube root a surd. Surds may be expressed in decimals, with a degree of accuracy, sufficient for all practical purposes.

As there is a mode of *indicating* powers; so likewise there is an appropriate corresponding method of indicating roots. As in powers, *a whole number*, equal to the number of the power, is placed at the right; so in roots, a *Fraction* whose denominator is the number of the root, is employed in a similar manner. Thus $\frac{1}{2}$ is the index of the 2d. root, $\frac{1}{3}$, of the 3d., and so on. The square root of 4 is then expressed $4^{\frac{1}{2}}$; the 8th. root of 7, $7^{\frac{1}{8}}$, &c. The numerator of the Fraction, in such cases, is always 1.

There is, likewise, another mode of indicating roots. The character, $\sqrt{}$, prefixed to a number denotes the square root, and may be employed, instead of the index $\frac{1}{2}$. The same character is used to denote other roots, the number of the root being written over it. Thus, $\sqrt[3]{}$, $\sqrt[4]{}$, $\sqrt[5]{}$, denote the third, fourth and fifth roots, respectively, and are used, instead of the indices $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.

The process of finding roots, is commonly called the EXTRACTION OF ROOTS; and, when a root is ascertained by an arithmetical operation, it is said to be EXTRACTED.

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EXTRACTION OF THE SQUARE ROOT.

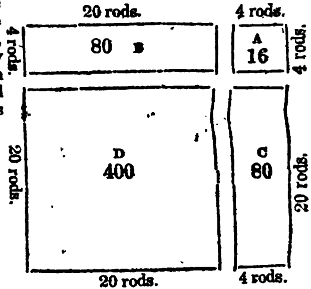
§ C. When two numbers are multiplied together, the product never contains more figures than both factors together, nor fewer than the same number, less one. For if 99, for example, (the greatest number, consisting of two figures,) be multiplied by 100, the product is 9,900. If, instead of 100, the multiplier had been 99 also, or 98, or any less number, the product would of course been less than 9,900. But 9,900 contains only four places. Hence, it is evident, that no two factors, consisting of two figures each, can produce a product, containing more than four figures; that is, more than there are in both factors. The same mode of proof will apply to any other case.

On the other hand, 10 and 10 are the smallest numbers, consisting each of two figures. But 10×10=100, which contains three figures, or one less than both factors. The same mode of proof will apply to other cases. Hence, no square can contain more figures than twice as many as its root contains, nor fewer than the same number, less one. Hence, when a square root is required, we can always determine how many figures it will contain. The most convenient mode of doing this, is to begin at the right of the given square, and point off the number into periods of two figures each. Thus, how many figures in the square root of 36372961? Put a point over units, and afterwards over each second figure, thus, 36372961. There are four points: therefore the root contains four figures. In extracting the square root, the first thing to be done is to point off the given number in this manner.

1. In a square court, are contained 576 square rods. How many rods in length is one side of the court?

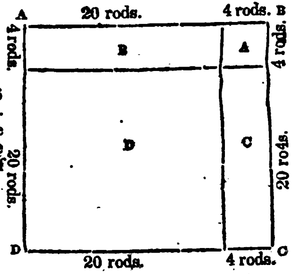
We will tell you in this instance, at first, that the side of the square is 24. Now involve 24, and set down the product of every two figures separately, thus. 4 times 4 units are 16 units; 4 times 2 tens are 8 tens, or 80; 2 tens into 4 units are 8 tens, or 80, and 2 tens into 2 tens are 4 hundreds=400. The little square A represents the first product, the long figure B, the second, the long figure C, the third, and the large square D, the fourth. Let them all be added, and the square A B C D below, will represent the total product, 576. Now suppose we did not know the root. On pointing off as above directed, we should know that it consisted of two figures, tens and units, because there are two points.

24
24
—
16
80
80
400
—
576



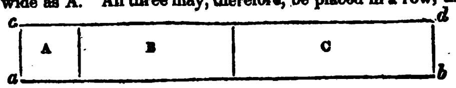
Process.

576(20   576(24
400 4   4
—   —
40+4=44)176 24   44)176
176   176
—   —
0   0



The involution above shows the 576 to consist of the square of the units' figure (A); the square of the tens' figure (D); the product of the units' into the the tens' figure (B); and the product of the tens' into the units' figure (C).

Now the square of the tens' figure is hundreds, and will of course be the greatest square to be found in the left hand period. This is 400, and its root 20. This square (=the figure D.) being taken away, 176 remain (=the figures A, B, and C.) B and C are each 20 rods long, (being equal in length to D,) and are just as wide as A. All three may, therefore, be placed in a row, thus,



If this figure, *a b c d*, were divided by the length of the side *a b*, the quotient would be *a c* (the side of A,) that is, the units' figure of the root. Now we do not know the length of A, but we know that B and C are each 20 rods long, and, of course, both together are 2×20 =40 rods long. 40, then, will be sufficiently near to divide by. 176÷40 =4 (neglecting remainder.) We *suppose* 4, then, to be the units' figure, or the side of A. This 4 we add to 40, to get the length of *a b c d*, which is 44. Then, the length, 44×4 the width, =176, the size of the figure *a b c d*. This agrees with its actual size, and hence we *know* 4 to be the units' figure of the root. Therefore 24 rods is the length of one side of a court containing 576 square rods.

The above operations show the process here illustrated. The large square, 400, is first subtracted; then the remainder is divided by 40, which is *twice* 20, the root already found. The quotient found is 4. This is added to 40 making 44, and the sum multiplied by 4 produces 176, which, being subtracted, leaves no remainder.

In one of the operations the cyphers are retained at the right of the square number 400, and of its root 20. In the other, these cyphers are dropped, as in common division. When they are dropped, however, a cypher must evidently be *understood* at the right of the *divisor*.

The intelligent pupil will easily see that when the root has more figures, the process is similar. For the first two periods, it is exactly the same. The third period must then be brought down, *the whole root already found* doubled for a divisor, (understanding a cypher at the right, as above,) and so on as before. All that is to be observed in this case is, that each divisor must be obtained *by doubling the whole root already found.* This may be proved by a diagram, as above. Another set of figures like A, B and C must be constructed outside of those. If the pupil does not succeed in the demonstration, it is recommended to the teacher to exhibit it.

It is almost needless to remark that we may sometimes have a remainder after the last period is brought down. Thus, in the example above, if the number were 580, instead of 576, 24 would still be the nearest whole number root, and a remainder of 4 would be left. To such a remainder, we may annex periods of cyphers, and continue the root to decimals. Each period, so annexed, must of course contain two cyphers. Likewise, if any dividend is too small to contain the divisor, we must put a cypher in the root, and bring down another period.

2. Extract the square root of 5,499,025.

```
  5499025(2345 ROOT.
  4
43|149
 3|129
464|2090
  4|1856
4685|23425
    |23425
        00
```

3. Find the square root of 2.

```
2(1.4142+ ROOT.
1
24|100
 4| 96
281| 400
  1| 281
2824|11900
   4|11296
28282|60400
     |56564
      3836
```

In extracting the root of 2, we are obliged to annex periods of cyphers, after obtaining one figure in the root. Such a root as this, must, of course, be surd; for every dividend ends with a cypher, and the first figure of each subtrahend is the product of some figure by itself, since the last figure of every divisor is the same as the quotient figure by which it is multiplied. But no significant figure, multiplied by itself, produces a product ending with a cypher. Hence, there will always be a remainder, and the root will be, of course, infinite. If, then, there is a remainder, when all the significant figures of any number have been employed in Evolution, the root of that number is a surd. The same is true in case of other roots, as well as of the square.

It will be seen, that, as we carry a root to decimals, by annexing periods of cyphers below the units' place, so we should annex periods of significant decimals, if they were in the given number. Hence, decimals are to be pointed off by *twos*, from the units' place.

4. A farmer wished to lay out a field, in the form of a square, to contain 529 square rods; how long must he have made one side? A. 23 rods.

5. A square floor contains 961 sq. ft. What is the length of one side? A. 31 ft.

6. Find the sq. root of 784. A. 28. Of 676. A. 26. Of 625. A. 25. Of 487,204. A. 698. Of 638,401. A. 779. Of 556,516. A. 746. Of 441. A. 21. Of 1,024. A. 32. Of 1,444. A. 38. Of 2,916. A. 54. Of 6,241. A. 79. Of 9,801. A. 99. Of 17,956. A. 134. Of 32,761. A. 181. Of 39,601. A. 199. Of 488,601. A. 699.

7. Find the sq. root of 69. A. 8.3066239. Of 83. A. 9.1104336. Of 97. A. 9.8488578. Of 299. A. 17.2916165. Of 222. A. 14.8996644. Of 282. A. 16.7928556. Of 394. A. 19.8494332. Of 351. A. 18.7349940. Of 699. A. 26.4386081. Of 979. A. 31.2889757. Of 989. A. 31.4483704. *Of 999. A. 31.6069613.* Of 397. A. 19.9248588. Of 687. A. 26.2106848. Of 892. A. 29.8663690.

To find the root of a fraction, take the root both of numerator and denominator, or, if this cannot be done, reduce the fraction to a decimal, and extract its root. The same may be done with mixed numbers.

8. What is the sq. root of $\frac{25}{36}$? A. $\frac{5}{6}$. Of $\frac{160801}{249001}$? A. $\frac{401}{499}$. Of $\frac{237169}{480249}$? A. $\frac{487}{693}$. Of $\frac{430336}{483025}$? A. $\frac{656}{695}$. Of $\frac{616225}{617796}$? A. $\frac{785}{786}$. Of $\frac{606841}{942841}$? A. $\frac{779}{971}$.

9. Find the sq. root of $\frac{3}{4}$. A. .8660254. Of $\frac{5}{12}$. A. .645497. Of $17\frac{3}{8}$. A. 4.168333. Of $\frac{3}{80}$ A. .193649167. Of $\frac{9}{13}$. A. .83205. Of $\frac{5}{60}$. A. .288617394+

From the above illustrations and examples, we have the rule,

I. HAVING POINTED OFF, SUBTRACT FROM THE HIGHEST PERIOD THE GREATEST SQUARE CONTAINED IN IT, PLACE THE ROOT IN THE QUOTIENT, AND TO THE REMAINDER BRING DOWN THE NEXT PERIOD FOR A DIVIDEND.

II. DOUBLE THE ROOT ALREADY FOUND, (UNDERSTANDING A CYPHER AT THE RIGHT,) FOR A DIVISOR, AND DIVIDE THE DIVIDEND BY IT, FOR THE NEXT FIGURE OF THE ROOT.

III. ANNEX THIS FIGURE TO THE DIVISOR, WHICH, SO INCREASED, MULTIPLY BY THE SAME FIGURE FOR A SUBTRAHEND.

IV. SUBTRACT THE SUBTRAHEND FROM THE DIVIDEND, TO THE REMAINDER BRING DOWN THE NEXT PERIOD FOR A NEW DIVIDEND, AND SO PROCEED.

The proof is by Involution.

NOTE. The roots of many powers may be found by repeated extractions of the square root. Thus, the square root of the square root is the 4th root; the square root of the 4th root the 8th root, and so on. The same may be done by means of the cube root, and by the square and cube roots combined.

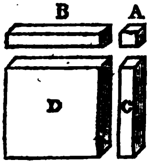
EXTRACTION OF THE CUBE ROOT.

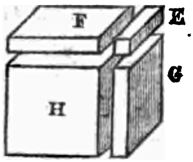
§. CI. By similar reasoning to that in § C. it may be shown, that, when three numbers are multiplied together, the product can never consist of more figures than all the factors together, nor of fewer than the same number less two. Hence, we infer, in like manner, that if any number is pointed off into periods of three figures each, from the units' place, the number of *periods* will be equal to the number of figures in its cube root. Thus, how many figures in the cube root of 27054036008? Point, thus, 27054036008 There are four periods, and, of course, the root consists of four figures.

1. A cubic solid contains 13,824 cubic feet. How many feet in length is one edge of the solid?

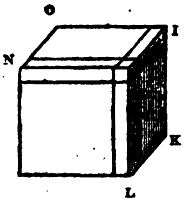
13,824 is the cube of 24. One side of the solid, then, is 24 feet long. This number, 24, is the same as that used in ex. 1, of the last §. Let it be involved to the cube, the products of the digits composing it, being preserved distinct. In § C. we have already performed the first multiplication, and the several products were 400+80+80+16, represented by distinct diagrams. Multiply by 24 again, and each product may be represented by a solid.

$$\begin{array}{rr} & 400+80+80+16=576 \\ & 20+4=\ \ 24 \\ \hline & 1600+320+320+64=2304 \\ 8000+1600+1600+320 & =1152 \\ \hline & 13824 \end{array}$$





Thus, (taking 4, the units' figure,) 16, (=the square, A, § C.)×4=64=the cube A. 80 (=long figure B, § C,) ×4=320=the solid B. 80 (=figure C, § C,) ×4=320=the solid C. 400 (=square D.) ×4 =1,600=the solid D. Then, (taking the 2 tens,=20) 16×20=320=the solid E. 80×20=1,600=the solid F. 80×20=1,600=the solid G. 80×400=8,000=the cube H.



Now, if the solids A, B, C, and D, should be placed immediately in front of E, F, G, and H, respectively, and the whole brought close together, a cube, I, K, L, M, N, O, would be formed=24^3=13,824. In this cube, the solid H,=8,000, is the cube of the tens in the root; the solids, B, C, E,=320 each, are products of the tens into the square of the units; the solids, D, F, G,=1,600 each, are products of the units into the square of the tens; and the solid A,=64, is the cube of the units. Let the given number now be pointed off, thus,

Operation I. $1\dot{3}82\dot{4}(20$

$8000 \quad 4$

$20^2\times3=1200)5824 \quad 24$

$20^2\times4\times3=4800 \quad 5824$

$4^2\times20\times3=\ 960$

$4^3=\ 64 \quad 0000$

Subtrahend, 5824

Operation II. $1\dot{3}82\dot{4}(24$

8

$2^2\times3=\quad 12)5824$

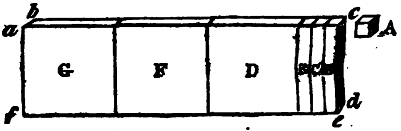
$12\times4=48 \quad 5824$

$4^2\times2\times3=\ 96$

$4^3=\ 64 \quad 0000$

Subtrahend 5824

The cube of the tens' figure is thousands, and will, of course, be the greatest cube to be found in the left hand period. This is 8,000, and its root, 20. This cube (the figure H) being taken away, 5,824 remains, (=the figures, A, B, C, D, E, F and G.) As B, C, D, E, F and G, are each, on one side, as long as H, (that is, 20 feet,) they may all be placed side by side, in one solid, thus,



The little solid, A, must, for the present, stand by itself. If this solid, *a b c d e f*, were divided by its whole upper surface, we should obtain its thickness, *a b*, which is the units' figure of the root. The upper surfaces of G, F and D, we know, because each surface is equal to 20×20, or the square of the root already found. Neglecting E, C, B, and also the little cube A, then, $20\times20\times3=1,200$, will be sufficiently near for a divisor. $5,824\div1,200=4$, (neglecting remainder.) We *suppose*, then, that 4 is the units' figure, or the thickness, *a b*, which is equal to the side of A. This 4, we multiply by 20^2, for each of the solids G, F, D; we then square 4, and multiply it by 20, for each of the solids, E, C, B; and finally cube it, for the solid, A. Then $20^2\times4\times3+4^2\times20\times3+4^3=5,824$. This agrees with what we know to be the solidity of A, B, C, D, E, F and G; and hence, we *know* 4 to be the units' figure of the root. Therefore, 24 is the side of a cube, containing 13,824 solid feet.

The above operations show the process here illustrated. The large cube, 8,000, is first subtracted, and the remainder, 5,824, is then divided by 1,200, which is 3 times the square of 20, (the root already found.) We then multiply 3 times 20^2, or 1,200, by the quotient 4, 3 times 4^2 by 20, and, finally, cube 4. These three results, added, form the subtrahend.

In the second operation above, the cyphers are omitted. It is evident, however, that when a divisor is obtained from the root already found, a cypher must be understood after that root. This, by squaring, will bring *two* cyphers at the right of the divisor. For the same reason, when a *subtrahend* is obtained as above directed, the first of the products which compose it, will have *two* cyphers, the second, one, and the third, none at the right. All these cyphers may be omitted, if they are *understood*, as in operation 2d, and the numbers arranged for addition, exactly as though the cyphers were *expressed*. Perhaps it will be better for pupils, at first, to write them out in full, as in operation 1st. When there are *more* than two periods, the operation is similar. A single period is all that need be brought down at once. Periods of cyphers, (of three places each,) may be annexed at the right, if necessary, and the root carried to decimals. In like manner, significant decimals may be pointed off towards the right, from the separatrix.

It may sometimes happen that the subtrahend found as above, will be larger than the dividend. This occurs, because the divisor is smaller than the whole surface of the solid, *abcdef*. When this is the case, the quotient, or figure of the root last found, must be diminished, and a new subtrahend found. When no subtrahend can be obtained, smaller than the dividend, a cypher must be placed in the root, and another period brought down.

2. Extract the cube root of 48,228,544.

$$
\begin{array}{lrl}
 & & 4\dot{8}22\dot{8}54\dot{4}(364\ \text{ROOT.}\\
 & & \underline{27}\\
\text{Cyphers} & 30^2\times3=2700) & 21228\ \text{Dividend.}\\
\text{retained.} & \left.\begin{array}{r}30^2\times6\times3=16200\\+30\times6^2\times3=3240\\+6^3=216\end{array}\right\}= & \underline{19656}\ \text{Subtrahend.}\\
 & 36^2\times3=3888) & 1572544\ \text{Dividend.}\\
\text{Cyphers omitted.} & \left.\begin{array}{r}36^2\times4\times3=15552\\+36\times4^2\times3=1728\\+4^3=64\end{array}\right\}= & \underline{1572544}\ \text{Subtrahend.}\\
 & & 00
\end{array}
$$

3. In making an excavation, there were thrown out 616,295,051 solid feet of earth. If it were all formed into a cubic mass, what would be the length of one side? A. 851 ft.

4. A box in the form of a cube, contains 9,261 cubic inches. What is the length of one side? A. 1 ft. 9 in.

5. From a cubical cellar were thrown out 510,082,399 ft. of earth. What was one side of the cellar? A. 799.

6. What is the side of a cubical solid, containing 988,047,936 cubic feet? A. 996 ft.

7. Find the cube root of 941,192,000. A. 980. Of 958,585,256. A. 986. Of 478,211,768. A. 782. Of 494,913,671. A. 791. Of 445,943,744. A. 764. Of 196,122,941. A. 581. Of 204,336,469. A. 589. Of 57,512,456. A. 386. Of 6,751,269. A. 189. Of 39,651,821. A. 341. Of 42,508,549. A. 349. Of 510,082,399. A. 799. Of 469,097,433. A. 777.

8. Find the cube root of 7. A. 1.912933. Of 41. A. 3.448217. Of 49. A. 3.659306. Of 94. A. 4.546836.

Of 97. A. 4.610436. Of 199. A. 5.838272. Of 179. A. 5.635741. Of 389. A. 7.299893. Of 364. A. 7.140037. Of 499. A. 7.931710. Of 699. A. 8.874809. Of 686. A. 8.819447. Of 886. A. 9.604569. Of 981. A. 9.936261.

Proceed with fractions as in extracting the square root.

9. Find the cube root of $\frac{27}{1331}$. A. $\frac{3}{11}$. Of $\frac{13824}{29791}$. A. $\frac{24}{31}$. Of $\frac{456533}{970299}$. A. $\frac{77}{99}$. Of $\frac{7301384}{26730899}$. A. $\frac{194}{299}$. Of $\frac{20346417}{62570773}$. A. $\frac{273}{397}$.

10. Find the cube root of $\frac{5}{8}$. A. .8549879. Of $\frac{7}{40}$. A. .5593445. Of $\frac{24}{250}$. A. .4578857. Of $\frac{19}{200}$. A. .4562903. Of $\frac{124}{125}$. A. .9973262.

From these illustrations and examples, we derive the rule,

I. HAVING POINTED OFF, SUBTRACT FROM THE HIGHEST PERIOD, THE GREATEST CUBE CONTAINED IN IT, PLACE THE ROOT IN THE QUOTIENT, AND, TO THE REMAINDER, BRING DOWN THE NEXT PERIOD FOR A DIVIDEND.

II. SQUARE THE ROOT ALREADY FOUND, (UNDERSTANDING A CYPHER AT THE RIGHT,) AND MULTIPLY IT BY 3 FOR A DIVISOR. DIVIDE THE DIVIDEND BY THE DIVISOR FOR THE NEXT FIGURE OF THE ROOT.

III. MULTIPLY THE DIVISOR BY THE QUOTIENT, MULTIPLY 3 TIMES THE SQUARE OF THE QUOTIENT BY THE PART OF THE ROOT PREVIOUSLY FOUND; FINALLY, CUBE THE QUOTIENT, AND ADD THESE THREE RESULTS TOGETHER FOR A SUBTRAHEND.

IV. SUBTRACT THE SUBTRAHEND FROM THE DIVIDEND, TO THE REMAINDER, BRING DOWN THE NEXT PERIOD FOR A NEW DIVIDEND, AND SO PROCEED.

NOTE. The pupil's attention should be particularly called to the effect of *understanding a cypher* in II, and III. If he does not fully understand this, he will be liable to fall into error. We have preferred the above rule, to the one usually given, because it keeps the *principles* of the operation before the mind, and, indeed, obliges the pupil to act upon them at almost every step. Perhaps it is not as easy as the other, *in the beginning*, but it will be found far more useful, and in fact, *easier in the end.*

The proof is by Involution.

◆

ARITHMETICAL PROGRESSION.

§ CII. The natural series of numbers, 1, 2, 3, 4, 5, &c., consists of numbers increasing by a continual addition of 1 to the number preceding. The series, 1, 3, 5, 7, 9, &c., increases by a continual addition of 2. The series, 15, 12, 9, 6, &c., decreases by a continual subtraction of 3. Any rank, or series of numbers, consisting of more than two terms, increasing or decreasing, like the above, by a common difference, is called an ARITHMETICAL SERIES OR PROGRESSION. The numbers forming the series, are called TERMS. The *first* and *last* terms, are called the EXTREMES, the others, the MEANS.

When the series *increases*, or is formed by a continual *addition* of the common difference, it is called an ASCENDING SERIES. When it *decreases*, or is formed by a continual *subtraction*, of the common difference, it is called a DESCENDING SERIES.

1. How many strokes do the clocks in Venice, which go from 1 to 24 o'clock, strike in the course of a day?

The answer might be found by addition. But by attending to the following, an easier mode may be discovered. Suppose another clock made to strike downward from 24 to 1. The two clocks would strike equal numbers of blows in a day. The order of their striking would be as follows:

1, 2, 3, 4, 5, 6, 7, 8, 9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24
24,23,22,21,20,19,18,17,16,15,14,13,12,11,10, 9, 8, 7, 6, 5, 4, 3, 2, 1
25,25,25,25,25,25,25,25,25,25,25,25,25,25,25,25,25,25,25,25,25,25,25,25

Thus, at every hour, the two clocks together would strike the same number of blows, viz. 25, and, in the course of the day, both would strike 24×25=600 blows. One would strike half this number= 300 *Ans.* It will be seen that 25=24+1=the sum of the extremes. Hence, the extremes and number of terms being given, to find the sum of all the terms,

MULTIPLY THE SUM OF THE EXTREMES BY THE NUMBER OF TERMS, AND HALF THE PRODUCT WILL BE THE ANSWER.

2. The first term of a series is 1, the last term 29, and the number of terms 14. What is the sum of the series? A. 210.

3. 1st. term, 2, last term, 51, number of terms, 18. Required the sum of the series, A. 477.

4. Find the sum of the natural terms 1, 2, 3, &c. to 10,000.
Ans. 50,005,000.

5. A man travelled from Hartford, going 3 miles the first day, and increasing each day by an equal excess. His 12th day's journey was 58 miles. What was the daily increase, and the distance he travelled from Hartford?

As he travelled 12 days, he increased his journey 11 times, by an equal addition. 58, then, is 11 times the daily increase, more than 3. Therefore 58—3=55÷11=5 *daily increase.* By last rule, 366 mls. distance from H.

Hence, the extremes and number of terms being given, to find the common difference,

DIVIDE THE DIFFERENCE OF THE EXTREMES BY THE NUMBER OF TERMS LESS 1.

6. Extremes 3 and 19; number of terms 9. Required the com. dif. A, 2.

7. Extremes 4 and 56; number of terms 14. Required the com. dif. A. 4.

8. A man had 15 houses, increasing equally in value, from the first, worth $700, to the 15th worth $3,500. What was the difference in value between the first and second? *Ans.* 200.

9. The ages of 5 persons were in arithmetical progression, the youngest being 15 yrs. old, and the com. dif. 2. What was the eldest

one's age? It was evidently 4 times the com. dif. more than that of the youngest. Hence, $15+4\times2=23$ *Ans.*

Hence, the least term, the number of terms, and the common difference being given, to find the greatest term,

TO THE LEAST TERM, ADD THE COMMON DIFFERENCE, MULTIPLIED BY THE NUMBER OF TERMS LESS 1.

4. 100 stones are a yard apart in a straight line, and the first one a yard from a basket. How far must a man go, to gather them, one by one, and return with them singly to the basket?

Ans. 5 mls. 5 fur. 36 rds. 2 yds.

NOTE. The cases of Arithmetical Progression are numerous, and of too little practical utility to warrant us in devoting to them much space. Any three of the following terms being given, the other two may be found. 1. The first term. 2. The last term. 3. The number of terms. 4. The common difference. 5. The sum of all the terms.

◆

GEOMETRICAL PROGRESSION.

§ CIII. The series 1, 3, 9, 27, 81, &c., consists of numbers, each of which is 3 times the preceding. The series 64, 32, 16, 8, 4, &c., consists of numbers, each of which is half the preceding. Any rank or series of numbers, of more than two terms, increasing, like the above, by a common multiplier, or decreasing by a common divisor, is called a GEOMETRICAL SERIES, or PROGRESSION.

The common multiplier or divisor is called the RATIO. The numbers which form the series are called TERMS.

The distinction between increasing or ascending, and decreasing or descending series, is made as in Arithmetical Progression.

Any *three* of the *five* following terms being given, the *other two* may be found. 1. The first term. 2. The last term. 3. The number of terms. 4. The ratio. 5. The sum of all the terms.

1. A man bought 5 sheep, giving $1 for the first; $3 for the second; $9 for the third, and so on, in geometrical progression. What did he give for the whole?

Write the whole series, thus, 1, 3, 9, 27, 81
Multiply by the ratio, 3, thus, 3, 9, 27, 81, 243
—1 243

We here obtain a new series, having 4 terms like the last. Let the first series be subtracted from the second, and all the similar terms vanish, leaving 1, the 1st. term of the 1st. series, to be subtracted from 243, the last term of the last. $243-1=242$. Now as the last series was 3 times the first, and we have subtracted *once* the first, the remainder, 242, must be *twice the first series*. HENCE, $242\div2=121$=sum of 1st. series, *Ans.*

The same result would have been obtained, if we had multiplied only the last term, and taken the first from the product, since the intermediate terms vanish, in the subtraction. Hence, the rule, the extremes and ratio being given, to find the sum of all the terms.

MULTIPLY THE GREATEST TERM BY THE RATIO, FROM THE PRODUCT SUBTRACT THE LEAST TERM, AND DIVIDE THE REMAINDER BY THE RATIO, LESS 1.

2. Given the first term, 1; the last term, 2,187; and the ratio, 3: required the sum of the series. A. 3,280

3. Extremes, 1 and 65,536; ratio 4: required the sum of the series. A. 87,381

4. Extremes 1,024 and 59,049: required as above. A. 175,099.

5. What is the sum of the series 16, 4, 1, $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, and so on, to an infinite extent? A. 21$\frac{1}{3}$.

NOTE. The last term is evidently 0.

6. A man had a debt to pay as follows: the first month $3; the second, $6; the third, $12, and so on, in a twofold ratio. In 12 months the debt was paid. What was the last payment?

The second payment is found by multiplying the first by the ratio, 2, *once*; the third, by multiplying by 2, *twice*; and so on, where it will be seen that the ratio is always used as a multiplier, *a single time less than the number of terms.* Hence, the 12th. term=the first term×the ratio eleven times successively, or, in other words, the eleventh power of the ratio, thus,

$3\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2=3\times2^{11}=6{,}144$ *Ans.*

Hence, the first term, ratio, and number of terms being given, to find the last term,

MULTIPLY THE FIRST TERM BY THAT POWER OF THE RATIO, WHOSE INDEX IS 1 LESS THAN THE NUMBER OF TERMS.

NOTE. *In involving the ratio, it will be seen that the process may often be abridged by multiplying together two powers already obtained. Thus, the 3d power×the 2d power=the 5th power, &c.*

7. If the first term is 2, the ratio, 2, and the number of terms, 13, what is the last term? A. 8,192.

8, Find the 12th. term of a series, whose 1st. term is 3, and ratio, 3. A. 531,441.

9. First term, 1, ratio 2; required the 23d. term. A. 4,194,304.

10. A man bought a horse, giving 1 ct. for the first nail in his shoes, three for the second, and so on; there were 32 nails; what cost the horse? A. 9,265,100,944,259.20.

11. A man works for a farmer 40 years, receiving 1 kernel of corn for the first year, 10 for the second, and so on; what do his wages amount to, allowing 1,000 kernels to a pt., and supposing corn worth 50 cts. per bu.?

A. $8,680,555,555,555,555,555,555,555,555,555,555.555555

12. A young man agreed to work eleven years with a farmer, on condition of receiving the produce of one wheat corn, the first year; the produce of that quantity, sowed the second year; and so on, to the end of the time. How much wheat was there due for his service, and what would it come to at $1 per bu., allowing the yearly increase to have been tenfold, and 7,680 corns to make a [illegible]

A. 226,056$\frac{1}{8}$ bu. at $226,056.1[illegible]

NOTE. The cases of Geometrical Progression are numerous. *Ten* different ones may be stated, with *two* requisitions in each case, making, in fact, 20 different cases. The same is true in Arithmetical Progression. It would require much space to give a clear understanding of all these cases, and as they are of little practical importance, we pass them by. The terms *Arithmetical* and *Geometrical*, are used without regard to their proper signification, (viz. belonging to Arithmetic and to Geometry,) but only to distinguish these different kinds of Progression.

ANNUITIES.

§ CIV. 1. A pension is allowed a man and his heirs, forever, of $600. He is willing to dispose of this pension at a fair price. What ought he to receive for it, allowing money to draw 6 per ct. interest?

It is evident he ought to receive a sum which would produce an annual interest, equal to the pension. $600 then is the interest, and the principal is acquired at 6 pr. ct. Hence, (§ LXXXI.) $600÷.06=$10,000 *Ans.*

A sum of money, payable periodically, for a certain length of time, or forever, is called an ANNUITY.

An annuity, in the proper sense of the word is a sum, payable *annually*. Payments, made at greater or less periods, are, however called annuities. To annuities belong pensions, salaries, rents, &c.

When an annuity remains unpaid after it is due, it is said to be *in arrears*. The sum of the annuities in arrears, with the interest on each, is called the AMOUNT.

The sum, which ought to be paid for an annuity *yet to come*, is called the PRESENT WORTH.

From the above example, it is evident that, *to find the present worth of an annuity to continue forever*, we must

DIVIDE THE ANNUITY BY THE RATE PER CENT.

2. What should be paid for a perpetual annuity of $40, discounting at 5 pr. ct.? *Ans.* 800.

3. What is an estate worth, which brings in $7,500 a year allowing 6 pr. ct.? *Ans.* $125,000.

ANNUITIES AT SIMPLE INTEREST.

1. If a rent of $600 be in arrears 5 years, what will be due at 4 per cent.?

On the rent of every year but the 5th., interest is due; for the 4th. year, one year's interest, for the 3d, two years' interest, and so on, in Arithmetical Progression. The question falls, therefore, under § CII. $24=the com. dif. 600+24×4=696 largest term. 696+600×5÷2=$3,240 *Ans.*

Hence, *to find the amount of an annuity in arrears*,

FIND THE SUM OF AN ARITHMETICAL PROGRESSION, OF WHICH THE ANNUITY IS THE FIRST TERM, THE NUMBER OF YEARS, THE NUMBER OF TERMS, AND THE ANNUAL INTEREST, THE RATIO; AND IT WILL BE THE AMOUNT REQUIRED.

2. Find the amount of an annuity of $50, for 7 yrs. at 5 per ct. *Ans.* $402.50.

3. What is the amount of an annuity of $250 for 7 yrs. at 6 per ct. payable half yearly? *Ans.* $2,091.25.

NOTE. As the annuity is payable half yearly, it amounts to the same sum it would, if the years were twice as many, and the rate half as great.

4. If a rent of $200 remain unpaid 8 years, what will it amount to? *Ans.* $1,936.

5. A man sold a pension of $100 to continue 5 years, allowing the buyer 6 per ct. What did he receive?

We may consider the annuity as five separate debts, due at the end of 1, 2, or 3 years, &c. Discounting at 6 per ct. on each, (§ LXXXII) we have 1st. year, present worth, $94.3396; 2d. year, $89.2857; 3d. $84.7457; 4th. $80.6451; 5th. $76.9230. The amount of these, =425.9391, is the amount of the annuity, required.

Hence, to find the present worth of an annuity, for a given time,

FIND THE PRESENT WORTH FOR EACH YEAR SEPARATELY. THE SUM OF THE PRESENT WORTHS THUS FOUND WILL BE THE ANSWER.

6. Find the present worth of a pension of $500 for 4 years at 5 per cent. *Ans.* $1,782.185+

7. Find the present worth of a salary of $200, to continue 3 yrs. at 4 per ct. *Ans.* $556.063.

NOTE. The estimation of the present worths of Annuities at Simple Interest, may easily be shown to be unreasonable and unjust. For the price of an annuity of $100 found in this way, for 40 years, at 6 per ct., will amount to a sum, which, put at interest, *would draw a greater annuity than the given one*; and one which would last *forever*, instead of *forty years*. It is therefore most equitable to allow Compound Interest.

ANNUITIES AT COMPOUND INTEREST.

1. An annuity of $100 was left 4 years unpaid. What was then due on it at 6 per ct. compound interest?

In finding an *amount* at compound interest, we multiply continually by the rate per ct.+1, (§ LXXXVIII) till the number of multiplications is equal to the number of years. Now, in calculating an annuity, we have for the last year, the annuity without interest; for the next preceding, the amount of the annuity for 1 yr.; for the next preceding still, the amount for 2 yrs., and so on. Thus, at compound interest, an annuity in arrears forms a Geometrical Progression, whose ratio is the rate per ct.+1, and whose terms equal the years in number. In the above example then, $100\times\overline{1.06}^{3}=119.101$. $\overline{119.101}\times\overline{1.06}-100\div.06=\$437.45$ *Ans.* Hence, to find the amount of an annuity in arrears at compound interest,

FIND THE SUM OF A GEOMETRICAL SERIES, WHOSE FIRST TERM IS THE ANNUITY, AND WHOSE RATIO, THE RATE PER CENT.+1, FOR AS MANY TERMS AS THERE ARE YEARS. THIS WILL BE THE AMOUNT REQUIRED.

2. Find the amount of $150 annuity, for 4 years, at 10 per cent. A. $696.15

3. Find the amount of an annuity of $40, for 5 years, at 5 per cent. A. $221.02525.

4. Find the amount $50 annuity, for 7 years, at 4 per cent.
A. $394.915—

For convenience, has been calculated the following

TABLE OF MULTIPLIERS,

FOR FINDING THE AMOUNT OF AN ANNUITY, FOR ANY NUMBER OF YEARS, FROM 1 TO 40, AT 6 PER CENT.

Yrs.	6 per cent.	Yrs.	6 per cent.	Yrs.	6 per cent.	Yrs.	6 per cent.
1	1.0000	11	14.9716	21	39.9927	31	84.8016
2	2.0600	12	16.8699	22	43.3922	32	90.8897
3	3.1836	13	18.8821	23	46.9958	33	97.3431
4	4.3746	14	21.0150	24	50.8155	34	104.1837
5	5.6371	15	23.2759	25	54.8645	35	111.4347
6	6.9753	16	25.6725	26	59.1563	36	119.1200
7	8.3938	17	28.2123	27	63.7057	37	127.2681
8	9.8974	18	30.9056	28	68.5281	38	135.9042
9	11.4913	19	33.7599	29	73.6397	39	145.0584
10	13.1807	20	36.7855	30	79.0581	40	154.7619

5. Find the amount of an annuity of $150, for 3 years, at 6 per cent. A. $477.54.

6. Find the amount of $500 annuity, for 24 years, at 6 per cent.
A. $25,407.75.

7. Find the amount of $100 annuity, for 49 years, at 6 per cent.
A. $26,172.08.

NOTE. If the time given be greater than 40 yrs., calculate for 40, and consider this amount a debt at compound interest, for the remaining time, which calculate accordingly. Then calculate the amount for the remaining time, as though the annuity commenced again, and add this sum to the last.

8. A pension of $100 for 4 years, was sold, the buyer being allowed 6 per ct. compound interest, for his money. What did the seller receive?

It is evident that the present worth is a sum, which, at compound interest would, in 4 years, produce the amount of the given annuity, for the same time. This amount is $437.46. To find an amount at compound interest, we multiply a sum by the rate per cent. +1, as many times successively as there are years, or, in other words, *we multiply by that power of the rate per cent.+1, whose index is the number of years.*

To find a present worth, we must, manifestly, reverse this process, and *divide by the same power.* Then $\$437.46 \div 1.26247$ $(=1.06^4)$ $=\$346.511$ *Ans.*

Hence, *to find the present worth of an annuity,*

FIND THE AMOUNT IN ARREARS FOR THE GIVEN TIME, AND DIVIDE IT BY THAT POWER OF THE RATE PER CENT.+1, WHOSE INDEX IS THE NUMBER OF YEARS.

NOTE. This power may be found in the table of multipliers for compound interest. (§ LXXXVIII.)

9. Find the present worth of a $40 annuity, to continue 5 yrs. at 5 per cent. A. $173.173.

10. Find the present worth of $100 annuity, for 20 yrs. at 5 per cent. A. $1,246.22.

For convenience, has been calculated the following

TABLE OF MULTIPLIERS,

FOR FINDING THE PRESENT WORTH OF AN ANNUITY FOR ANY NUMBER OF YEARS, FROM 1 TO 20, AT 5 AND 6 PER CENT.

Years.	5 per cent.	6 per cent.	Years.	5 per cent.	6 per cent.
1	0.95238	0.94339	11	8.30641	7.88687
2	1.85941	1.83339	12	8.86325	8.38384
3	2.72325	2.67301	13	9.39357	8.85268
4	3.54595	3.46510	14	9.89864	9.29498
5	4.32948	4.21236	15	10.37966	9.71225
6	5.07569	4.91732	16	10.83777	10.10589
7	5.78637	5.58238	17	11.27407	10.47726
8	6.46321	6.20979	18	11.68958	10.82760
9	7.10782	6.80169	19	12.08532	11.15811
10	7.72173	7.36008	20	12.46221	11.46992

11. Find the present worth of an annuity of $21.54, for 7 yrs. at 6 per cent. A. $120.244+

12. Find the present worth of an annuity of $100, to continue 12 years, at 6 per cent. A. $838.384.

13. Find the present worth of an annuity of $936, for 20 yrs. at 5 per cent. A. $11,664.629—

As any annuity multiplied by one of the numbers in the last table will give the present worth of that annuity, so it is evident that, any *present worth*, *divided* by the same number, will give the *annuity itself*. Hence, if I wish to discover of what annuity any given sum is the present worth, that is, *what annuity any given sum will buy*, I have only to use the above table, as a table of *divisors*, instead of multipliers.

14. What annuity, to continue 5 years, will $432,948 purchase, when money is worth 5 per cent.? A. $100,000.

15. What annuity, to continue 19 years, will $6,694.866 purchase, when money will bring 6 per cent.? A. $600.

When an annuity is to commence at some future time, it is said to be *in reversion.* An annuity in reversion, is evidently not worth as much as one of the same amount, which commences immediately. For if the present worth of the annuity be calculated as usual, it will be what the annuity is worth at the time it commences; and as that time is still future, we must discount for the intervening space, in order to obtain the true present worth. Hence, *to find the present worth of an annuity in reversion,*

FIND THE PRESENT WORTH AS USUAL, AND DISCOUNT UPON IT FOR THE TIME OF REVERSION.

NOTE. Of course, the discount should be made as directed in § LXXXVIII, *page* 230, *article* COMPOUND INTEREST.

Or, the present worth may be calculated as though the annuity were to *commence immediately*, and to continue to the end of the time of the given annuity: if, from this sum, the present worth for the time of reversion be subtracted, the remainder will be the present worth required.

16. If an annuity of $100 be 14 years in reversion, to continue 20 years afterwards, what is its present worth, discounting at 5 per cent.? A. $629.426.

17. What is the present worth, at 6 per cent. of an annuity of $120, to continue forever? A. $2,000.

NOTE. The answer is evidently a sum whose annual interest is $120.

18. Which is preferable, an annuity of $100 for 15 years, to commence immediately, or the reversion of the same annuity, forever, after the 15 years have expired? also, what is the difference?

A. The term of 15 yrs. is better than the reversion forever after, by $75.928+

NOTE. If the time extend beyond the limits of the table, calculate as far as the table will allow, and consider the rest as an annuity in reversion.

19. Find the present worth of an annuity of $400, to continue 34 yrs. at 6 per cent. A. $6,477.16.

20. Find the present worth of a $70 annuity, to continue 59 yrs. at 5 per cent. A. $1,321.3021.

NOTE. To give a complete developement of the subject of annuities, is not the province of arithmetic. Contingent annuities, or those whose continuance depends on uncertainties, as the duration of the life, or lives, of one, or of several persons, involve the doctrines of CHANCES, and are, in many cases, complex and tedious in calculation.

PERMUTATION AND COMBINATION.

§ CV. The two letters, A B, may be written A B, or B A. Any two things, therefore, have two *orders of succession*, or relative positions, in which they may be placed, in a single line. The word *permutation*, means change, and in mathematics,

CHANGES IN THE ORDER IN WHICH THINGS SUCCEED EACH OTHER, ARE CALLED PERMUTATIONS.

1. What number of permutations can be made on the letters A B C?

If C be left out, A and B, as seen above, admit of 2 permutations. So, if B be left out, A and C admit of 2 permutations. And if A be left out, B and C admit of 2 permutations. But before each of these permutations, the letter left out may be placed; and as there were 2 permutations, 3 times, there are 6 in the whole. The pupil may make them for himself.

From the above it will be seen that, of 2 things, there may be $1\times2=2$ permutations; of 3, $1\times2\times3=6$ permutations, and by the same mode of reasoning, it may be shown that, THE PERMUTATIONS, WHICH CAN BE MADE OF ANY NUMBER OF THINGS, ARE EQUAL TO THE CONTINUED PRODUCT OF THE NATURAL SERIES OF NUMBERS, FROM 1, UP TO THE NUMBER OF THINGS GIVEN.

2. Four gentlemen agreed to remain together, as long as they could arrange themselves differently at dinner. How many days did they remain? A. 24 days.

3. 10 gentlemen made the same agreement, but they all died before it could be fulfilled. The last survivor lived 53 yrs. 98 days, after the agreement. How much did the bargain then want of being fulfilled, allowing 365 days to the year? A. 9,888 yrs. 237 d.

4. How many years will it take to ring all the possible changes on 12 bells, supposing that 10 can be rung in a minute, and that the year contains 365 d. 5 h. 49 m.? A. 91 yrs. 26 d. 22 h. 41 m.

5. How many permutations may be made of the figures, **1, 2, 3, 4, 5,** taken two at a time?

Let **1** be placed by itself. To this, each other figure may be joined, making 4 permutations. Then **2** may be taken in the same way; and so with every other figure, there being 4 permutations each time. Then, as there are 5 figures, there will be $5\times4=20$ permutations of two figures.

6. How many permutations can be made on the figures above, taken three at a time?

Here, if we set apart each arrangement of 2 figures, found as above, we may join to every one, each of the 3 remaining figures, which will make 3 times as many permutations. Now the permutations by *twos*, we have seen, are 5×4, and 3 times this number $=5\times4\times3=60$ permutations of three figures.

By extending this mode of reasoning, we obtain the following.

THE PERMUTATIONS WHICH CAN BE MADE OF ANY NUMBER OF THINGS, TAKEN A GIVEN NUMBER AT A TIME, ARE EQUAL TO THE CONTINUED PRODUCT OF A DECREASING NATURAL SERIES, WHOSE GREATEST TERM IS THE WHOLE NUMBER OF THINGS, AND WHOSE NUMBER OF TERMS, THE NUMBER TO BE TAKEN AT A TIME.

7. How many numbers can be expressed by the nine digits, taken four at a time? A. 3,024.

8. How many words of five letters each, may be made from an alphabet of 26 letters, supposing that a number of consonants may make a word? A. 7,893,600.

From the letters, A, B and C, we can make three assemblages of two letters, of which no one shall contain exactly the same letters as another. These are A B, A C and B C.

A combination means a collection of things, and in mathematics,

COLLECTIONS OF WHICH NO TWO ARE EXACTLY ALIKE, CONSISTING EACH OF A GIVEN NUMBER OF THINGS, ARE CALLED COMBINATIONS.

9. How many combinations of two letters can be made from A B C D?

The *permutations* of two, we have seen to be $4\times3=12$. But on each *combination* of two, we have likewise seen, there can be made

$1 \times 2 = 2$ *permutations* of two. Hence, the permutations are *twice as many* as the combinations. $12 \div 2 = 6$ *Ans.*

10. How many combinations of three can be made of A B C D E F? The permutations of three are $6 \times 5 \times 4 = 120$, which, as before, are $1 \times 2 \times 3 = 6$ times the number of combinations. $120 \div 6 = 20$ *Ans.*

Hence, to find the number of combinations, which can be made of a given number of things taken from a given set,

FIND THE NUMBER OF PERMUTATIONS WHICH CAN BE MADE FROM THE PROPOSED SET, TAKING THE GIVEN NUMBER OF THINGS AT A TIME, AND DIVIDE IT BY THE NUMBER OF PERMUTATIONS WHICH CAN BE MADE ON ANOTHER SET, CONSISTING ONLY OF AS MANY THINGS AS ARE TO BE TAKEN AT A TIME.

11. How many combinations of two letters can be made from 24? A. 276.

12. A successful general was asked by his king, what reward he should give him for his services. The general's modesty only permitted him to ask a cent for every file of 12 men which he could make with 100 men. The king graciously granted the request, requiring only that the general should actually *parade his several files* in front of the palace, before payment was made. What was the amount of the reward, and how long was the general obliged to wait for it, allowing 2 minutes to draw up a file, 6 hours a day to the parade, and 365 days to the year, Sundays being excepted?

A. Reward $10,504,210,511,067.—Time 186,528,344 centuries, 94 yrs. 196 d. 2 h. 20 m.

On the principles above explained, are formed *combination lotteries*.

13. How many tickets in a lottery formed by ternary combination, from 60 numbers? A. 34,220.

14. In the same lottery, are 9 drawn numbers; how many prize tickets, having each 3 drawn numbers on them? A. 84.

◆

§ CVI. We have now illustrated all the fundamental principles of ARITHMETIC, and investigated those subjects which fall within the scope of a practical treatise.

It will be seen, that one part of the preceding exercises consists of problems to be performed by numerical operations, and of rules to serve as guides in performing them. These, taken by themselves, constitute what is called PRACTICAL ARITHMETIC.

Another part investigates the principles of numbers, or demonstrates their properties and relations; and this is denominated THEORETIC ARITHMETIC. Hence, the subject may properly be said to embrace both an art and a science, of which,

THEORETIC ARITHMETIC IS THE SCIENCE WHICH TREATS OF NUMBERS, and

PRACTICAL ARITHMETIC IS THE ART OF COMPUTING BY NUMBERS.

Theoretic arithmetic, then, calls into exercise the reasoning powers, and is well adapted to mental discipline; practical arithmetic exercises the judgment in an inferior degree, but is of great utility in the transactions of business.

OBSERVATIONS ON VARIOUS TOPICS.

The learner has, no doubt, perceived that many of our present advantages for calculation, result from our system of notation. The ancient Greeks and Romans made their calculations by means of small pebbles, as is probable from the frequent use of the word meaning pebble, by the classic writers, as connected with calculation. The Romans afterwards employed, in their ordinary calculations, a small board or table, on which beads were strung on wires. This was called the ABACUS. The Chinese use something very similar at the present day, called the SWAN-PAN.

The properties of numbers are of two kinds, the essential, and the accidental; the essential, existing in numbers from their very nature, and the accidental, depending on the mode of representing them. Thus, it is an essential property of 9, that it is a square number, and of 7, that it is prime. It is an accidental property of 9, that it will divide a number, when it will divide the sum of the figures composing it. It is from the accidental properties, that we derive the greatest advantage in calculation. Among the Greeks, the disciples of **PYTHAGORAS** turned their attention to the essential properties of numbers. They divided them into many different classes, as perfect or imperfect, redundant or deficient, &c. Perfect numbers, are those which are equal to the sum of their aliquot parts: 6, 28 and 496, are examples; only 10 are known. Besides this, they entertained the most absurd notions, with respect to the qualities of numbers. They considered *even* numbers, as feminine, and terrestrial in their nature; while *odd* numbers were esteemed masculine, and celestial. The sum of the first four even, and of the first four odd numbers, viz. 36, was supposed to combine all virtue, celestial and terrestrial; and being, at the same time, the square of 6, the first perfect number was thought to possess wonderful properties. To swear by the TETRACTYS OR QUATERNION, as this number was called, "was to contract the most solemn of all obligations."

Before the introduction of Arabic figures into England, arithmetical operations, (which, of course, were performed by means of Roman characters,) were difficult, particularly those of division. The science had, however, considerably advanced, and treatises were written upon it, of one of which, ALCUIN, a disciple of the "venerable BEDE," who, himself, wrote on the subject, was the author. In this work, were first proposed the well known puzzles, *of conveying three jealous husbands, with their wives, across a river, in a boat which can carry but two at a time, so that no woman shall be in company with any of the men, unless her own husband be present; of dividing equally, among three persons,* 21 *casks, of which,* 3 *are full,* 3 *half full, and* 3 *are empty;* and many others of the same nature.

About the middle of the 15th century, a man named **EMANUEL MOSCOPULIUS** invented MAGIC SQUARES, of which the following are examples.

31	22	28	13
40	1	43	10
19	34	16	25
4	37	7	46

22	47	16	41	10	35	4
5	23	48	17	42	11	29
30	6	24	49	18	36	12
13	31	7	25	43	19	37
38	14	32	1	26	44	20
21	39	8	33	2	27	45
46	15	40	9	34	3	28

22	48	14	40	6
8	24	50	16	32
34	10	26	42	18
20	36	2	28	44
46	12	38	4	30

If the columns of these squares be added, either perpendicularly, horizontally, or diagonally, the sum will be the same, throughout each square. The numbers arranged in them, form, in each square, a complete arithmetical progression. Any arithmetical progression, in which the *number of terms* is a *square number*, will admit of a similar arrangement.